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The translation invariant uniform approximation property for compact groups

by

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Abstract. The translation invariant analogue in commutative harmonic analysis of the uniform approximation property of Banach spaces was introduced by M. Bożejko and A. Pełczyński (1978). This paper contains noncommutative analogues of their results.

§0. Introduction. Recall (see [1], [5]) that a Banach space X is said to have the *uniform bounded approximation property*, abbreviated ubap, if there exist a $k \geqslant 1$ and a positive sequence $q(m)$ such that given a finite-dimensional subspace $E \subset X$, there exists an operator $T: X \to X$ satisfying the following conditions:

(i) $T(x) = x$ for $x \in E$.
(ii) $\|T\| \leqslant k$.
(iii) $\dim T(X) \leqslant q(\dim E)$.

It is known ([4], [5]) that L^p-spaces, $C(K)$-spaces and reflexive Orlicz spaces have the ubap. The definition can be modified for function spaces on compact groups by, roughly speaking, assuming that X, E and T are translation invariant. In this way we obtain the translation invariant analogue of the uniform bounded approximation property introduced in [1] where the case of compact Abelian groups is considered.

§1. Preliminaries. In the sequel G is a compact group, Σ its dual object called also the *hypergroup*, λ the normalized Haar measure on G.

For $g \in G$ the left translation operator is defined by $(l_g f)(x) = f(g^{-1}x)$ and the right translation operator by $(r_g f)(x) = f(xg)$ for f λ-measurable and $x \in G$. A vector space X of λ-equivalence classes of λ-measurable functions is *left translation invariant* if $l_g X \subset X$, *right translation invariant* if $r_g X \subset X$ and *conjugate translation invariant* if $l_g r_g X \subset X$ for all $g \in G$. We call X *translation invariant* if it is both left and right translation invariant.

By $L^p(G)$ $(p \geqslant 1)$ we denote as usual the Banach space of the λ-equivalence classes of λ-measurable functions f with the norm $\|f\|_p = \left(\int |f(x)|^p \, d\lambda(x)\right)^{1/p}$. For $f, g \in L^1(G)$ we define the convolution $f * g \in L^1(G)$ by

$$(f * g)(x) = \int_G f(y^{-1}x) g(y) \, d\lambda(y).$$

This section is written for the translation invariant Banach spaces but it can be easily modified for the left, right or conjugate translation invariant Banach spaces.

(1.1) Definition. A translation invariant Banach space X is called *regular* if:

(h.0) $X \subset L^1(G)$ with continuous inclusion.

(h.1) The translations l_g, r_g: $X \to X$ are isometries.

(h.2) For every $f \in X$ the maps $g \to l_g f$, $g \to r_g f$ (from G into X) are continuous.

Remarks. 1° The spaces $L^p(G)$ $(p \geqslant 1)$ are translation invariant and regular.

2° Every closed translation invariant subspace of a regular translation invariant Banach space is regular.

3° Every finite-dimensional translation invariant subspace E of a regular translation invariant Banach space is of the form

$$E = \{f = \sum_{\sigma \in R_E} d_\sigma \operatorname{tr}(A_\sigma U^\sigma) \mid A_\sigma \text{ are } d_\sigma \times d_\sigma \text{ complex matrices}, \quad R_E \text{ is a finite subset of } \Sigma\}$$

where U^σ denotes the matrix of the representation σ in a (fixed) orthonormal basis of the representation space H_σ with $\dim H_\sigma = d_\sigma$.

Next we have

(1.2) Proposition. *Let X be a regular translation invariant Banach space. For every central function g define the operator T_g of convolution with g by the (X-valued) integral*

$$T_g(f) = \int l_x f \cdot g(x)\, d\lambda(x) \quad \text{for } f \in X.$$

Then T_g: $X \to X$ is a bounded translation invariant linear operator with $\|T_g\| \leqslant \|g\|_1$ and $T_g(f) = f * g$ for $f \in X$.

Recall that $g \in L^1(G)$ is *central* if $f * g = g * f$ for all $f \in L^1(G)$.

