APPLICATIONES MATHEMATICAE
25,1 (1998), pp. 19–28

J. DUDKIEWICZ (Kielce)

COMPOUND POISSON APPROXIMATION FOR EXTREMES OF MOVING MINIMA IN ARRAYS OF INDEPENDENT RANDOM VARIABLES

Abstract. We present conditions sufficient for the weak convergence to a compound Poisson distribution of the distributions of the kth order statistics for extremes of moving minima in arrays of independent random variables.

1. Introduction. Let $\{X_{n,i} : i = 1, \ldots, n, n = 1, 2, \ldots\}$ be an array of independent random variables with a common distribution function F_n for fixed n. We define

$$V_{n,j} = \min_{j \le i < j+m_n} X_{n,i}, \quad j = 1, \ldots, n - m_n + 1, \tag{1}$$

where m_n is a sequence of positive integers. The array $\{V_{n,j} : j = 1, \ldots, n - m_n + 1,\ n = 1, 2, \ldots\}$ is stationary and $(m_n - 1)$-dependent in each row. Denote by

$$\begin{aligned} \min(V_{n,j} : j = 1, \ldots, n - m_n + 1) &= M_{n,m_n}^{(n-m_n+1)} \le M_{n,m_n}^{(n-m_n)} \\ \le \ldots \le M_{n,m_n}^{(1)} &= \max(V_{n,j} : j = 1, \ldots, n - m_n + 1) \end{aligned} \tag{2}$$

the order statistics of the sequence $V_{n,1}, \ldots, V_{n,n-m_n+1}$. In [2] E. R. Canfield and W. P. McCormick have obtained a limit law for $M_{n,m_n}^{(1)}$. They showed that if

$$\frac{m_n}{\ln n} \to d \ge 0 \quad \text{as } n \to \infty, \tag{3}$$

then

$$P\{M_{n,m_n}^{(1)} \le u_n\} \to e^{-\theta\lambda} \quad \text{as } n \to \infty, \tag{4}$$

1991 *Mathematics Subject Classification*: 60F, 60E.
Key words and phrases: compound Poisson distribution, order statistics, moving minima, consecutive-m-out-of-n system.

where $\theta = 1 - \exp(-1/d)$, while $\lambda > 0$ and the sequence $\{u_n : n = 1, 2, \ldots\}$ of real numbers are related by

$$nP^{m_n}\{X_{n,1} > u_n\} = \lambda. \tag{5}$$

In this paper we extend (4) to the case of any kth order statistic. The limit law will be represented in terms of a compound Poisson distribution. Our result is also a generalization of [4] where Zubkov's method (see [7]) was used to obtain weak convergence of the distributions of the kth order statistics (2) to the Poisson law under the condition

$$m_n / \ln n \to 0 \quad \text{as } n \to \infty.$$

The proofs of the main result of this paper are based on Stein's method (see [1]).

The problems considered have a connection with reliability theory. The random variables $M^{(1)}_{n,m_n}$ can be interpreted as lifetimes of consecutive-m-out-of-n systems. Such a system fails if and only if at least m consecutive components out of n linearly ordered components fail. Some examples of applications to telecommunication and oil pipelines modelling may be found in [3] and [5].

2. Definitions and preliminary results. We say that a discrete random variable W has a *compound Poisson distribution* if

$$M(t) = E\exp(-tW) = \exp\Big(-\sum_{n=1}^{\infty} c_n(1 - e^{-tn})\Big) \tag{6}$$

for all $t > 0$, where $c_n \geq 0,\ n = 1, 2, \ldots$, are such that $0 < \sum_{n=1}^{\infty} c_n < \infty$. Note that the corresponding distribution function is

$$G(x, \{c_n\}) = \sum_{s \leq x} p_s(\{c_n\}), \qquad x \in \mathbb{R}, \tag{7}$$

where

$$p_s(\{c_n\}) = \begin{cases} \exp\Big(-\sum_{n=1}^{\infty} c_n\Big), & s = 0, \\ \exp\Big(-\sum_{n=1}^{\infty} c_n\Big) \displaystyle\sum_{\substack{k_1 + 2k_2 + \ldots + sk_s = s \\ k_j \geq 0,\, j = 1, \ldots, s}} \frac{c_1^{k_1} c_2^{k_2} \ldots c_s^{k_s}}{k_1! k_2! \ldots k_s!}, & s = 1, 2, \ldots \end{cases} \tag{8}$$

