

Denote the norm in B by $\|\cdot\|_1$. Using inequality (7), we can show by an argument similar to that in [14, p. 273] that there is a norm reducing algebra isomorphism T' of A onto a dense subalgebra of B . This completes the proof.

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Factoring the identity operator on a subspace of l_n^∞

by

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Abstract. It is proved that a “random n -dimensional subspace E of l_n^∞ ” has the property that for every factorization of the identity on E through a Banach space Y , $S_1: E \rightarrow Y$ and $S_2: Y \rightarrow E$, one has $\text{bc}(Y) \|S_1\| \|S_2\| \geq cn^{3/2}/\dim Y$, where $c > 0$ is a numerical constant.

A. Pełczyński ([4], Prop. 11.1) proved that for every n -dimensional Banach space X_n there is a Banach space Y with basis constant $\text{bc}(Y) = 1$, $\dim Y \leq n^{3/2}$, and there are operators $S_1: X_n \rightarrow Y$, $S_2: Y \rightarrow X_n$, such that $S_2 S_1 = \text{Id}_{X_n}$ and $\|S_1\| \|S_2\| \leq 3$. In this context he asked whether the estimate on the dimension of Y is optimal. Essentially the same proof yields the following more general result:

For every n -dimensional Banach space X_n and every $m \leq \text{nb}(X_n)$ there is an m -dimensional Banach space Y_m and operators $S_1: X_n \rightarrow Y_m$, $S_2: Y_m \rightarrow X_n$ such that $S_2 S_1 = \text{Id}_{X_n}$ and

$$(1) \quad \text{bc}(Y_m) \|S_1\| \|S_2\| \leq 3n \text{bc}(X_n)/m.$$

One can ask whether the estimate (1) is optimal. S. J. Szarek ([6], Prop. 5.1), using the technique introduced in [1] and developed in [5], proved that there are real n -dimensional Banach spaces X_n such that for every factorization of identity on X_n through an m -dimensional Banach space one has (in the notation above)

$$(2) \quad \text{bc}(Y_m) \|S_1\| \|S_2\| \geq \frac{cn}{m} \log^{3/2} n,$$

where $c > 0$ is a numerical constant. The complex variant of (2) was done in a similar way in [2] by the author. The aim of this note is to show that (both in the real and complex case) the estimate (1) is optimal “up to a multiplicative numerical constant” even if we restrict our interest to the case when X_n is an n -dimensional subspace of l_n^∞ with basis constant of order \sqrt{n} . The same argument yields that (1) is optimal “up to a multiplicative numerical constant” for n -dimensional subspaces of l_n^p with basis constant of order $n^{1/2-1/p}$ for $p \geq 2$.

1. Preliminaries. Our notation and terminology are standard. To fix the notation we shall consider real Banach spaces only. However, exactly the same argument yields the complex case. We shall deal mainly with linear operators acting on \mathbf{R}^n equipped with different norms. If $x = (x_1, \dots, x_n) \in \mathbf{R}^n$, then $\|x\|_p = (\sum_{i=1}^n |x_i|^p)^{1/p}$ for $p \in [1, \infty]$. If $\|\cdot\|$ is a norm on \mathbf{R}^n and $X_n = (\mathbf{R}^n, \|\cdot\|)$, then for $T \in L(\mathbf{R}^n)$ we shall denote by $\|T\|_{X_n}$ the norm of T as an operator from X_n into itself, while $\|T\|_p$ will stand for the norm of T as an operator from $\mathbf{R}^n = (\mathbf{R}^n, \|\cdot\|_p)$ into itself. If E is a linear subspace of \mathbf{R}^n and $T: E \rightarrow E$, then $\text{tr} T$ will denote the trace of T , and $\|T\|_{\text{HS}}$ will stand for the Hilbert–Schmidt norm of T . Note that in such a case both $\text{tr} T$ and $\|T\|_{\text{HS}}$ depend only on E and T (i.e. do not depend on the way we isometrically embed E in \mathbf{R}^n). In particular, we may identify E with $\mathbf{R}^{\dim E}$. For a linear subspace E of \mathbf{R}^n we shall denote by E^\perp and P_E the orthogonal complement of E and the orthogonal projection on E in \mathbf{R}^n , respectively. For a subset $A \subset \mathbf{R}^n$ by $[A]$ we shall mean the linear hull of A .

Let us recall the following definition ([5], cf. also [2]).