Proof. It follows from (h.2) that the integral $\int l_x f \cdot g(x)\, d\lambda(x)$ exists, and by (h.0) it is equal to $f * g$. Since g is central T_g is translation invariant. Finally,

$$\|T_g(f)\| \leqslant \int \|l_x f\|\, |g(x)|\, d\lambda(x) = \|g\|_1 \|f\|.$$

Following the ideas of Bożejko and Pełczyński ([1]) we shall prove a generalization of their theorem on the translation invariant ubap for the translation invariant Banach spaces on an arbitrary compact group. The result of Travaglini ([6]) shows that there is no analogue of the translation invariant ubap for the conjugate translation invariant Banach spaces on a compact connected semisimple Lie group.

§2. **The main result.** We recall ([1]) that a translation invariant Banach space X is said to have the *invariant uniform bounded approximation property* if there exist a $k \geqslant 1$ and a positive sequence $q(m)$ such that given a finite-dimensional invariant subspace $E \subset X$, there exists a translation invariant operator T: $X \to X$ satisfying the following conditions:

(i) $T(x) = x$ for $x \in E$.

(ii) $\|T\| \leqslant k$.

(iii) $\dim T(X) \leqslant q(\dim E)$.

Now we are ready to state the main result of this paper.

(2.1) Theorem. *Every regular translation invariant Banach space on a compact group has the invariant ubap.*

To prove Theorem 2.1 it is enough to establish it for the space $L^1(G)$ which is in fact equivalent to a result in harmonic analysis (see below). Recall that the *Fourier transform* of $g \in L^1(G)$ is the matrix-valued function $\hat{g}$ on Σ defined by

$$\hat{g}(\sigma) = \int_G g(x)\, U^\sigma_{x^{-1}}\, d\lambda(x)$$

where U^σ_x is the matrix of σ evaluated at x. If $R \subset \Sigma$ is finite then $\nu(R) = \sum_{\sigma \in R} d_\sigma^2$ where d_σ is the degree of σ. If G is Abelian, $\nu(R)$ is the cardinality of R.

(2.2) Theorem. *For every $k > 1$ there exists a positive sequence $q_k(r)$ such that for every finite set $R \subset \Sigma$ there exists a central function $g \in L^1(G)$ such that:*

(j) $\hat{g}(\sigma) = I_{d_\sigma}$ *for* $\sigma \in R$.

(jj) $\|g\|_1 \leqslant k$.

(jjj) $\nu(\operatorname{supp} \hat{g}) \leqslant q_k(\nu(R))$.

To derive Theorem 2.1 from Theorem 2.2 fix k and a translation invariant finite-dimensional subspace E of X.

By Remark 3° after Definition 1.1, $E = \{f \in L^1(G) \mid \operatorname{supp} \hat{f} \subset R_E\}$ and E is spanned by all coordinate functions of matrices U^σ with $\sigma \in R_E$. Clearly $\dim E = \nu(R_E)$. Now pick g satisfying (j)–(jjj) for R_E and $T = T_g$. Then (j) implies (i), (jj) via Proposition 1.2 implies (ii), and (jjj) and (h.0) imply (iii).

Remark. Note that when X is a left (or right) regular translation invariant Banach space, then we still have an analogue of Theorem 2.1. In this case condition (iii) follows from (jjj) and the fact that a finite-dimensional left (right) translation invariant subspace E is spanned by the coordinate functions which fill the columns (rows) of the matrices U^σ with $\sigma \in R_E$.

For the proof of Theorem 2.2 we first introduce more notation. A function F on Σ is said to be a *matrix function* if for every $\sigma \in \Sigma$, $F(\sigma)$ is a $d_\sigma \times d_\sigma$ complex matrix. For $R \subset \Sigma$ we define the *matrix characteristic function* ξ_R of R by $\xi_R(\sigma) = I_{d_\sigma}$ if $\sigma \in R$ and $\xi_R(\sigma) = 0_{d_\sigma}$ otherwise. By $E^p(\Sigma)$ ($p \geqslant 1$) we denote the Banach space of matrix functions F on Σ with the norm

$$\|F\|_p = \sup_R \big(\sum_{\sigma \in R} d_\sigma \|F(\sigma)\|_p^p\big)^{1/p}$$

where the supremum is taken over all finite subsets of Σ and $\|F(\sigma)\|_p$ is the l^p-norm of the sequence of the eigenvalues of the matrix $|F(\sigma)|$.