The total variation distance between two probability measures F and G is defined by

$$d(F, G) = \sup_E |F(E) - G(E)|,$$

where the supremum is taken over all measurable subsets E of the real line. Denote by $L(X)$ the law of a random variable X and recall (see [6]) that if $d(L(X_n), L(X)) \to 0$ as $n \to \infty$ then $X_n \xrightarrow{w} X$ (weak convergence; see [6]). The following lemma will be used in the next section.

LEMMA 1. *Let* $\{X_{n,i} : i = 1, \ldots, n,\ n = 1, 2, \ldots\}$ *be an array of independent random variables with a common distribution function* F_n *for fixed* n. *If the sequence* $\{m_n : n = 1, 2, \ldots\}$ *of positive integers is such that*

$$\lim_{n\to\infty} m_n/\ln n = d, \qquad d \ge 0, \tag{9}$$

and

$$\lim_{n\to\infty} n[1 - F_n(u_n)]^{m_n} = \lambda \tag{10}$$

where $\{u_n\}$ *is a sequence of real numbers, then*

$$\lim_{n\to\infty} F_n(u_n) = 1 - e^{-1/d}. \tag{11}$$

Proof. From (10) we obtain

$$\lim_{n\to\infty} \ln[n(1 - F_n(u_n))^{m_n}] = \ln\lambda$$

Since

$$\lim_{n\to\infty} \frac{\ln n + m_n \ln[1 - F_n(u_n)]}{m_n} = \lim_{n\to\infty} \frac{\ln\lambda}{m_n} = 0,$$

we conclude from (9) that

$$\lim_{n\to\infty} [1 - F_n(u_n)] = e^{-1/d}.$$

3. The main results. Let $\{X_{n,i} : i = 1, \ldots, n,\ n = 1, 2, \ldots\}$ be an array of independent random variables with a common distribution function F_n for fixed n, and let $\{V_{n,j} : j = 1, \ldots, n - m_n + 1,\ n = 1, 2, \ldots\}$ be defined by (1). Consider an array $\{I_{n,j} : j = 1, \ldots, n - m_n + 1,\ n = 1, 2, \ldots\}$ of zero-one random variables $I_{n,j} = I_{\{V_{n,j} > u_n\}}$, where u_n is a sequence of real numbers and I_A denotes the indicator function of the set A. This last array is stationary and $(m_n - 1)$-dependent in each row and

$$\begin{aligned} P\{I_{n,j} = 1\} &= P\{I_{n,1} = 1\} = P\{V_{n,1} > u_n\} \\ &= P\{X_{n,1} > u_n, X_{n,2} > u_n, \ldots, X_{n,m_n} > u_n\} \\ &= [1 - F_n(u_n)]^{m_n}. \end{aligned} \tag{12}$$

Let us observe that $(m-1)$-dependence is a special case of local dependence defined in [1] with

$$\begin{aligned} A_\alpha &= \{\beta \in I : |\alpha - \beta| < m\}, \\ B_\alpha &= \{\beta \in I : |\alpha - \beta| \le 2(m-1)\}, \qquad I = \{1, \ldots, n\}. \end{aligned}$$