DEFINITION. For an operator $T \in L(\mathbf{R}^n)$ we write $T \in M_n(\alpha, \beta)$ with $\alpha, \beta > 0$ iff there is a linear subspace $E \subset \mathbf{R}^n$ with $\dim E \geq \alpha$ such that

$$\|P_{E^\perp} T x\|_2 \geq \beta \|x\|_2 \quad \text{for every } x \in E.$$

Observe that $T \in M_n(\alpha, \beta)$ for some $\alpha, \beta > 0$ iff $T - \lambda \text{Id}_{\mathbf{R}^n} \in M_n(\alpha, \beta)$ for every $\lambda \in \mathbf{R}$ iff $T - \lambda \text{Id}_{\mathbf{R}^n} \in M_n(\alpha, \beta)$ for some $\lambda \in \mathbf{R}$.

2. Technical proposition. The following lemma concerning the properties of operators on \mathbf{R}^n which do not belong to $M_n(\frac{1}{32}n, \frac{1}{8})$ seems to be crucial for the results of this paper. However, a similar argument can be found in [2], and the lemma itself (with different numerical constants) can easily be deduced in the real case from the results in [5] and in the complex case from the results in [7]. The proof presented here works in both cases. In the complex case one has only to use a complex version of results in [2].

LEMMA 2.1. Let $T \in L(\mathbf{R}^n)$ be such that for every $\lambda \in \mathbf{R}$ and for every linear subspace $E \subset \mathbf{R}^n$ with $\dim E \geq \frac{7}{8}n$ we have

$$\|(T - \lambda \text{Id}_{\mathbf{R}^n})|_E\|_2 \geq 1.$$

Then $T \in M_n(\frac{1}{32}n, \frac{1}{8})$.

Proof. Take $T \in L(\mathbf{R}^n)$ satisfying the assumption of the lemma and assume to the contrary that $T \notin M_n(\frac{1}{32}n, \frac{1}{8})$. Let $F \subset \mathbf{R}^n$ be a linear subspace of maximal dimension satisfying

$$\|P_{F^\perp} T x\|_2 \geq \frac{1}{8} \|x\|_2 \quad \text{for every } x \in F$$

and set $E = [F \cup TF \cup T^*F \cup T^*TF]^\perp$. Since $T \notin M_n(\frac{1}{32}n, \frac{1}{8})$ we infer that $\dim F < \frac{1}{32}n$ and therefore $\dim E \geq \frac{7}{8}n$. Let $\lambda = (\dim E)^{-1} \text{tr}(P_E T|_E)$ and define

$T_1 = T - \lambda \text{Id}_{\mathbf{R}^n}$. Note that by the definition of E it is enough to find an $x_0 \in E$, $\|x_0\| = 1$, such that

$$\|P_{[x_0]^\perp} T_1 x_0\|_2 \leq \frac{1}{8}$$

(for every $x \in \mathbf{R}^n$ we have $\|P_{[x]^\perp} T_1 x\|_2 = \|P_{[x]^\perp} T x\|_2$) to get a contradiction with the maximality of the dimension of F . Indeed, in such a case we would have

$$\|P_{F_1^\perp} T x\|_2 \geq \frac{1}{8} \|x\|_2 \quad \text{for every } x \in F_1,$$

where $F_1 = [F \cup x_0]$.

To this end consider $T_1|_E$. By the assumption $\|T_1|_E\|_2 \geq 1$. Let $x_0 \in E$, $\|x_0\| = 1$, be such that $\|T_1 x_0\| \geq 1$. If $\|P_{E^\perp} T_1 x_0\|_2 \geq \frac{1}{4}$, then since

$$\|P_{[x_0]^\perp} T_1 x_0\|_2 \geq \|P_{E^\perp} T_1 x_0\|_2$$

we get a contradiction. Hence $\|P_E T_1 x_0\|_2 \geq \frac{3}{4}$. Now, if there is an $x_1 \in E$, $\|x_1\| = 1$, such that $\|P_E T_1 x_0\|_2 < \frac{1}{4}$ then, applying Lemma 2.6 in [2] for the operator $P_E T_1|_E$ considered as an operator on $\mathbf{R}^{\dim E}$, we infer that there exists an $x_2 \in E$, $\|x_2\|_2 = 1$, such that

$$\|P_{[x_2]^\perp \cap E} P_E T_1 x_2\|_2 \geq \frac{1}{8}.$$

Since trivially

$$\|P_{[x_2]^\perp} T_1 x_2\|_2 \geq \|P_{[x_2]^\perp \cap E} T_1 x_2\|_2 = \|P_{[x_2]^\perp \cap E} P_E T_1 x_2\|_2$$

we get a contradiction once again. Thus the only remaining case is that $\|P_E T_1 x\|_2 \geq \frac{1}{4} \|x\|_2$ for every $x \in E$. But in this case we have