For $F \in E^1(\Sigma)$ the *inverse Fourier transform* of F is the function $\check{F}$ on G defined by

$$\check{F}(x) = \sum_{\sigma \in \Sigma} d_\sigma \operatorname{tr}\big(F(\sigma) U_x^\sigma\big).$$

For $\sigma \in \Sigma$ let $\bar{\sigma}$ be the representation conjugate to σ and χ_σ the character of σ. If $\sigma, \tau \in \Sigma$ we write $\chi_\sigma \chi_\tau = \sum_{\eta \in \Sigma} n_{\sigma\tau}(\eta) \chi_\eta$ corresponding to the decomposition of the tensor product $\sigma \otimes \tau$. The (finite) support of $n_{\sigma\tau}$ is denoted by $\sigma \times \tau$. A set $P \subset \Sigma$ such that $\sigma \times \tau \subset P$ and $\bar{\sigma} \in P$ whenever $\sigma, \tau \in P$ is called a *subhypergroup* of Σ. Finally, if R and S are subsets of Σ then $R \times S = \bigcup \{\sigma \times \tau \mid \sigma \in R,\ \tau \in S\}$ and $\bar{R} = \{\bar{\sigma} \mid \sigma \in R\}$.

The proof of Theorem 2.2 is based upon the following lemmas. The next lemma is in fact due to Dooley ([2]).

(2.3) LEMMA. *Let $\varepsilon > 0$. Assume that for a finite set $R \subset \Sigma$ there exists a finite set S such that*

$$\nu(R \times \bar{S}) \leqslant (1+\varepsilon)\nu(S). \tag{1}$$

Let $g = \nu(S)^{-1} \check{\xi}_S \cdot \check{\xi}_{R \times \bar{S}}$. Then g is central and

$$\hat{g}(\sigma) = I_{d_\sigma} \quad \text{for } \sigma \in R, \qquad \|g\|_1 \leqslant (1+\varepsilon)^{1/2},$$
$$\nu(\operatorname{supp} \hat{g}) \leqslant (1+\varepsilon)\nu(S)^2.$$

Proof. We have

$$\hat{g}(\sigma) = \nu(S)^{-1} \sum_{\eta \in S} d_\eta \Big[\sum_{\zeta \in R \times \bar{S}} d_\zeta n_{\eta\zeta}(\sigma)\Big] d_\sigma^{-1} I_{d_\sigma}.$$

Since $n_{\eta\zeta}(\sigma) = n_{\sigma\bar{\eta}}(\zeta)$ the sum in square brackets is equal to $d_\sigma d_\eta$ for $\sigma \in R$. Thus $\hat{g}(\sigma) = I_{d_\sigma}$ for $\sigma \in R$. Clearly g is central. Now,

$$\|g\|_1 \leqslant \nu(S)^{-1} \|\xi_S\|_2 \|\xi_{R \times \bar{S}}\|_2 = \left[\frac{\nu(R \times \bar{S})}{\nu(S)}\right]^{1/2} \leqslant (1+\varepsilon)^{1/2}.$$

Finally, note that $\operatorname{supp} \hat{g} = \bigcup \{\eta \times \zeta \mid \eta \in S,\ \zeta \in R \times \bar{S}\}$ and $\nu(\eta \times \zeta) \leqslant d_\eta^2 d_\zeta^2$. Hence

$$\nu(\operatorname{supp} \hat{g}) \leqslant \nu(S)\nu(R \times \bar{S}) \leqslant (1+\varepsilon)\nu(S)^2.$$

To complete the proof of Theorem 2.2 in view of Lemma 2.3 we have to construct for a given set $R \subset \Sigma$ a set $S \subset \Sigma$ so that (1) is satisfied and $\nu(S)$ depends on $\nu(R)$ only. Without loss of generality one may assume that R contains the trivial representation 1 of Σ. For $\sigma \in \Sigma$ we write $\sigma^0 = 1$ and $\sigma^{n+1} = \sigma^n \times \sigma$.

(2.4) LEMMA. *Let $R = \{\sigma_1, \ldots, \sigma_r\}$ with $\sigma_1 = 1$. Let $S_n = A_n(\bar{\sigma}_1) \times \ldots \times A_n(\bar{\sigma}_r)$ where $A_n(\sigma) = \bigcup \{\sigma^i \mid 0 \leqslant i \leqslant n\}$. Then for n sufficiently large*

$$\nu(R \times \bar{S}_n) \leqslant \left(1 + \frac{\nu(R)^2}{n+1}\right)\nu(S_n), \qquad \nu(S_n) \leqslant n^{\nu(R)}.$$

Lemma 2.4 is an easy consequence of the following one.