Set

$$S_n = \sum_{i=1}^{n-m_n+1} I_{n,i}.$$

We define, as in [1],

$$Y_{n,\alpha} = \sum_{\substack{|\alpha-\beta|<m_n \\ \alpha\neq\beta}} I_{n,\beta}, \qquad \alpha = 1,\ldots,n-m_n+1,$$

and

$$\lambda_{n,i} = \frac{1}{i}\sum_{\alpha=1}^{n-m_n+1} P\{I_{n,\alpha}=1,\ Y_{n,\alpha}=i-1\}, \qquad i=1,\ldots,2m_n-1.$$

Let $M_{n,m_n}^{(n-m_n+1)} \le \ldots \le M_{n,m_n}^{(1)}$ be the order statistics of the sequence $V_{n,1},\ldots,V_{n,n-m_n+1}$ defined by (2).

Lemma 2. *For $k=1,2,\ldots,$*

$$(13)\qquad |P\{M_{n,m_n}^{(k)} \le u_n\} - G(k-1,\{\lambda_{n,i}\})| \le 2(1\wedge\lambda_{n,1}^{-1})\exp\Big(-\sum_{i=1}^{\infty}\lambda_{n,i}\Big)\sum_{\alpha=1}^{n-m_n+1}\sum_{\beta\in B_{n,\alpha}} P_{n,\alpha}P_{n,\beta},$$

where

$$a\wedge b = \min(a,b),$$
$$P_{n,\alpha} = P\{I_{n,\alpha}=1\} = [1-F_n(u_n)]^{m_n},$$
$$B_{n,\alpha} = \{\beta\in\{1,\ldots,n-m_n+1\} : |\alpha-\beta| \le 2(m_n-1)\}.$$

Proof. This follows from the equality $P\{M_{n,m_n}^{(k)} \le u_n\} = P\{S_n < k\} = P\{S_n \le k-1\}$ and Theorem 8 of [1].

Lemma 3. *If*

$$(14)\qquad \lim_{n\to\infty} n[1-F_n(u_n)]^{m_n} = \lambda, \qquad \lambda > 0,$$

and

$$(15)\qquad \lim_{n\to\infty} m_n/\ln n = d \ge 0$$

then

$$\lim_{n\to\infty}\lambda_{n,i} = \lambda_i, \qquad i=1,2,\ldots,$$

where

$$\lambda_1=\lambda, \qquad \lambda_i = 0, \qquad i=2,3,\ldots, \qquad \text{for } d=0,$$

and

$$\lambda_i = \lambda\theta^2 e^{-(i-1)/d}, \qquad i=1,2,\ldots, \qquad \text{for } d>0.$$

P r o o f. We fix $i,\ i=1,\ldots,2m_n-1$. For each n we divide the integers $1,\ldots,n-m_n+1$ in three parts:

$$\begin{aligned} J_{n,1} &= \{1,\ldots,m_n-1\},\\ J_{n,2} &= \{m_n,\ldots,n-2m_n+2\},\\ J_{n,3} &= \{n-2m_n+3,\ldots,n-m_n+1\}. \end{aligned}$$

Because the array $\{I_{n,j}\}$ is stationary, we have

$$\begin{aligned} \lambda_{n,i} = \frac{1}{i}\Big(&\sum_{\alpha=1}^{m_n-1} P\{I_{n,\alpha}=1,\ Y_{n,\alpha}=i-1\}\\ &+(n-3m_n+3)\sum_{j=0}^{i-1} P\Big\{\sum_{k=1}^{m_n-1} I_{n,k}=j,\ I_{n,m_n}=1,\\ &\qquad\qquad\qquad\qquad \sum_{k=m_n+1}^{2m_n-1} I_{n,k}=i-1-j\Big\}\\ &+\sum_{\alpha=n-2m_n+3}^{n-m_n+1} P\{I_{n,\alpha}=1,\ Y_{n,\alpha}=i-1\}\Big). \end{aligned}$$

Define

$$R_{n,j} = \Big\{\sum_{k=1}^{m_n-1} I_{n,k}=j,\ I_{n,m_n}=1, \sum_{k=m_n+1}^{2m_n-1} I_{n,k}=i-1-j\Big\},\qquad j=0,\ldots,i-1.$$

