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C_p-Estimates for certain kernels on local fields

by

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Abstract. We give necessary and sufficient conditions for certain operators defined on $L^2(\boldsymbol{K})$ ($\boldsymbol{K}$ a local field) to belong to Schatten–von Neumann ideals. The operators considered are defined by a type of integral kernels.

1. The purpose of this paper is to extend to the case of a local field $\boldsymbol{K}$ the results proved in [9] for the real case. They concern necessary and sufficient conditions for certain kernels to give rise to operators (on $L^2(\boldsymbol{K})$) belonging to Schatten–von Neumann classes (for the theory of Schatten–von Neumann classes, see, for instance, [1]). Though the main ideas are the same as in [9], their actual application needs several adaptations to the new context.

In order to present the results, we have first to establish the notation and to remind some facts from the theory of local fields; the basic reference for this topic is [7].

Let $\boldsymbol{K}$ be a local field, that is, a locally compact, nondiscrete, totally disconnected field with the valuation $|\ |$. We write $\mathfrak{O} = \{x \in \boldsymbol{K}, |x| \leqslant 1\}$, $\mathfrak{O}^* = \{x \in \boldsymbol{K}, |x| = 1\}$, $\mathfrak{P} = \{x \in \boldsymbol{K}, |x| < 1\}$. It is known that there exists $\mathfrak{p} \in \mathfrak{P}$ such that $\mathfrak{P} = \mathfrak{p}\mathfrak{O}$ (this $\mathfrak{p}$ will be fixed in the sequel). The residue space $\mathfrak{O}/\mathfrak{P}$ is a finite field; let Q be a complete set of representatives for it. If $\operatorname{card} Q = q$, then the image of $\boldsymbol{K}^*$ in $(0, \infty)$ under the valuation $|\ |$ is the multiplicative subgroup of $(0, \infty)$ generated by q; also $|\mathfrak{p}| = q^{-1}$. We have $\mathfrak{P}^k = \{x \in \boldsymbol{K}, |x| \leqslant q^{-k}\}$, and we will write $S_k = \{x \in \boldsymbol{K}, |x| = q^{-k}\}$; Φ_k will be the characteristic function of $\mathfrak{P}^k$. $\mathscr{S} = \mathscr{S}(\boldsymbol{K})$ will denote the space of finite linear combinations of characteristic functions of balls.

The Fourier transform on $\boldsymbol{K}$ is defined as follows: let χ be a fixed character on $\boldsymbol{K}$ that is trivial on $\mathfrak{O}$ but is nontrivial on $\mathfrak{P}^{-1}$. Then, for $f \in L^1(\boldsymbol{K})$,

$$\hat{f}(x) = \int_{\boldsymbol{K}} f(\xi)\,\overline{\chi(x\xi)}\,d\xi.$$

The standard properties of the Fourier transform can be found in [7, Chap. II].

Caution: the sign $|\ |$ is used to denote the valuation on $\boldsymbol{K}$ as well as the modulus of a complex number.

The author thanks Dan Voiculescu for many useful discussions.

2. In the sequel we shall recapture the main results of [9]. We shall consider operators given by kernels of the form

$$A(x, y)\,\hat{\varphi}(x-y) \tag{1}$$

where the main condition imposed on the continuous function A is its p-homogeneity:

$$A(\mathfrak{p}x, \mathfrak{p}y) = A(x, y), \qquad \forall\, x, y \in \boldsymbol{K}. \tag{2}$$

Further restrictions on A will be stated when necessary.

Let us also introduce the equivalent of Besov spaces on $\boldsymbol{K}$. If $R_k(x) = q^{-k}\Phi_{-k}(x)$, then $\hat{R}_k = \Phi_k$; we define, for $p, q \geqslant 1$, $s \in \boldsymbol{R}$,

$$\dot{B}^s_{pq} = \dot{B}^s_{pq}(\boldsymbol{K}) = \{f \in \mathscr{S}',\ \{q^{-sk}\|f * (R_k - R_{k-1})\|_p\}_{k\in\boldsymbol{Z}} \in l^q\}.$$

These spaces appear in [7] (actually, we use their "homogeneous" version).