(2.5) LEMMA. *Let $1 \in T \subset \Sigma$ with $\nu(T) < \infty$ and let $\sigma \in \Sigma$. Let $T_0 = T$, $T_n = A_n(\sigma) \times T$. Then for n sufficiently large*

$$\nu\big((\sigma \times T_n) \setminus T_n\big) \leqslant \frac{d_\sigma^4}{n+1} \nu(T_n). \tag{2}$$

Moreover,

$$\nu\big(A_n(\sigma)\big) \leqslant (1-1/n)^{-1} n^{d_\sigma^2}. \tag{3}$$

Proof([1]). Let $d = d_\sigma^2$ and let $u_1, \ldots, u_d$ be all coordinate functions of σ. Let L^n be the linear subspace of $L^2(G)$ spanned by all functions of the form $u_1^{k_1} \ldots u_d^{k_d}$ with $k_1 + \ldots + k_d \leqslant n$, where k_i is a nonnegative integer for $i = 1, \ldots, d$. Then let $N_0 = 1$ and for $j = 1, \ldots, d$ let N_j be the number of the products $u_1^{k_1} \ldots u_d^{k_d}$ such that $k_i = 0$ for $i < j$ and $k_j \neq 0$. It follows that $N_j \leqslant n^{d-j+1}$ and thus

$$\dim L^n \leqslant \sum_{j=0}^{d} N_j \leqslant (1-1/n)^{-1} n^d.$$

Now, L^n is spanned by all coordinate functions of the tensor products $\sigma \otimes \ldots \otimes \sigma = \sigma^{\otimes j}$ with $j = 0, 1, \ldots, n$ which can be written (via the Clebsch–Gordon formulas) in terms of the coordinate functions of all $\eta \in A_n(\sigma)$. Hence

$$\dim L^n = \sum_{\eta \in A_n(\sigma)} d_\eta^2 = \nu\big(A_n(\sigma)\big)$$

and by the above([2]) we obtain (3).

Now, let M be the subspace of $L^2(G)$ spanned by the coordinate functions of all $\tau \in T$. Let $L^n M$ be the subspace of $L^2(G)$ spanned by all

([1]) The idea of using linear subspaces of $L^2(G)$ is adapted from [2].

([2]) In general no estimate from below can be given since the functions $u_1^{k_1} \ldots u_d^{k_d}$ can be linearly dependent (take σ to be a torsion element of Σ).

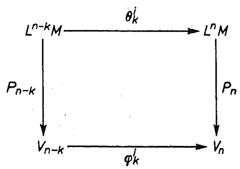
functions of the form $f\cdot\psi$, $f\in L^n$ and $\psi\in M$. Then by the same argument

$$\dim L^n M = \nu(A_n(\sigma)\times T) = \nu(T_n).$$

Clearly $L^{n-1}M\subset L^n M$. Thus $L^n M$ can be decomposed into the orthogonal sum (in the sense of $L^2(G)$) $L^n M = V_n\oplus L^{n-1}M$. Observe that V_n is spanned by the coordinate functions of all $\tau\in T_n\setminus T_{n-1}$ and hence $\dim V_n = \nu(T_n\setminus T_{n-1})$.

For every n let P_n be the orthogonal projection of $L^n M$ onto V_n. It follows that V_n is spanned by all functions of the form $P_n(u_1^{k_1}\dots u_d^{k_d}\psi)$, $\psi\in M$, with $k_1+\dots+k_d=n$.

For $i=1,\dots,d$ and $1\leqslant k\leqslant n$ define linear mappings θ_k^i from $L^{n-k}M$ into $L^n M$ by $\theta_k^i(f)=u_i^k\cdot f$, $f\in L^{n-k}M$. It follows that $\theta_k^i(\operatorname{Ker} P_{n-k})\subset \operatorname{Ker} P_n$. Hence there exists a unique linear mapping φ_k^i such that the following diagram commutes:



Now fix n and let m be the first natural number such that $n/d^2\leqslant m$. For $k=1,\dots,m$ consider the linear mappings π_k from $W_k=\bigoplus_{i=1}^d V_{n-(d-i)m-k}$ into V_n defined by $\pi_k=\bigoplus_{i=1}^d \varphi^i_{(d-i)m+k}$.

We shall show that for $k=1,\dots,m$ the mapping π_k is an epimorphism.

