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S_g spaces and vanishing of the functor Ext

by

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Abstract. $S_g(a, r)$ spaces are introduced by V. V. Kashirin [4]. We derive some properties of this class of Köthe spaces and obtain necessary and sufficient conditions for an $S_g(a, 1)$ space E to satisfy $\mathrm{Ext}^1(E, E) = 0$ when E is of type d_1 or d_2. In [7, 8], $\lambda_\Phi(a)$ spaces are introduced and shown to be the only d_1 Köthe spaces which satisfy $\mathrm{Ext}^1(E, E) = 0$. We show that the class of $S_g(a, \infty)$ spaces coincides with the subclass of $\lambda_\Phi(a)$ spaces which consists of those $\lambda_\Phi(a)$ spaces where $\Phi = (\varphi, \varphi, \ldots)$, i.e., generated by a single function φ. V. V. Kashirin, in [4], asked whether every regular nuclear Köthe space E of type d_1 is representable as an $S_g(b, \infty)$ space, which was answered in the negative by M. Kocatepe (Alpseymen) [5]. We show that even under the additional assumption that $\mathrm{Ext}^1(E, E) = 0$ this is not possible.

Preliminaries. Unless otherwise stated, throughout this work, the letters E, F, ... etc. will denote nuclear Köthe spaces $K(a_{kn})$ which have a continuous norm and whose generating matrix (a_{kn}) satisfies $0 < a_{kn} \leqslant a_{k+1,n}$ $\forall k, n$. Following E. Dubinsky [1] we say that $E = K(a_{kn})$ is *of type* (d_i), $i = 0, 1, 2, 5$, if it is generated by a matrix (a_{kn}) which satisfies the corresponding condition below:

(d_0) For each k, $a_{k+1,n}/a_{kn}$ is nondecreasing in n (in this case E is also called *regular*).

(d_1) $\exists p \ \forall k \ \exists m \quad \sup_n a_{kn}^2/(a_{pn} a_{mn}) < +\infty$.

(d_2) $\forall k \ \exists m \ \forall r \quad \sup_n a_{kn} a_{rn}/a_{mn}^2 < +\infty$.

(d_5) $\exists M > 0 \ \forall k \ \forall n \quad a_{k+1,n}/a_{kn} \leqslant (a_{k+2,n}/a_{k+1,n})^M$.

An $L_f(a, r)$ *space*, also called a *Dragilev space*, is the Köthe space $E = K(\exp f(r_k a_n))$ where f is an increasing, odd, logarithmically convex function (i.e. $\ln f(\exp x)$ is convex for $x \geqslant 0$), $a = (a_n)$ is an exponent sequence (a nondecreasing sequence of positive real numbers which approaches infinity rapidly enough to make E nuclear), and (r_k) a strictly increasing sequence with limit $r \in \boldsymbol{R} \cup \{+\infty\}$. An $L_f(a, r)$ space is isomorphic to an L_f space of type -1 (resp. 1) if $r < 0$ (resp. $0 < r < +\infty$). Hence basically there are four types of L_f spaces: $r = -1, 0, 1, +\infty$. For any $A > 1$ the function $f(Ax)/f(x)$

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is increasing and either has a finite limit for all $A>1$ or approaches infinity for all $A>1$ as $x\to\infty$; accordingly, f is called *slowly increasing* resp. *rapidly increasing*. If f is slowly increasing then $L_f(a,r)$ is isomorphic to the *power series space of finite type* (which is d_2) $\Lambda_0(a)=L_{\mathrm{id}}(a,0)$ if $r<+\infty$ or to the *power series space of infinite type* (which is d_1) $\Lambda_\infty(a)=L_{\mathrm{id}}(a,\infty)$, where id denotes the identity function. If f is rapidly increasing, then for all $A>1$, $f^{-1}(Ax)/f^{-1}(x)$ decreases to 1, while $f(Ax)/f(x)$ and $f(x)/x$ increase to $+\infty$. In this case $L_f(a,r)$ is of type d_1 (resp. d_2) if $r=1,\infty$ (resp. $r=-1,0$). All Dragilev spaces are regular and independent of the choice of the sequence (r_k).