Now, let $T(A, \varphi)$ be the operator whose kernel is $A(x, y)\,\hat{\varphi}(x-y)$, and $T(A)$ the operator whose kernel is $A(x, y)$. The supplementary conditions on A which will appear below will tend towards establishing, for a fixed A, the equivalence "$T(A, \varphi) \in C_p$ if and only if $\varphi \in \dot{B}^{1/p}_{pp}$".

LEMMA 1. *Let $E = \{(x, y) \in \boldsymbol{K} \times \boldsymbol{K},\ |x-y| = 1\}$, and let $\chi(x, y)$ be the characteristic function of E. Define*

$$a_1(A) = \|T(\chi A)\|_{C_1}.$$

Then

$$\|T(A, \varphi)\|_{C_1} \leqslant C a_1(A) \|\varphi\|_{\dot{B}^1_{11}}.$$

The proof is similar to that of Lemma 1 in [9]. The functions ψ_k are given simply by $\psi_k = \Phi_k - \Phi_{k-1}$.

Also, with a proof similar to that of Lemma 2 in [9], we obtain

LEMMA 2. *Define*

$$a_2(A)^2 = \sup_{|t|=1} \int_{\boldsymbol{K}} |A(y+t, y)|^2\, dy.$$

Then

$$\|T(A, \varphi)\|_{C_2} \leqslant C a_2(A) \|\varphi\|_{\dot{B}^{1/2}_{22}}.$$

In order to apply Lemmas 1 and 2, we state the following result, which can be proved by methods analogous to those in [1, XI, 9].

PROPOSITION A. *Suppose $\xi, \eta \in \boldsymbol{K}$, $K(x, y)$ is a kernel defined on $(\xi + \mathfrak{O}) \times (\eta + \mathfrak{O})$, and $T(K)$ is the operator corresponding to K; suppose also $\alpha > \frac{1}{2}$.*

Define

$$|||K||| = \max\{\sup_{x,y} |K(x, y)|,\ \sup_{x,y,h} |h|^{-\alpha} |K(x, y+h) - K(x, y)|\}.$$

Then, if $|||K||| < \infty$, $T(K) \in C_1$, and

$$\|T(K)\|_{C_1} \leqslant C\,|||K|||.$$

We may now state

THEOREM 1. *Suppose A is continuous on $(\boldsymbol{K} \times \boldsymbol{K}) \setminus \{0\}$, satisfies (2) and, moreover, on $B = \{(x, y) \in \boldsymbol{K} \times \boldsymbol{K},\ \max\{|x|, |y|\} = 1\}$ we have*

(i) $|A(x, y)| \leqslant C|x-y|^\alpha$,

(ii) $|A(x, y+h) - A(x, y)| \leqslant C|h|^\alpha$,

where $\alpha > 1/p$, $1 \leqslant p \leqslant 2$.

Then there is a constant C (depending on A and p) such that

$$\|T(A, \varphi)\|_{C_p} \leqslant C\|\varphi\|_{\dot{B}^{1/p}_{pp}}.$$

Proof. We shall suppose $1 < p < 2$ (otherwise the proof is simpler). Consider the analytic family of kernels

$$A_\lambda(x, y) = \left[\frac{|x-y|}{\max\{|x|, |y|\}}\right]^{\lambda-\alpha} A(x, y) \tag{3}$$

defined for $\alpha p/2 \leqslant \operatorname{Re}\lambda \leqslant \alpha p$.

We will use interpolation between $\operatorname{Re}\lambda = \alpha p/2$ and $\operatorname{Re}\lambda = \alpha p$.

For $\operatorname{Re}\lambda = \alpha p/2$, the estimate is rather straightforward. If $|y| = q^k > 1$, and $|t| = 1$, we have, using condition (i) in the theorem,

$$\begin{aligned} |A_\lambda(y+t, y)| &= |y|^{-\alpha(p/2-1)} |A(y+t, y)| = q^{k\alpha(1-p/2)} |A(\mathfrak{p}^k(y+t), \mathfrak{p}^k y)| \\ &\leqslant C q^{k\alpha(1-p/2)} q^{-k\alpha} = C q^{-k/2} q^{-k(\alpha p-1)/2}. \end{aligned}$$

Therefore, for $|t| = 1$,

$$\begin{aligned} \int_{\boldsymbol{K}} |A_\lambda(y+t, y)|^2\, dy &= C + \int_{|y|>1} |A_\lambda(y+t, y)|^2\, dy = C + \sum_{k\geqslant 1} \int_{|y|=q^k} |A_\lambda(y+t, y)|^2\, dy \\ &\leqslant C\Big(1 + \sum_{k\geqslant 1} q^k q^{-k} q^{-k(\alpha p-1)}\Big) < \infty \end{aligned}$$

whence $a_2(A_\lambda) < \infty$.