Since V_n is spanned by the functions of the form $f=P_n(u_1^{k_1}\dots u_d^{k_d}\psi)$, $\psi\in M$, with $k_1+\dots+k_d=n$ it is enough to find for every such f a $g\in W_k$ such that $\pi_k(g)=f$.

Fix k and take f as above. There is a j, $1\leqslant j\leqslant d$, such that $k_j\geqslant(d-j)m+k$ since otherwise for $n\geqslant(d+1)d^2/(d-1)$ (and for $n\geqslant 1$ if $d=1$) we have

$$\sum_{i=1}^d k_i<\sum_{i=1}^d((d-i)m+k)=m\frac{d(d-1)}{2}+dk\leqslant n$$

contrary to the assumption on f. Choose a j with $k_j\geqslant q_j=(d-j)m+k$ and put

$$g=P_{n-q_j}(u_1^{k_1}\dots u_{j-1}^{k_{j-1}}u_j^{k_j-q_j}u_{j+1}^{k_{j+1}}\dots u_d^{k_d}\psi).$$

Then $g\in V_{n-q_j}\subset W_k$ and we get

$$\pi_k(g)=\varphi^j_{q_j}(g)=P_n(\theta^j_{q_j}(u_1^{k_1}\dots u_j^{k_j-q_j}\dots u_d^{k_d}\psi))=P_n(u_1^{k_1}\dots u_d^{k_d}\psi)=f.$$

Hence π_k maps W_k onto V_n as claimed.

Now, note that $L^n M$ can be decomposed into the direct sum $V_n\oplus V_{n-1}\oplus\dots\oplus V_1\oplus LM$. In particular, we have

$$\bigoplus_{k=1}^m W_k=\bigoplus_{k=1}^m\bigoplus_{i=1}^d V_{n-(d-i)m-k}\subset L^{n-1}M.$$

Since the mappings π_k are epimorphisms, $\dim V_n\leqslant\dim W_k$ for $k=1,\dots,m$. Hence we get

$$m\dim V_n\leqslant\sum_{k=1}^m\dim W_k\leqslant\dim L^{n-1}M.$$

The last line implies that

$$\dim V_{n+1}\leqslant\frac{d^2}{n+1}\dim L^n M.$$

It follows from the remarks at the beginning of the proof that the latter is equivalent to

$$\nu(T_{n+1}\setminus T_n)\leqslant\frac{d^2}{n+1}\nu(T_n).$$

In order to obtain (2) it suffices to note that

$$(\sigma\times T_n)\setminus T_n\subset T_{n+1}\setminus T_n.$$

Proof of Lemma 2.4. Fix n and for $j=1,\dots,r$ put

$$T^j=A_n(\bar\sigma_1)\times\dots\times A_n(\bar\sigma_{j-1})\times A_n(\bar\sigma_{j+1})\times\dots\times A_n(\bar\sigma_r).$$

Then

$$S_n=A_n(\bar\sigma_j)\times T^j\quad\text{for } j=1,\dots,r.$$

Thus applying Lemma 2.5 for $\sigma=\bar\sigma_j$ and $T=T^j$ we get

$$\nu((\bar\sigma_j\times S_n)\setminus S_n)\leqslant\frac{d^4_{\sigma_j}}{n+1}\nu(S_n).$$

Thus

$$\nu(R\times\bar S_n)\leqslant\nu(S_n)+\nu((R\times\bar S_n)\setminus\bar S_n)\leqslant\nu(S_n)+\sum_{j=1}^r\nu((\sigma_j\times\bar S_n)\setminus\bar S_n)$$
$$\leqslant\left(1+\frac{\nu(R)^2}{n+1}\right)\nu(S_n).$$

Since ν is submultiplicative (cf. the proof of Lemma 2.3) and since $\sigma_1 = 1$ we get

$$\nu(S_n) \leqslant \prod_{j=1}^{r} \nu\big(A_n(\sigma_j)\big) \leqslant n^{\nu(R)}.$$

Proof of Theorem 2.2. Put $\varepsilon = k^2-1$. Let $R \subset \Sigma$ with $1 \in R$ and $r = \nu(R) < \infty$ be given. Pick n so that $r^2/(n+1) \leqslant \varepsilon$, say([3]) $n = [r^2/\varepsilon] = [r^2/(k^2-1)]$. If S_n is that of Lemma 2.4, then $\nu(S_n) \leqslant n^r$. For R and S_n construct g as in Lemma 2.3. Then $\|g\|_1 \leqslant (1+\varepsilon)^{1/2} = k$ and

$$\nu(\operatorname{supp}\hat{g}) \leqslant (1+\varepsilon)\,\nu(S_n)^2 \leqslant k^2 n^{2r} \leqslant k^2\left(\frac{r^2}{k^2-1}\right)^{2r} = q_k(r).$$

§ 3. **Final remarks.** 1° A routine argument using the duality between $L^1(G)$ and $L^\infty(G)$ (see [3]) yields that the assertion of Theorem 2.2 is equivalent to the following.