A regular Köthe space $E=K(a_{kn})$ is called *weakly stable* (resp. *unstable*) if the generating (regular) matrix (a_{kn}) satisfies

$$\forall s\ \exists p\ \forall q\ \exists r\quad \sup_n a_{qn}a_{s,n+1}/(a_{pn}a_{r,n+1})<+\infty$$

$$(\text{resp. } \exists s\ \forall p\ \exists q\ \forall r\quad \lim_n a_{p,n+1}a_{rn}/(a_{q,n+1}a_{sn})=0).$$

For concepts not defined and as general references we refer the reader to [1], [6], or [10].

1. S_g spaces. An $S_g(a,r)$ is defined similarly to an $L_f(a,r)$, but the logarithmic convexity of f is replaced by the convexity of g for $x\geq 0$. Below some properties of S_g spaces are stated. The proofs, being immediate consequences of the convexity of g, are omitted.

PROPOSITION 1.1. *Let $E=S_g(a,r)$. Then E is regular and:*

(i) *If $r<0$ then $E\cong S_g(b,-1)$ and E is d_2.*

(ii) *If $r=0$ then E is d_2.*

(iii) *If $0<r<+\infty$ then $E\cong S_g(b,1)$ and E is d_5.*

(iv) *If $r=+\infty$ then E is d_1.*

PROPOSITION 1.2. *Every $L_f(a,r)$ space is an $S_g(a,r)$ space for some g.*

Proof. The proofs for $r=1,\infty$ will be given. The cases $r=-1,0$ can be similarly proved.

Let $E=L_f(a,\infty)$; $r_k\uparrow+\infty$. Define $g(2^n)=f(2^n)$ and extend g to an odd function on $\boldsymbol{R}$ via linear segments; hence g is strictly increasing. Since

$$\big(g(2^{n+1})-g(2^n)\big)/2^n=\big(f(2^n)/2^n\big)\big(f(2^{n+1})/f(2^n)-1\big)$$

and each factor in the latter product is nondecreasing in n, it follows that g is also convex. Suppose $2^m\leq r_ka_n<2^{m+1}$. Then

$$f(r_ka_n)\leq f(2^{m+1})=g(2^{m+1})\leq g(2r_ka_n),$$

$$g(r_ka_n)\leq g(2^{m+1})=f(2^{m+1})\leq f(2r_ka_n)$$

so $L_f(a,\infty)=S_g(a,\infty)$ as sets and the topologies coincide.

Let $E=L_f(a,1)$. Since the case $f=$ identity is trivial, assume f increases rapidly. Let $r_k\uparrow 1$. Let $p_n=f^{-1}(2^n)$, define $g(p_n)=f(p_n)=2^n$ and extend g to an odd function on $\boldsymbol{R}$ via linear segments; hence g is strictly increasing. Then

$$\big(g(p_{n+1})-g(p_n)\big)/(p_{n+1}-p_n)=\big(2^n/f^{-1}(2^n)\big)/\big(f^{-1}(2^{n+1})/f^{-1}(2^n)-1\big)$$

and the latter quotient increases with n since f is rapidly increasing. We conclude that g is convex. Now suppose $p_m\leq r_ka_n<p_{m+1}$. Then for $r_l>r_k$ and for large n we have

$$g(r_ka_n)<g(p_{m+1})=2^{m+1}=2f(p_m)<2f(r_ka_n)\leq f(r_la_n),$$

$$f(r_ka_n)<f(p_{m+1})=g(p_{m+1})\leq g\left(\frac{p_{m+1}}{p_m}r_ka_n\right)\leq g(r_la_n)$$

where the last inequality follows from $\lim_m p_{m+1}/p_m=1$. Therefore $L_f(a,1)=S_g(a,1)$ as sets and the topologies coincide.

Remark 1.3. The above propositions show that S_g spaces generalize L_f spaces and moreover exhibit similar properties as L_f spaces regarding the types except for $S_g(a,1)$ spaces. V. V. Kashirin in [4] gives an example of an $S_g(a,\infty)$ space which is not isomorphic to any L_f space and concludes that S_g spaces form a strictly larger class. A close inspection reveals that $S_g(a,1)$ spaces may be of type d_1 or d_2 (in which case they are isomorphic to some power series space of finite type; see [1]), or even a cross product of a d_1 space by a d_2 space, which cannot occur for any L_f space.