$$\|P_E T_1|_E\|_{\text{HS}} \geq \frac{1}{4} (\dim E)^{1/2},$$

while $\text{tr}(P_E T_1|_E) = 0$. Hence, again considering $P_E T_1|_E$ as an operator on $\mathbf{R}^{\dim E}$, by [2], Cor. 2.2, we obtain

$$\|P_{[x]^\perp \cap E} P_E T_1 x\|_2 \geq \frac{1}{4} \frac{(\dim E)^{1/2}}{(2 + \dim E)^{1/2}} \geq \frac{1}{8}$$

for some $x \in E$. This in the same way as above yields a contradiction and completes the proof.

In the sequel we shall need the following basic result on properties of “random quotients of l_{2n}^1 ” (= Gluskin spaces) due to S. J. Szarek ([5], Th. 1.4).

PROPOSITION 2.2. There is a numerical constant $c_1 > 0$ such that for every $n \geq 2$ there is a norm $\|\cdot\|_{X_n}$ on \mathbf{R}^n such that

- (i) the Banach space $X_n = (\mathbf{R}^n, \|\cdot\|_{X_n})$ is isometrically isomorphic to a quotient of l_{2n}^1 ,
- (ii) $\|x\|_2 \leq \|x\|_{X_n} \leq \|x\|_1 (\leq \sqrt{n} \|x\|_2)$ for every $x \in \mathbf{R}^n$,
- (iii) $\|T\|_{X_n} > c_1 \sqrt{n}$ for every $T \in M_n(\frac{1}{32}n, \frac{1}{8})$.

Combining Lemma 2.1 and Prop. 2.2 we are able to prove the following technical fact which summarizes the basic properties of random quotients of l_{2n}^1 and seems to be of independent interest (e.g. it is a starting point for the construction in [3]).

PROPOSITION 2.3. *There is a numerical constant $c > 0$ such that for every $n \geq 2$ there is a norm $\|\cdot\|_{X_n}$ on \mathbf{R}^n such that*

- (i) $X_n = (\mathbf{R}^n, \|\cdot\|_{X_n})$ is isometrically isomorphic to a quotient of l_{2n}^{cn} ,
- (ii) $\|x\|_2 \leq \|x\|_{X_n} \leq \|x\|_1$ for every $x \in \mathbf{R}^n$,
- (iii) for every $T \in L(\mathbf{R}^n)$ there are $\lambda_T \in \mathbf{R}$, $V_T \in L(\mathbf{R}^n)$ and a linear subspace $E_T \subset \mathbf{R}^n$, with $\dim E_T > \frac{7}{8}n$, such that

$$(a) \quad T = \lambda_T \text{Id}_{\mathbf{R}^n} + V_T,$$

$$(b) \quad |\lambda_T| \leq c \|T\|_{X_n},$$

$$(c) \quad \|V_T|_{E_T}\|_2 \leq c \|T\|_{X_n} / \sqrt{n}.$$

Proof. Fix $n \geq 2$ and let $\|\cdot\|_{X_n}$ be the norm on \mathbf{R}^n from Prop. 2.2. Then (i) and (ii) are automatically satisfied. To prove (iii) let T be an arbitrary operator in $L(\mathbf{R}^n)$. Set

$$T_1 = \frac{c_1 \sqrt{n}}{\|T\|_{X_n}} T.$$

We have $\|T_1\|_{X_n} = c_1 \sqrt{n}$. Thus, by Prop. 2.2(iii), $T_1 \notin M_n(\frac{1}{32}n, \frac{1}{8})$. Hence, Lemma 2.1 implies that there are $\lambda_{T_1} \in \mathbf{R}$ and a linear subspace $E_{T_1} \subset \mathbf{R}^n$ with $\dim E_{T_1} \geq \frac{7}{8}n$ such that

$$\|(T_1 - \lambda_{T_1} \text{Id}_{\mathbf{R}^n})|_{E_{T_1}}\|_2 < 1.$$

Set $V_{T_1} = T_1 - \lambda_{T_1} \text{Id}_{\mathbf{R}^n}$ and define

$$\lambda_T = \frac{\|T\|_{X_n}}{c_1 \sqrt{n}} \lambda_{T_1}, \quad V_T = \frac{\|T\|_{X_n}}{c_1 \sqrt{n}} V_{T_1}$$

and $E_T = E_{T_1}$. Then we have $T = \lambda_T \text{Id}_{\mathbf{R}^n} + V_T$ and