For every $k > 1$ there exists a sequence $q_k(r)$ such that for every finite subset $R \subset \Sigma$ there exists a set S with $R \subset S \subset \Sigma$ and $\nu(S) \leqslant q_k(\nu(R))$ such that if $h \in L^\infty(G)$ and $\hat{h}(\sigma) = 0_{d_\sigma}$ for $\sigma \in S \setminus R$ then

$$\Big|\sum_{\sigma \in R} d_\sigma \operatorname{tr}\hat{h}(\sigma)\Big| \leqslant k\|h\|_\infty.$$

2° If for some $R \subset \Sigma$ with $1 \in R$ there exists a central function $g \in L^1(G)$ such that $\hat{g}(\sigma) = I_{d_\sigma}$ for $\sigma \in R$, $\nu(\operatorname{supp}\hat{g}) < \infty$ and $\|g\|_1 = 1$, then R is contained in a finite subhypergroup Σ_0 of Σ with $\nu(\Sigma_0) \leqslant \nu(\operatorname{supp}\hat{g})$.

Proof. Let $\varphi_n = \sum_{j=0}^{n}(n+1)^{-1}\hat{g}^j$. Then φ_n tends (in $E^1(\Sigma)$) to the characteristic matrix function of a subset, say Σ_0, of Σ. Clearly $R \subset \Sigma_0 \subset \operatorname{supp}\hat{g}$ and $\|\xi_{\Sigma_0}\|_1 = \lim_n \|\check{\varphi}_n\|_1 \leqslant \|g\|_1 = 1$. Thus $\|\xi_{\Sigma_0}\|_1 = 1$. Now, let

$$f = \nu(\Sigma_0)^{-1}\xi_{\Sigma_0} = \nu(\Sigma_0)^{-1}\sum_{\sigma \in \Sigma_0} d_\sigma \chi_\sigma.$$

Then $\|f\|_2 = \nu(\Sigma_0)^{-1/2}$ and $\|f\|_\infty = 1$. In other words, we have

$$\int_G |f(x)|^2\, d\lambda(x) = \int_G |f(x)|\, d\lambda(x)$$

and $|f(x)| \leqslant 1$ for $x \in G$. Since f is continuous we see that $|f|$ admits only values 0 and 1. Since $f = \nu(\Sigma_0)^{-1}\sum_{\sigma \in \Sigma_0} d_\sigma\chi_\sigma$ with $1 \in \Sigma_0$ we conclude that f admits no values other than 0 and 1. Let $G_0 = \{x \in G \mid U_x^\sigma = I_{d_\sigma} \text{ for } \sigma \in \Sigma_0\}$. Then G_0 is a normal subgroup of G and $f\,d\lambda$ is the normalized Haar measure on G_0. Since $\|f\|_1 = \nu(\Sigma_0)^{-1}$ we see that $H = G/G_0$ is a finite group

([3]) It is essential for the following estimates that n is sufficiently large and this leads to the assumption $\varepsilon < 1$. Note also that if $\varepsilon \geqslant 1$ the theorem still holds.

of order $\nu(\Sigma_0)$. For every $\sigma \in \Sigma_0$, U^σ is constant on cosets of G_0 and thus σ can be identified with a representation of H. Moreover, with this identification, $\xi_{\Sigma_0} = \nu(\Sigma_0)f$ is the character of the regular representation of H. Thus Σ_0 is isomorphic to the dual object of H. Hence Σ_0 is a subhypergroup of Σ.

3° It follows from 2° that for some compact groups Theorem 2.2 cannot be extended to the case $k = 1$. In fact, we have:

(3.1) Proposition. *If a compact group G satisfies the assertion of Theorem 2.2 with $k = 1$ then G is 0-dimensional.*

4° Except for the Abelian case (see [1]) no characterization of such groups seems to be known.

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