An $L_f(a,1)$ space is d_1 iff f is rapidly increasing. The next lemma shows that if an $S_g(a,1)$ space is d_1 then g resembles a rapidly increasing function, but only at the points a_n so somewhat "locally".

LEMMA 1.4. *If $E=S_g(a,1)$ is d_1 and $0<r_k\uparrow 1$ then g satisfies:*

(i) $\forall M>0\ \forall k\ \exists p,n_0\quad M\leq g(r_pa_n)/g(r_ka_n)\quad \forall n\geq n_0$.

(ii) $\forall A>1\quad \lim_n g(a_n)/g(Aa_n)=\lim_n g(a_n/A)/g(a_n)=0$.

(iii) $g(x)/x\uparrow+\infty$.

Proof. The proofs for (ii) and (iii) will be omitted since they follow from (i). To prove (i) let E be d_1. Then we may assume that $2g(r_ka_n)\leq g(r_1a_n)+g(r_{k+1}a_n)$ $\forall k$ and for $n\geq n_k$. By successive applications we obtain

$$g(r_{k+1}a_n)\leq \tfrac12\big(g(r_1a_n)+g(r_{k+2}a_n)\big)\leq\ldots$$

$$\leq g(r_1a_n)\sum_{j=1}^m 2^{-j}+2^{-m}g(r_{k+m+1}a_n)$$

$$\leq g(r_1a_n)+2^{-m}g(r_{k+m+1}a_n)\qquad\text{for } n\geq n_{k+m}.$$

By the convexity of g and using the above estimate we have for $n\geq n_{k+m}$

$$r_{k+1}g(r_ka_n)/r_k\leq g(r_{k+1}a_n)\leq g(r_ka_n)+2^{-m}g(r_{k+m+1}a_n).$$

Therefore with $p = k+m+1$ where m is chosen large enough to satisfy $M \leqslant 2^m (r_{k+1}-r_k)/r_k$ and for $n_0 = n_p$ we obtain the result.

We next show that the classes of S_g spaces for $r = 1$, $r = \infty$ are not disjoint. In the next section this intersection will be completely characterized.

Proposition 1.5. *Any unstable $S_f(a, \infty)$ space E is diagonally isomorphic to some $S_g(b, 1)$ space.*

Proof. The unstability of E is equivalent to $\lim_n a_{n+1}/a_n = +\infty$. Let $d_n = \inf_{m \geqslant n} a_{m+1}/a_m$. Then $d_n \uparrow +\infty$. By modifying (a_n) if necessary assume $d_3 > 16$. Define r_k by $r_1 = 1/4$, $r_2 = 1/2$, $r_k = 1-d_k^{-1/2}$ for $k > 2$. Then $(1-r_k)^{-2} \leqslant d_k \leqslant a_{k+1}/a_k$, $k > 2$ (to avoid repetitions, modify (r_k) slightly so that the inequalities are still valid). Define (s_k) by $s_1 = 0$, $s_k = (1-r_k)^{-1}$ for $k > 1$. Then $s_k \uparrow +\infty$ strictly. We have:

(1) $(s_{k+1}-s_k)/(r_{k+1}-r_k)$ increases with k.

(2) $(s_{k+1}-s_k)/(r_{k+1}-r_k) \leqslant d_{k+1} \quad \forall k > 1$.

Let $b_1 = 1$, $b_{n+1} = 4r_n b_n$ for $n > 1$. Clearly $b_n \uparrow +\infty$. Define g successively at the points

$$\to r_2 b_n \to r_3 b_n \to \ldots \to r_n b_n = r_1 b_{n+1} \to r_2 b_{n+1} \to \ldots$$

by

$$g(r_k b_n) = \sum_{j=1}^{n-1} f(s_j a_j) + f(s_k a_n) \quad \text{for } k = 1, \ldots, n$$

and extend g to an odd function on $\boldsymbol{R}$ via linear segments. Clearly g is strictly increasing and $g(r_n b_n) = g(r_1 b_{n+1})$ since $s_1 = 0$. Let $m(x, y)$ denote the slope of the segment joining $(x, g(x))$ to $(y, g(y))$ for $x < y$. To show that g is convex we distinguish two cases:

Case 1: $k+1 < n$. Using (1) and the convexity of f we have

$$\begin{aligned}(r_{k+2}-r_{k+1})/(r_{k+1}-r_k) &\leqslant (s_{k+2}-s_{k+1})/(s_{k+1}-s_k)\\ &\leqslant (f(s_{k+2} a_n)-f(s_{k+1} a_n))/(f(s_{k+1} a_n)-f(s_k a_n))\end{aligned}$$

from which it follows that

$$m(r_k b_n, r_{k+1} b_n) \leqslant m(r_{k+1} b_n, r_{k+2} b_n).$$

Case 2: $k+1 = n$. We have

$$\frac{b_{n+1}}{b_n} \cdot \frac{r_2-r_1}{r_n-r_{n-1}} = \frac{r_n}{r_n-r_{n-1}} \leqslant \frac{d_n}{s_n-s_{n-1}} \leqslant \frac{a_{n+1}}{(s_n-s_{n-1})\, a_n}.$$

By the convexity of f, this is

$$\leqslant \frac{f(2a_{n+1})-f(a_{n+1})}{f(s_n a_n)-f(s_{n-1} a_n)} \leqslant \frac{f(s_2 a_{n+1})}{f(s_n a_n)-f(s_{n-1} a_n)} = \frac{g(r_2 b_{n+1})-g(r_1 b_{n+1})}{g(r_n b_n)-g(r_{n-1} b_n)}$$

from which $m(r_{n-1} b_n, r_n b_n) \leqslant m(r_n b_n, r_1 b_{n+1})$ follows.

Letting

$$t_n = \exp\Big[\sum_{j=1}^{n-1} f(s_j a_j)\Big]$$

we obtain $t_n \exp[f(s_k a_n)] = \exp[g(r_k b_n)]$ and hence conclude that $S_f(a, \infty) \cong S_g(b, 1)$ diagonally.

Kashirin in [4] has shown the following result:

Proposition 1.6 *Any unstable d_1 Köthe space is diagonally isomorphic to some $S_g(a, \infty)$ space.*

Let $K(a_{kn})$ be a Köthe space, and (n_i) a subsequence of N. The space $K(b_{kn})$ where $b_{kn} = a_{ki}$ for $n_i \leqslant n < n_{i+1}$ will be called a *repeated form* of $K(a_{kn})$. In the case of a $S_g(a, r)$ space this is equivalent to repeating terms of $a = (a_n)$, that is, $b_n = a_i$ if $n_i \leqslant n < n_{i+1}$, and $b = (b_n)$ will be called a repeated form of (a_n). Combining Proposition 1.6 with Proposition 1.5 we obtain

Proposition 1.7. *Any repeated form of an unstable d_1 Köthe space is diagonally isomorphic to some $S_g(a, 1)$ and $S_f(b, \infty)$ space.*

Proof. Since in Propositions 1.5 and 1.6 the isomorphisms are diagonal, repeating a coordinate a finite number of times does not disturb the isomorphism if we repeat the corresponding coordinate of (a_n) resp. (b_n) the same number of times. This, in turn, still gives us an S_g space of the same type.

2. Vanishing of the functor Ext **for S_g spaces.** For the definition of the functor $\operatorname{Ext}(E, F) = \operatorname{Ext}^1(E, F)$ on the category of Fréchet spaces we refer the reader to Vogt [12]. Consider the following conditions for two Köthe spaces $E = K(a_{kn})$, $F = K(b_{kn})$:

(S_1) $\exists p\ \forall u\ \exists k\ \forall m, K, R > 0\ \exists n, S > 0\ \forall i, j$

$$a_{mi}/b_{kj} \leqslant \max\{S a_{ni}/b_{Kj},\ a_{pi}/R b_{uj}\}.$$

(S) $\forall u\ \exists p, k\ \forall m, K\ \exists n, S > 0\ \forall i, j$

$$a_{mi}/b_{kj} \leqslant S \max\{a_{ni}/b_{Kj},\ a_{pi}/b_{uj}\}.$$

Write $(E, F) \in S_1$ resp. $(E, F) \in S$ if the corresponding condition is satisfied. To simplify notation we shall write $\operatorname{Ext}(E)$ for $\operatorname{Ext}(E, E)$. We state the following result of Krône–Vogt [8].