Suppose now $\operatorname{Re}\lambda = \alpha p$. We have to estimate $a_1(A_\lambda) = \|T(\chi A_\lambda)\|_{C_1}$, where χ is the characteristic function of the set $E \subset \boldsymbol{K} \times \boldsymbol{K}$, $E = \{(x, y),\ |x-y| = 1\}$.

We shall need some preliminary notation. Suppose $x \in \boldsymbol{K}$. Then $x = \sum_{k=k_0}^{\infty} \mathfrak{p}^k x_k$, where $x_k \in Q$; that is, x_k belongs to a set of q elements. Moreover, in this case $|x| = q^{-k_0}$. Define a function $\iota\colon \boldsymbol{K} \to \boldsymbol{K}$ by putting $\iota(x)$

$= \bar{x}$, where $\bar{x} = \sum_{k=k_0}^{-1} \mathfrak{p}^k x_k$ (in case $k_0 > -1$, we put $\bar{x} = 0$). Then $\imath(\boldsymbol{K})$ is a denumerable set of elements. Define $I_0 = \{0\}$ and, for $k \geqslant 1$, $I_k = \{x \in \imath(\boldsymbol{K}),\ |x| = q^k\}$.

Then $\imath(\boldsymbol{K}) = \bigcup_{k=0}^{\infty} I_k$, and the number of elements in I_k is less than q^k. Also, $|x-y| = 1$ is equivalent to $\imath(x) = \imath(y)$ and $\imath(\mathfrak{p}^{-1}x) \neq \imath(\mathfrak{p}^{-1}y)$. If $B(x, \varepsilon)$ denotes the (closed) ball of center x and radius ε, then, for $\xi \in \imath(\boldsymbol{K})$, we have

$$B(\xi, 1)\ (= \xi + \mathfrak{D}) = \bigcup_{\zeta \in Q} B(\xi + \mathfrak{p}\zeta, q^{-1}),$$

and we may write the set E as a disjoint union

$$E = \bigcup_{\xi \in \imath(\boldsymbol{K})} \bigcup_{\substack{\zeta,\eta \in Q \\ \zeta \neq \eta}} B(\xi + \mathfrak{p}\zeta, q^{-1}) \times B(\xi + \mathfrak{p}\eta, q^{-1})$$

$$= \bigcup_{k=0}^{\infty} \bigcup_{\xi \in I_k} \bigcup_{\substack{\zeta,\eta \in Q \\ \zeta \neq \eta}} B(\xi + \mathfrak{p}\zeta, q^{-1}) \times B(\xi + \mathfrak{p}\eta, q^{-1}).$$

Write

$$E_{\xi;\zeta,\eta} = B(\xi + \mathfrak{p}\zeta, q^{-1}) \times B(\xi + \mathfrak{p}\eta, q^{-1}), \qquad \chi_{\xi;\zeta,\eta} = \chi_{E_{\xi;\zeta,\eta}},$$

and let us estimate $\|T(\chi_{\xi;\zeta,\eta} A_\lambda)\|$ (the estimate below is actually valid for $k \geqslant 1$; I_0 can be treated similarly).

Recall that A_λ is given by formula (3); however, when $(x, y) \in E_{\xi;\zeta,\eta}$, we have $|x-y| = 1$ and $\max\{|x|, |y|\} = q^k$.