Observe that events of the form $\{\ldots,V_{n,i}>u_n,\ V_{n,i+1}\le u_n,\ V_{n,i+2}> u_n,\ldots\}$ are impossible because $\{X_{n,i+m_n}\le u_n\}$ and $\{X_{n,i+m_n}>u_n\}$ are mutually exclusive. Thus

$$\begin{aligned} R_{n,j} = \{&V_{n,1}\le u_n,\ldots,V_{n,m_n-j-1}\le u_n,\\ &V_{n,m_n-j}>u_n,\ldots,V_{n,m_n}>u_n,\ldots,V_{n,m_n+i-j-1}>u_n,\\ &V_{n,m_n+i-j}\le u_n,\ldots,V_{n,2m_n-1}\le u_n\}. \end{aligned}$$

We fix $j=0,\ldots,i-1$. By the definition of $\{I_{n,j}\}$ and $\{V_{n,j}\}$, and the assumptions on $\{X_{n,i}\}$, we have

$$\begin{aligned} P\{R_{n,j}\} = \sum_{l=0}^{m_n-j-2}\ \sum_{p=0}^{m_n-i+j-1} P\Big\{&\sum_{k=1}^{m_n-j-2} I_{\{X_{n,k}>u_n\}}=l,\\ &I_{\{X_{n,m_n-j-1}>u_n\}}=0, \end{aligned}$$

$$\sum_{k=m_n-j}^{2m_n+i-j-2} I_{\{X_{n,k}>u_n\}} = m_n + i - 1,\ I_{\{X_{n,2m_n+i-j-1}>u_n\}} = 0,$$
$$\left.\sum_{k=2m_n+i-j}^{3m_n-2} I_{\{X_{n,k}>u_n\}} = p\right\}$$
$$= [1 - F_n(u_n)]^{m_n+i-1} F_n^2(u_n).$$

Because $P\{R_{n,j}\}$ does not depend on j, for each $0 \le j,\, k \le i-1$ we have

$$P\{R_{n,j}\} = P\{R_{n,k}\}. \tag{16}$$

Next, set

$$K_n(i) = (n - 3m_n + 3)\sum_{j=0}^{i-1} P\{R_{n,j}\}.$$

From (16) we obtain

$$\begin{aligned} K_n(i) &= i(n - 3m_n + 3)P\{R_{n,0}\} \\ &= i(n - 3m_n + 3)[1 - F_n(u_n)]^{m_n+i-1} F_n^2(u_n). \end{aligned}$$

Hence

$$\begin{aligned} \frac{1}{i}K_n(i) = (n - 3m_n + 3)&[1 - F_n(u_n)]^{m_n} \\ &\times F_n^2(u_n)[1 - F_n(u_n)]^{i-1}. \end{aligned} \tag{17}$$

Using Lemma 1 and (14) we have the following result: for $d = 0$,

$$\lim_{n\to\infty} \frac{1}{i}K_n(i) = \begin{cases} 0, & i > 1, \\ \lambda, & i = 1, \end{cases}$$

and

$$\lim_{n\to\infty} \frac{1}{i}K_n(i) = \lambda\theta^2 e^{-(i-1)/d} \qquad \text{for } d > 0.$$

Now let

$$\begin{aligned} L_n^{(1)}(i) &= \sum_{\alpha=1}^{m_n-1} P\{I_{n,\alpha} = 1,\ Y_{n,\alpha} = i-1\}, \\ L_n^{(2)}(i) &= \sum_{\alpha=n-2m_n+3}^{n-m_n+1} P\{I_{n,\alpha} = 1,\ Y_{n,\alpha} = i-1\}. \end{aligned}$$