Theorem 2.1. *Let $E = K(a_{kn})$ be a Schwartz space satisfying $a_{kn} > 0$ $\forall k, n$. Then the following conditions are equivalent*:

(i) $(E, E) \in S$.

(ii) $\operatorname{Ext}(E) = 0$.

(iii) *Every exact sequence* $0 \to E \to F \to E \to 0$, *where F is a Fréchet space, splits.*

If E is further d_1 then these conditions are also equivalent to [8]:

(iv) $(E, E) \in S_1$.

Combining Theorem 2.1 and modified forms of some results of M. Kocatepe [5] resp. J. Hebbecker [3] with Proposition 1.7 we obtain the main result of this section:

THEOREM 2.2. *Let $E = S_g(a, 1)$ be d_1. Then the following conditions are equivalent*:

(i) $\mathrm{Ext}(E) = 0$.

(ii) $E \cong S_f(b, \infty)$.

(iii) $E \cong S_g(c, 1)$ *where c is a repeated form of some d with* $\liminf d_{n+1}/d_n > 1$.

Proof. (iii) is equivalent to the condition that 1 is an isolated point among the limit points of $\{d_n/d_m\colon n, m \in N\}$, hence in view of isomorphism also of $\{a_n/a_m\colon n, m \in N\}$. For a discussion of the limit points of (a_n/a_m) see [2]. In [3], Satz 2.6, this condition is shown to be equivalent to (i) for $L_f(a, 1)$ spaces, but in view of Lemma 1.4(i) this result generalizes to d_1 $S_g(a, 1)$ spaces. (i) $\Leftrightarrow$ (iii) can also be obtained from Proposition 2 in [5] with obvious modifications for d_1 $S_g(a, 1)$ spaces. (iii) $\Rightarrow$ (ii) is Proposition 1.7, and finally (ii) $\Rightarrow$ (i) follows from the corresponding results for L_f spaces in [5] or [7] by using Lemma 1.4.

COROLLARY 2.3. *Let $E = S_g(a, \infty)$. Then $E \cong S_f(b, 1)$ for some b and f iff $S_g(a, \infty) = S_g(c, \infty)$ where c is a repeated form of some d with $\lim d_{n+1}/d_n = +\infty$.*

Proof. If $E \cong S_f(b, 1)$ then $\mathrm{Ext}(S_f(b, 1)) = 0$ and by Theorem 2.2(iii) it follows that $S_f(b, 1)$ is a repeated form of some unstable $S_f(c, 1)$. Then the same is true for E. The converse implication is Proposition 1.7.

COROLLARY 2.4. *The intersection class (up to isomorphism) of S_g spaces of type 1 resp. ∞ consists precisely of repeated forms of unstable d_1 Köthe spaces.*

Proof. This follows by Proposition 1.7 and Theorem 2.2.

The corresponding results in the case of a d_2 $S_g(a, 1)$ space are already known (see Nyberg [9] and in particular Hebbecker [3] for a complete treatment); in fact, a d_2 $S_g(a, 1)$ space, being also of type d_5 by Proposition 1.1(iii), is isomorphic to some $\Lambda_1(b)$ (see [1]). We cite Satz 1.11 in [3] in the language of Theorem 2.2:

THEOREM 2.5. $\mathrm{Ext}(\Lambda_1(a)) = 0$ *iff $a = (a_n)$ is equivalent to a repeated form of some unstable exponent sequence.*

3. λ_Φ spaces and S_g spaces. The $\lambda_\Phi(a)$ spaces are introduced in [7].

DEFINITION. Let $\Phi = \{\varphi_k\}$ be a sequence of positive increasing functions satisfying for all $x > 0$ and k

$$\varphi_{k+1}(x) \geqslant \varphi_k(x) \geqslant \varphi_1(x) \geqslant x^2.$$

Then the Köthe space $K(a_{kn})$ with $a_{1n} = \varphi_1(a_n)$, $a_{k+1,n} = \varphi_{k+1}(a_{kn})$, $k \geqslant 1$, where $a = (a_n)$ is some exponent sequence, is denoted by $\lambda_\Phi(a)$.