$$(3) \quad \|V_T|_{E_T}\|_2 < c_1^{-1} \|T\|_{X_n} / \sqrt{n},$$

which completes the proof of (iii)(a) and (c). To prove (iii)(b) assume that $\lambda_T > (c_1^{-1} + 1) \|T\|_{X_n}$. Then for every $x \in \mathbf{R}^n$ we have

$$\begin{aligned} \|V_T x\|_{X_n} &= \|(T - \lambda_T \text{Id}_{\mathbf{R}^n})x\|_{X_n} > ((c_1^{-1} + 1) \|T\|_{X_n} - \|T\|_{X_n}) \|x\|_{X_n} \\ &\geq c_1^{-1} \|T\|_{X_n} \|x\|_{X_n}. \end{aligned}$$

On the other hand, by (ii) and (3), for every $x \in E_T$ we have

$$\|V_T x\|_{X_n} \leq \|V_T x\|_1 \leq \sqrt{n} \|V_T x\|_2 \leq \sqrt{n} \|V_T|_{E_T}\|_2 \|x\|_2 \leq c_1^{-1} \|T\|_{X_n} \|x\|_{X_n},$$

a contradiction which completes the proof of the proposition with $c = c_1^{-1} + 1$.

3. Main results. The theorem below states that in the Banach spaces from Prop. 2.3 it is impossible to represent the identity operator as a sum of a “small number of operators with small rank” in such a way that all partial sums will have “small norms”. More precisely, we have

THEOREM 3.1. *There is a numerical constant $c_0 > 0$ such that for each $n \geq 2$ there is a norm $\|\cdot\|_{X_n}$ on \mathbf{R}^n such that the Banach space $X_n = (\mathbf{R}^n, \|\cdot\|_{X_n})$ is isometrically isomorphic to a quotient of l_{2n}^1 and has the property:*

- (*) whenever $\text{Id}_{X_n} = \sum_{i=1}^k T_i$, with $\text{rank } T_i < \frac{7}{8}n$ for $i = 1, \dots, k$, then

$$\max_{1 \leq k_1 \leq k} \left\| \sum_{i=1}^{k_1} T_i \right\|_{X_n} \geq c_0 \sqrt{n}/k.$$

Proof. Fix $n \geq 2$ and let $X_n = (\mathbf{R}^n, \|\cdot\|_{X_n})$ be the space from Prop. 2.3. Let $\text{Id}_{X_n} = \sum_{i=1}^k T_i$ with $\text{rank } T_i < \frac{7}{8}n$ for $i = 1, \dots, k$. Set

$$b = \max_{1 \leq k_1 \leq k} \left\| \sum_{i=1}^{k_1} T_i \right\|_{X_n}$$

and observe that if $T_i = \lambda_i \text{Id}_{X_n} + V_i$ for $i = 1, \dots, k$ are the representations of T_i 's as in Prop. 2.3(iii) then for $x \in \ker T_i \cap E_{T_i} (\neq \{0\})$ with $\|x\|_2 = 1$ we have

$$0 = \|T_i x\|_2 = \|(\lambda_i \text{Id}_{X_n} + V_i)x\|_2 \geq |\lambda_i| - 2cbn^{-1/2}.$$

Thus we have $|\lambda_i| \leq 2cbn^{-1/2}$ for $i = 1, \dots, k$. Therefore

$$\|T_i|_{E_{T_i}}\|_2 \leq |\lambda_i| + 2cbn^{-1/2} \leq 4cbn^{-1/2}$$

for $i = 1, \dots, k$.

CLAIM. *For every $j = 1, \dots, k$ there are $\tilde{\lambda}_j$ and a linear subspace $\tilde{E}_j \subset \mathbf{R}^n$ with $\dim \tilde{E}_j \geq \frac{7}{8}n$ such that*

$$\sum_{i=1}^j T_i = \tilde{\lambda}_j \text{Id}_{X_n} + \tilde{V}_j$$

with $|\tilde{\lambda}_j| \leq 6jcbn^{-1/2}$ and $\|\tilde{V}_j|_{\tilde{E}_j}\|_2 \leq cbn^{-1/2}$.