Our purpose is to show that

$$\lim_{n\to\infty} L_n^{(r)}(i) = 0, \qquad r = 1, 2.$$

Set

$$A_{n,\alpha}(j)=\Big\{\sum_{k=1}^{\alpha-1}I_{n,k}=j,\ I_{n,\alpha}=1,\ \sum_{k=\alpha+1}^{\alpha+m_n-1}I_{n,k}=i-j-1\Big\},$$
$$\alpha=1,\dots,m_n-1,\ 0\le j\le i-1.$$

Then

$$L_n^{(1)}(i)=\sum_{\alpha<i}\sum_{j=0}^{i-1}P\{A_{n,\alpha}(j)\}+\sum_{i\le\alpha\le m_n-1}\sum_{j=0}^{i-1}P\{A_{n,\alpha}(j)\}.$$

Note that

$$A_{n,1}(0)=\{V_{n,1}>u_n,\dots,V_{n,i}>u_n,V_{n,i+1}\le u_n,\dots,V_{n,m_n}\le u_n\},$$
$$\begin{aligned}A_{n,\alpha}(j)=\{&V_{n,1}\le u_n,\dots,V_{n,\alpha-j-1}\le u_n,\\ &V_{n,\alpha-j}>u_n,\dots,V_{n,\alpha}>u_n,\dots,V_{n,\alpha+i-1-j}>u_n,\\ &V_{n,\alpha+i-j}\le u_n,\dots,V_{n,\alpha+m_n-1}\le u_n\}.\end{aligned}$$

Now it is easy to see that $A_{n,\alpha}(j)\subset C_{n,\alpha}(j)$ where

$$\begin{aligned}C_{n,\alpha}(j)=\{&V_{n,\alpha-j}>u_n,\dots,V_{n,\alpha}>u_n,\dots,V_{n,\alpha+i-1-j}>u_n,\\ &V_{n,\alpha+i-j}\le u_n,\dots,V_{n,\alpha+m_n-1}\le u_n\}.\end{aligned}$$

From the stationarity of the array $\{V_{n,j}\}$,

$$P\{C_{n,\alpha}(j)\}=P\{D_{n,\alpha}(j)\},$$

where

$$\begin{aligned}D_{n,\alpha}(j)=\{&V_{n,1}>u_n,\dots,V_{n,j+1}>u_n,\dots,V_{n,i}>u_n,\\ &V_{n,i+1}\le u_n,\dots,V_{n,m_n+j}\le u_n\}.\end{aligned}$$

It is obvious that $D_{n,\alpha}(j)\subset A_{n,1}(0)$ so

$$P\{A_{n,\alpha}(j)\}\le P\{A_{n,1}(0)\}\tag{18}$$

for $j=0,\dots,\alpha-1$ if $\alpha<i$, and for $j=0,\dots,i-1$ otherwise. From (18) and the assumed properties of $\{X_{n,j}\}$ and $\{V_{n,j}\}$ we obtain

$$\begin{aligned}L_n^{(1)}(i)&\le(m_n-1)i[1-F_n(u_n)]^{m_n+i-1}F_n(u_n)\\ &\quad\times\sum_{s=0}^{m_n-i-1}\binom{m_n-i-1}{s}[1-F_n(u_n)]^sF_n^{m_n-i-1-s}(u_n)\\ &=\frac{m_n-1}{n}\cdot i\cdot n[1-F_n(u_n)]^{m_n}F_n(u_n)[1-F_n(u_n)]^{i-1}.\end{aligned}\tag{19}$$

Note that in view of (15), $m_n=o(n)$, which together with the assumption (14) and Lemma 1 implies that the right-hand side of (19) tends to zero as $n\to\infty$. The same is true for $L_n^{(2)}(i)$.

