About the space $\cap l_p$, $p > 0$.

by

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Abstract. We give a few properties of the space of sequences $(a_n)$ such that $\sum |a_n|^p = r_p(a)$ is finite for all positive $p$, with the Fréchet locally pseudo-convex topology determined by the pseudo norms $r_p$. The reader may find the following observations about this space amusing.

The elements of $l_{\infty}$ are the sequences whose decreasing rearrangements belong to the space $s$ of rapidly decreasing sequence. If we equip $s$ with the usual topology determined by the norms $\sup |a_n|$, the identity mapping $s \to l_{\infty}$ is continuous. Permutations of $N$ induce on $l_{\infty}$ an equicontinuous family of linear transformations. A translation invariant topology $T$ on $l_{\infty}$ is weaker than the given one if it induces on $s$ a weaker topology than its usual one, and if permutations of $N$ induce on $l_{\infty}$ a $T$-equicontinuous system of transformations at the origin.

These facts are either trivial, well known, or follow from the observation that $|a_n| < en^{-1/p}$ if $(a_n)$ is the decreasing rearrangement of $(a_n)$ and $\sum |a_n|^p < \infty$.

Let $T : l_{\infty} \to l_{\infty}$ be a continuous linear transformation. Let $B_0$ be the set of sequences $(a_n) \in l_{\infty}$ such that $r_p((a_n)) \leq 1$. Then $B_0$ is closed, absolutely $p$-convex, and a neighbourhood of the origin. $T(B_0)$ is then also a closed, absolutely $p$-convex neighbourhood of the origin in $l_{\infty}$. Being a neighbourhood of the origin, it contains $\epsilon B_p$ for some $\epsilon > 0$, $p' > 0$. Further, the closed, absolutely $p$-convex hull of $B_p$ is $B_p$ when $p' < p$, so that $T B_p = \epsilon B_p$.

In other words, $T$ extends to a continuous linear transformation of $l_p$, for all $p$, $0 < p \leq 1$. We can define $r_p(T)$ by $r_p(T) = \sup (r_p(Tw) : |w(x) \leq 1)$.
Decompositions of non-contrative operator-valued representations of Banach algebras

by

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Abstract. The present paper deals with some decompositions of non-contrative operator-valued representations of Banach algebras. These decompositions are closely related to the abstract F. and M. Riesz property. An examination of the Boolean character of this property is basic for our purposes. This, when combined with the Sz Nagy-Fitler theorem concerning similarity of certain Boolean algebras of projections shows that the representation in question is similar to a suitable orthogonally decomposed representation.

Let \( T \) be the Hilbert space representation of a function algebra \( A \). There are results of Sarason [14] and of Mlak [7], [8] that to every Gleason part of \( A \) or intersection of peak sets of \( A \) there corresponds a projection which commutes with \( T \). This projection is orthogonal for contracitive \( T \). In this case a full decomposition of \( T \) with respect to the totality of all Gleason parts or to the Bishop decomposition of \( A \) is available.

In both cases an essential role is played by the F. and M. Riesz property. The point is that this property in an abstract form [13] gives rise to a homomorphism of a certain Boolean algebra of projections in the dual space onto a Boolean algebra of projections commuting with \( T \). It seems that this is one of the real reasons why such decompositions as in [7], [8], [14] are available.

Although our theory concerns representations of general non-commutative algebras, the examples of applications we give in the present paper are commutative. Non-commutative cases will be treated elsewhere.

1. Let \( B \) be a (not necessarily commutative) Banach algebra with the unit \( 1 \). The norm of \( u \in B \) is denoted by \( \|u\| \). \( B^* \) is the dual of \( B \). For \( u \in B \) and \( \mu \in B^* \) we write \( \langle \mu, u \rangle \) for \( \mu(u) \). \( I \) stands for the identity element in \( B^* \).

Let \( A \) be a closed subalgebra of \( B \) and assume \( 1 \in A \). If \( \langle \mu, u \rangle = 0 \) (for all \( u \in A \)) then we write \( \mu \perp A \). For \( u \in B \) and \( \mu \in B^* \) we define \( \mu u \) and \( \mu e \) as the elements of \( B^* \) given by the formulæ: \( \langle \mu u, v \rangle = \langle \mu, uv \rangle \), \( \langle \mu e, v \rangle = \langle \mu, ev \rangle \), \( u \in B \).