For $j = 1$ the claim has already been proved above. Assume that it is true for some $j \geq 1$. Then

$$\sum_{i=1}^{j+1} T_i = \sum_{i=1}^j T_i + T_{j+1} = \tilde{\lambda}_j \text{Id}_{X_n} + \tilde{V}_j + \lambda_{j+1} \text{Id}_{X_n} + V_{j+1}.$$

Thus, for every $x \in \tilde{E}_j \cap E_{T_{j+1}} (\neq \{0\})$, with $\|x\|_2 = 1$, by our assumption we have

$$(4) \quad \left\| \sum_{i=1}^{j+1} T_i x \right\|_2 \leq 6jcbn^{-1/2} + cbn^{-1/2} + 4cbn^{-1/2} = (6j+5)cbn^{-1/2}.$$

Now, if $\sum_{i=1}^{j+1} T_i = \tilde{\lambda}_{j+1} \text{Id}_{X_n} + \tilde{V}$ is the representation of $\sum_{i=1}^{j+1} T_i$ as in Prop. 2.3(iii) and \tilde{E}_{j+1} is the corresponding subspace with $\dim \tilde{E}_{j+1} \geq \frac{7}{8}n$ then for every $x \in \tilde{E}_{j+1}$ with $\|x\|_2 = 1$ we have

$$(5) \quad \left\| \sum_{i=1}^{j+1} T_i x \right\|_2 \geq |\tilde{\lambda}_{j+1}| - cbn^{-1/2}.$$

Combining (4) and (5) we deduce that for $x \in \tilde{E}_j \cap \tilde{E}_{j+1} \cap E_{j+1}$ with $\|x\|_2 = 1$ (note that $\dim(\tilde{E}_j \cap \tilde{E}_{j+1} \cap E_{j+1}) \geq n/2$) we have

$$|\tilde{\lambda}_{j+1}| - cbn^{-1/2} \leq \left\| \sum_{i=1}^{j+1} T_i x \right\|_2 \leq (6j+5)cbn^{-1/2},$$

which yields the desired estimate and concludes the proof of the claim.

Now, in particular we have

$$\sum_{i=1}^k T_i = \text{Id}_{X_n} = \tilde{\lambda} \text{Id}_{X_n} + \tilde{V}_k$$

with $|\tilde{\lambda}_k| \leq 6kcbn^{-1/2}$ and $\|\tilde{V}_k\|_2 < cbn^{-1/2}$. Thus $|\tilde{\lambda}_k| \geq 1 - cbn^{-1/2}$. Hence $7kcbn^{-1/2} \geq 1$, which completes the proof of the theorem with $c_0 = (7c)^{-1}$.

Remark 3.2. It is well known (cf. [3], [6]) and easy to see that for an m -dimensional Banach space X_n the property (*) from Theorem 3.1 formally implies the following:

(**) whenever Y_m is an m -dimensional Banach space with the basis constant $\text{bc}(Y_m)$ and $S_1: X_n \rightarrow Y_m$ and $S_2: Y_m \rightarrow X_n$ are linear operators such that $S_2 S_1 = \text{Id}_{X_n}$, then

$$\text{bc}(Y_m) \|S_1\| \|S_2\| \geq \tilde{c}_0 n^{3/2}/m$$

where \tilde{c}_0 is a numerical constant (\tilde{c}_0 depends on c_0).

The remark above and Theorem 3.1 by a standard duality argument yield

THEOREM 3.3. *There is a numerical constant $c > 0$ such that for every $n \in \mathbb{N}$ there is an n -dimensional subspace E_n of l_{2n}^∞ with the property:*

whenever Y_m is an m -dimensional Banach space with basis constant $\text{bc}(Y_m)$ and $S_1: E_n \rightarrow Y_m$, $S_2: Y_m \rightarrow E_n$ are linear operators such that $S_2 S_1 = \text{Id}_{E_n}$, then

$$\text{bc}(Y_m) \|S_1\| \|S_2\| \geq cn^{3/2}/m.$$

Remark 3.4. Since a “random n -dimensional quotient of l_{2n}^1 ” satisfies Prop. 2.2 (cf. [3]), we infer that the conclusion of Th. 3.2 is satisfied by a “random n -dimensional subspace of l_{2n}^∞ ”.

Remark 3.5. Using a standard argument one can deduce from Th. 3.2

and Remark 3.3 that for a “random n -dimensional subspace” E_n of l_{2n}^∞ , $p \in [2, \infty]$, one can get the following estimate:

$$\text{bc}(Y_m) \|S_1\| \|S_2\| \geq \frac{cn^{3/2-1/p}}{m} \geq \frac{cn d(E_n, l_n^2)}{m} \geq \frac{cn \text{bc}(E_n)}{m}$$

for factoring the identity operator on E_n through an arbitrary m -dimensional Banach space Y_m .

Remark 3.6. The results above remain true (with different constants) for “random n -dimensional subspaces of $l_{(1+\varepsilon)n}^p$ ”, for a fixed $\varepsilon > 0$ and $p \in [2, \infty]$.

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