Finally, for $d = 0$,

$$\lim_{n\to\infty} \lambda_{n,i} = \begin{cases} \lambda, & i = 1, \\ 0, & i = 2, \ldots,\ 2m_n - 1, \end{cases}$$

and for $d > 0$,

$$\lim_{n\to\infty} \lambda_{n,i} = \lambda\theta^2 e^{-(i-1)/d}, \qquad i = 1, \ldots, 2m_n - 1.$$

LEMMA 4. *We have*

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \lambda_{n,i} = \sum_{i=1}^{\infty} \lambda_i = \lambda\theta. \tag{20}$$

Proof. The second equality of (20) follows simply, since

$$\sum_{i=1}^{\infty} \lambda_i = \lambda\theta^2 \frac{1}{1 - e^{-1/d}} = \lambda\theta.$$

Next, for fixed n, because of (17) and (19), we obtain

$$\lambda_{n,i} \le n[1 - F_n(u_n)]^{m_n} F_n(u_n)[1 - F_n(u_n)]^{i-1}.$$

Hence, from Lemma 1,

$$\lambda_{n,i} \le \lambda(1 - e^{-1/d})e^{-(i-1)/d}.$$

Set

$$a_i = \lambda\theta e^{-(i-1)/d}$$

and note that

$$\sum_{i=1}^{\infty} a_i = \lambda\theta \frac{1}{1 - e^{-1/d}} = \lambda < \infty.$$

Hence the series $\sum_{i=1}^{\infty} \lambda_{n,i}$ is uniformly convergent and thus in view of Lemma 3 we have (20).

LEMMA 5. *If* (14) *and* (15) *hold, then*

$$\lim_{n\to\infty} \sum_{\alpha=1}^{n-m_n+1} \sum_{\beta\in B_{n,\alpha}} P_{n,\alpha} P_{n,\beta} = 0, \tag{21}$$

where $P_{n,\alpha}$ and $B_{n,\alpha}$ are as in Lemma 2.

Proof. Since $P_{n,\alpha} = [1 - F_n(u_n)]^{m_n}$, we have

$$\begin{aligned} \sum_{\alpha=1}^{n-m_n+1} \sum_{\beta\in B_{n,\alpha}} P_{n,\alpha} P_{n,\beta} &\le 2(n - m_n + 1)(m_n - 1)[1 - F_n(u_n)]^{2m_n} \\ &= 2\frac{n - m_n + 1}{n} \cdot \frac{m_n - 1}{n} n^2 [1 - F_n(u_n)]^{2m_n}. \end{aligned}$$

The right side converges to zero as $n \to \infty$ by (14) and (15).

The main result of this paper may now be readily proved.

THEOREM 1. *Let $\{m_n\}$ be a sequence of positive integers satisfying* (15) *and $\{u_n\}$ a sequence of real numbers satisfying* (14). *Then for $k = 1, 2, \ldots$,*

$$\lim_{n\to\infty} P\{M^{(k)}_{n,m_n} \le u_n\} = G(k-1, \{\lambda_i\}) \tag{22}$$

where the distribution function G is given by (7)–(8).

Proof. We have

$$\begin{aligned} |P\{M^{(k)}_{n,m_n} \le u_n\} - G(k-1, \{\lambda_i\})| &\\ \le |P\{M^{(k)}_{n,m_n} \le u_n\} - G(k-1, \{\lambda_{n,i}\})| &\\ + |G(k-1, \{\lambda_{n,i}\}) - G(k-1, \{\lambda_i\})|, \quad & k = 1, 2, \ldots \end{aligned} \tag{23}$$

From Lemma 2,

$$\begin{aligned} |P\{M^{(k)}_{n,m_n} \le u_n\} - G(k-1, \{\lambda_{n,i}\})| &\\ \le 2(1 \wedge \lambda_{n,1}^{-1}) \exp\Big(-\sum_{i=1}^{\infty} \lambda_{n,i}\Big) \sum_{\alpha=1}^{n-m_n+1} \sum_{\beta\in B_{n,\alpha}} P_{n,\alpha} P_{n,\beta}. \end{aligned} \tag{24}$$