Also, by homogeneity (relation (2)), we have

$$|A(x, y)| = |A(\mathfrak{p}^k x, \mathfrak{p}^k y)| \leqslant C|\mathfrak{p}^k x - \mathfrak{p}^k y| = Cq^{-k\alpha}$$

and, for $|h| < 1$,

$$|h|^{-\alpha}|A(x, y+h) - A(x, y)| = |h|^{-\alpha}|A(\mathfrak{p}^k x, \mathfrak{p}^k y + \mathfrak{p}^k h) - A(\mathfrak{p}^k x, \mathfrak{p}^k y)|$$
$$\leqslant C|h|^{-\alpha} q^{-k\alpha}|h|^{\alpha} = Cq^{-k\alpha}$$

(since $(x, y) \in E_{\xi;\zeta,\eta}$ implies $|\mathfrak{p}^k x| = |\mathfrak{p}^k y| = 1$, we have applied (i) and (ii) in the hypothesis).

By Proposition A, we have

$$\|T(\chi_{\xi;\zeta,\eta} A_\lambda\|_{C_1} \leqslant Cq^{-k\alpha}$$

and therefore

$$\|T(A_\lambda)\|_{C_1} \leqslant C \sum_{k=0}^{\infty} \sum_{\xi \in I_k} \sum_{\substack{\zeta,\eta \in Q \\ \zeta \neq \eta}} q^{-k\alpha(p-1)} q^{-k\alpha}$$

$$\leqslant C \sum_{k=0}^{\infty} q^k q^{-k\alpha p} < \infty$$

(by the condition $\alpha > 1/p$).

To end the proof, consider, for $\alpha p/2 \leqslant \operatorname{Re}\lambda \leqslant \alpha p$, the analytic family of operators $\mathscr{T}_\lambda$, which associate to the function φ the operator $T(A_\lambda, \varphi)$. Then, for $\operatorname{Re}\lambda = \alpha p/2$, $\mathscr{T}_\lambda$ maps $\dot{B}^{1/2}_{22}$ into C_2, while, for $\operatorname{Re}\lambda = \alpha p$, it maps $\dot{B}^1_{11}$ into C_1. The desired conclusion follows by interpolation.

3. We shall now treat the reverse problem. We rely on the following lemma, whose proof is similar to that of Corollary 1 in [9].

LEMMA 3. *Let A be some locally integrable kernel on $\boldsymbol{K} \times \boldsymbol{K}$; suppose $\alpha, \alpha' \in \mathscr{S}(\boldsymbol{K})$. Define the function a by its Fourier transform:*

$$\hat{a}(u) = \alpha'(u) \int_{\boldsymbol{K}} A(x+u, x)\alpha(x)\,dx.$$

Suppose $\operatorname{supp}\alpha$, $\operatorname{supp}\alpha + \operatorname{supp}\alpha' \subset \{x \in \boldsymbol{K}, |x| \leqslant R\}$, *and denote by P, P' the projections onto $L^2(\operatorname{supp}\alpha)$, $L^2(\operatorname{supp}\alpha + \operatorname{supp}\alpha')$, respectively. Then, for $1 \leqslant p \leqslant \infty$,*

$$\|a\|_{L^p} \leqslant CR^{1-1/p}\|P'T(A)P\|_{C_p}$$

where C depends on the multiplier norm (in L^1) of α' and on the uniform norm of α.

The following proposition, which we will use in the sequel, is an immediate consequence of [5, 3.Corollary 2].

PROPOSITION B. *If $m \in \dot{B}^{1/2}_{21}(\boldsymbol{K})$, then $\hat{m} \in L^1(\boldsymbol{K})$; therefore m is a multiplier in all $L^p(\boldsymbol{K})$, $1 \leqslant p \leqslant \infty$.*

We may now state the theorem.

THEOREM 2. *Let A be continuous on $\boldsymbol{K} \times \boldsymbol{K} \setminus \{0\}$, satisfying* (2). *Suppose that:*

(i) *There is $\alpha > \frac{1}{2}$ such that*

$$|A(x+h, y) - A(x, y)| \leqslant C|y|^{\alpha} \quad \text{for } |x| = |y| = 1,\ |h| < 1.$$

(ii) *For any $u \in K \setminus \{0\}$, there exists $x \in \boldsymbol{K}$ such that $A(x+u, x) \neq 0$.*