Note that $\lim_{n\to\infty} \lambda_{n,1} = \lambda\theta^2$ and since $\lim_{n\to\infty} \sum_{i=1}^{\infty} \lambda_{n,i} = \lambda\theta$ we have

$$\lim_{n\to\infty} \exp\Big(-\sum_{i=1}^{\infty} \lambda_{n,i}\Big) = \exp(-\lambda\theta). \tag{25}$$

Thus from (21) the right side of (24) tends to zero as $n \to \infty$. For $k = 1, 2, \ldots$ we also have

$$\begin{aligned} |G(k-1, \{\lambda_{n,i}\}) - G(k-1, \{\lambda_i\})| &\\ \le \sum_{s<k} \bigg| \exp\Big(-\sum_{i=1}^{\infty} \lambda_{n,i}\Big) \sum_{\substack{k_1+2k_2+\ldots+sk_s=s\\ k_j\ge 0,\, j=1,\ldots,s}} \frac{\lambda_{n,1}^{k_1}\lambda_{n,2}^{k_2}\ldots\lambda_{n,s}^{k_s}}{k_1!k_2!\ldots k_s!} &\\ - \exp(-\lambda\theta) \sum_{\substack{k_1+2k_2+\ldots+sk_s=s\\ k_j\ge 0,\, j=1,\ldots,s}} \frac{\lambda_1^{k_1}\lambda_2^{k_2}\ldots\lambda_s^{k_s}}{k_1!k_2!\ldots k_s!} \bigg|. \end{aligned}$$

Note that for fixed k we have a finite number of terms in the last two sums. Hence by (25) and Lemma 3 the right side of the inequality (23) converges to zero as $n \to \infty$.

As an immediate corollary of Theorem 1 we easily obtain the result of Canfield and McCormick [2].

COROLLARY 1. *Let* $\{m_n\}$ *and* $\{u_n\}$ *be sequences satisfying* (15) *and* (14) *respectively. Then*

$$\lim_{n\to\infty} P\{M^{(1)}_{n,m_n} \le u_n\} = e^{-\lambda\theta}.$$

Proof. Using Theorem 1 for $k = 1$ we obtain

$$\lim_{n\to\infty} P\{M^{(1)}_{n,m_n} \le u_n\} = G(0, \{\lambda_i\})$$

where

$$G(0, \{\lambda_i\}) = p_0(\{\lambda_i\}) = \exp\Big(-\sum_{i=1}^{\infty} \lambda_i\Big) = \exp(-\lambda\theta).$$

References

[1] A. D. Barbour, L. H. Y. Chen and W. L. Loh, *Compound Poisson approximation for nonnegative random variables via Stein's method*, Ann. Probab. 20 (1992), 1843–1866.

[2] E. R. Canfield and W. P. McCormick, *Asymptotic reliability of consecutive k-out-of-n systems*, J. Appl. Probab. 29 (1992), 142–155.

[3] O. Chryssaphinou and S. G. Papastavridis, *Limit distribution for a consecutive k-out-of-n:F system*, Adv. Appl. Probab. 22 (1990), 491–493.

[4] J. Dudkiewicz, *Asymptotic of extremes of moving minima in arrays of independent random variables*, Demonstratio Math. 29 (1996), 715–721.

[5] S. G. Papastavridis, *A limit theorem for the reliability of a consecutive-k-out-of-n system*, Adv. Appl. Probab. 19 (1987), 746–748.

[6] R. J. Serfling, *A general Poisson approximation theorem*, Ann. Probab. 3 (1975), 726–731.

[7] A. M. Zubkov, *Estimates for sums of finitely dependent indicators and for the time of first occurrence of a rare event*, Probabilistic Problems of Discrete Mathematics, Trudy Mat. Inst. Steklov. 177 (1986), 33–46, 207 (in Russian).

Jadwiga Dudkiewicz
Institute of Mathematics
Technical University of Kielce
Tysiąclecia PP 7
25-314 Kielce, Poland

Received on 11.7.1996;
revised version on 25.1.1997