Then

$$\|\varphi\|_{\dot{B}^{1/p}_{pp}} \leqslant C\|T(A, \varphi)\|_{C_p} \quad \text{for } 1 \leqslant p \leqslant \infty.$$

Proof. Let $\{\Omega_j\}$ be a finite open cover of $\mathfrak{O}^*$ (we may choose it to be open also in $\boldsymbol{K}$), D_j open sets in $\boldsymbol{K} \setminus \{0\}$, $\varkappa_j \in \boldsymbol{C}$, $|\varkappa_j| = 1$, such that $\operatorname{Re}(\varkappa_j A(x+u, x)) > 0$ for $u \in \Omega_j$, $x \in D_j$. Take $\Omega'_j \Subset \Omega_j$ such that $\{\Omega'_j\}$ is still an open cover of $\mathfrak{O}^*$, and choose positive functions α_j, $\alpha'_j \in \mathscr{S}(\boldsymbol{K})$ such that:

(i) $\alpha'_j(u) > 0$ for $u \in \Omega'_j$.

(ii) $\operatorname{supp}\alpha'_j \subset \Omega_j$, $\operatorname{supp}\alpha_j \subset D_j$.

(The possibility of this construction follows from (ii) in the statement of the theorem.)

Define now the functions b_{jk} by

$$\hat{b}_{jk}(u) = \hat{\varphi}(u)\alpha'_j(\mathfrak{p}^{-k}u) \int_{\boldsymbol{K}} A(x+u, x)\alpha_j(\mathfrak{p}^{-k}x)\,dx.$$

If P'_{jk}, P_{jk} are the corresponding projections, then we have, by Lemma 3,

$$\|b_{jk}\|_{L^p} \leqslant Cq^{-k(1-1/p)}\|P'_{jk}\,T(A,\,\varphi)\,P_{jk}\|_{\mathcal{C}_p}. \tag{5}$$

(Note that C depends on the multiplier norm of α'_j, and can therefore be chosen independently of k and j).

Write

$$\theta_{jk}(u) = \alpha'_j(\mathfrak{p}^{-k}u)\int_K A(x+u,\,x)\,\alpha_j(\mathfrak{p}^{-k}x)\,dx.$$

A change of variables yields

$$\theta_{jk}(u) = q^{-k}\theta_{j0}(\mathfrak{p}^{-k}u).$$

Now, define ψ, $\hat{\psi}_k$ by $\hat{\psi} = \sum_j \varkappa_j\theta_{j0}$, $\hat{\psi}_k(u) = \hat{\psi}(\mathfrak{p}^{-k}u)$. Note that $\operatorname{supp}\theta_{j0}$, $\operatorname{supp}\hat{\psi} \subset \mathfrak{O}^*$. Also, for $|u| = 1$, $|v| < 1$, we have, by condition (i),

$$|\theta_{j0}(u+v) - \theta_{j0}(u)| \leqslant C|v|^\alpha$$

and therefore

$$|\hat{\psi}(u+v) - \hat{\psi}(u)| \leqslant C|v|^\alpha.$$

But we have $\operatorname{Re}\hat{\psi} > 0$ on $\mathfrak{O}^*$, and therefore $1/\hat{\psi}$ satisfies a similar estimate on $\mathfrak{O}^*$:

$$|(1/\hat{\psi})(u+v) - (1/\hat{\psi})(u)| \leqslant C|v|^\alpha.$$

It follows easily, since $1/\hat{\psi}$ is supported on $\mathfrak{O}^*$, that it belongs to $\dot{B}^{1/2}_{21}(K)$, and we may apply Proposition B to conclude that it is a multiplier on any $L^p(K)$.

From (5) we obtain

$$q^{-k}\|\varphi * \psi_k\|_{L^p} \leqslant \sum_j \|b_{jk}\|_{L^p} \leqslant Cq^{-k(1-1/p)}\Big(\sum_j \|P'_{jk}\,T(A,\,\varphi)\,P_{jk}\|_{\mathcal{C}_p}\Big)$$

whence

$$q^{-k/p}\|\varphi * \psi_k\|_{L^p} \leqslant C\Big(\sum_j \|P'_{jk}\,T(A,\,\varphi)\,P_{jk}\|^p_{\mathcal{C}_p}\Big)^{1/p}.$$

Therefore

$$\sum_{k\in\mathbf{Z}} q^{-kd}\|\varphi * \psi_k\|^p_{L^p} \leqslant C\sum_{k\in\mathbf{Z}}\sum_j \|P'_{jk}\,T(A,\,\varphi)\,P_{jk}\|^p_{\mathcal{C}_p} \leqslant C\|T(A,\,\varphi)\|^p_{\mathcal{C}_p}, \tag{6}$$

since P'_{jk_1}, P'_{jk_2} and P_{jk_1}, P_{jk_2} are disjoint for $|k_1 - k_2|$ sufficiently large (depending only on the sets D_j).

But, since $1/\hat{\psi}_k$ are multipliers of uniformly bounded norm in L^p, it follows that

$$\|\varphi * \psi_k\|_{L^p} \geqslant C\|\varphi * (R_k - R_{k-1})\|_{L^p}$$

and, by (6),

$$\|\varphi\|_{\dot{B}^{1/p}_{pp}} \leqslant C\|T(A,\,\varphi)\|_{\mathcal{C}_p}.$$

4. Final remarks. 1. The results presented apply in particular to commutators of multiplication operators with singular integral operators of the type considered in [7, VI.4] (this is the analogue for local fields of the commutators considered in [4], [6]).

2. The results of [9] are proved for operators on $L^2(\boldsymbol{R}^n)$; thus one might think of extending them to $L^2(\boldsymbol{K}^n)$. Actually, no generality is obtained in this way, since, by [8], $\boldsymbol{K}^n$ has a natural local field structure, with valuation given by $|x| = \max|x_i|^n$ $(x = (x_1, \ldots, x_n))$. The difference in homogeneity implies that in $\boldsymbol{K}^n$ we would have to consider the Besov space $\dot{B}^{n/p}_{pp}$, while the condition $\alpha > 1/p$ becomes $\alpha > n/p$. The author thanks the referee for this remark and for bringing [8] to his attention.

3. In a recent work ([3]), Janson and Peetre extend the results of [2] and of [9]; by combining their method with the proof of Theorem 1 above, the range of the latter may be extended to the whole $1 < p < \infty$.

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Subspace mixing properties of operators in R^n with applications to Gluskin spaces

by

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Abstract. There is a constant $c > 0$ such that for every $n \in N$ there is an n-dimensional Banach space X_n with the property that whenever G is a compact group of operators acting on X_n then

$$\sup\{\|T\|_{X_n}\colon\ T \in G\} \geqslant \frac{c(n - t(G))}{n^{1/2}\log^{3/2} n}$$

where $t(G) = \inf\{|\operatorname{tr} T|\colon\ T \in G\}$.

E. D. Gluskin in [3] introduced a class of random n-dimensional Banach spaces in order to prove that the Banach–Mazur diameter of the set of all n-dimensional Banach spaces is of order n. Later the same author in [4] and independently S. J. Szarek [8] used different variants of spaces defined in [3] to prove the existence of finite-dimensional Banach spaces with the "worst possible" basis constants. Another variant of Gluskin spaces was used by the author in [5] to construct finite-dimensional Banach spaces with the "worst possible" symmetry constants. The importance of the notion of "subspace mixing operators on R^n" in the context of Gluskin spaces was implicit in [4] and "almost explicit" in [5]. The final step in this direction was done by S. J. Szarek [9], who proved that a "vast majority" of Gluskin spaces enjoy the property that every subspace mixing operator on such a space has large norm. The subspace mixing property was used in that paper to prove the existence of finite-dimensional Banach spaces with two essentially different complex structures. Later on, the techniques developed in [9] were used by the same author to construct infinite-dimensional Banach spaces with some pathological properties [10], [11]; however, the credit for the first use of Gluskin spaces to construct pathological infinite-dimensional Banach spaces should be given to J. Bourgain [1].

In this paper we study subspace mixing properties of operators in R^n with special attention turned to operators which belong to a compact group of operators. The main difference in the approach between [9] and this paper lies in the fact that in [9] the author studied the subspace mixing property of an operator T in terms of certain "distances" of T to the line $\{\lambda \mathrm{Id}\}_{\lambda \in R}$ while