Clearly a $\lambda_\Phi(a)$ space is of type d_1 and moreover, by replacing $\varphi_k(x)$ by $x\varphi_k(x)$ if necessary, one can assume that $\lambda_\Phi(a)$ is regular. Krone in [7] proves the following theorem:

THEOREM 3.1. *Let E be a d_1 Schwartz Köthe space. Then* $\mathrm{Ext}(E) = 0$ *iff $E \cong \lambda_\Phi(a)$ for some Φ and a.*

We next show that λ_Φ and S_g spaces of infinite type are closely related.

THEOREM 3.2. *The class of $S_g(a, \infty)$ spaces coincides with the subclass of $\lambda_\Phi(b)$ spaces where Φ consists of a single function, i.e. $\Phi = (\varphi, \varphi, \ldots)$.*

Proof. Let $E = \lambda_\Phi(a)$ with $\Phi = \{\varphi, \varphi, \ldots\}$. Without loss of generality assume that $\varphi(x) \geqslant x^4$ $\forall x$ and let $\varphi^{(k)}(x)$ denote $\varphi \circ \varphi \circ \ldots \circ \varphi(x)$ (k-fold composition). Let $a_{kn} = \varphi^{(k)}(a_n)$. Define (p_m) by $p_0 = 2$, $p_{m+1} = \varphi(p_m)$ for $m \geqslant 0$. Then (p_m) increases strictly. Let $b_n = 2^m$ if $p_m < a_n \leqslant p_{m+1}$. Finally define $g(2^n) = \log p_n$, extend via linear segments, and complete to an odd function. Then g increases strictly. To show that g is convex we consider

$$(g(2^{n+1}) - g(2^n))/(2^{n+1} - 2^n) = 2^{-n} \log(p_{n+1}/p_n).$$

Since

$$(p_{n+1}/p_n)^2 = (\varphi(p_n)/p_n)^2 < \varphi(p_n)^2 \leqslant \varphi \circ \varphi(p_n)/\varphi(p_n) = p_{n+2}/p_{n+1}$$

we see that the right-hand side of the considered equation increases with n, and hence g is convex.

Now suppose $p_m < a_n \leqslant p_{m+1}$ (so in particular $b_n = 2^m$). Then

$$\begin{aligned} a_{k-1,n} &= \varphi^{(k-1)}(a_n) \leqslant \varphi^{(k-1)}(p_{m+1}) = p_{m+k} \\ &= \exp[g(2^{m+k})] = \exp[g(2^k b_n)] \end{aligned}$$

and

$$\exp[g(2^k b_n)] = p_{m+k} = \varphi^{(k)}(p_m) < \varphi^{(k)}(a_n) = a_{kn}.$$

We conclude that $E \cong S_g(b, \infty)$.

Conversely, given $F = S_g(b, \infty)$, define for $x > 0$, $\varphi(x) = \exp[f(2f^{-1}(\log x))]$. Then it easily follows that $F \cong \lambda_\Phi(a)$ where $a_n = \exp[f(b_n)]$.

Remark 3.3. It now follows easily that the popular Köthe space $K(a_{kn})$ where $a_{kn} = \exp \circ \exp \circ \ldots \circ \exp(a_n)$ (k-fold composition) is not only an $S_g(b, \infty)$ space but also an $L_f(c, \infty)$ space. Here it is natural to ask whether every $\lambda_\Phi(a)$ space is isomorphic to some $S_g(b, \infty)$ space. Our next example shows that this is not so and hence that even if a d_1 Köthe space E satisfies $\mathrm{Ext}(E) = 0$, it still does not follow that $E \cong S_g(b, \infty)$. This gives a sharper negative answer to a question of Kashirin [4] whether every regular d_1

Köthe space is isomorphic to some $S_g(b, \infty)$ space, than that given by M. Kocatepe (Alpseymen) [5].

Example 3.4. In this example we construct a $\lambda_\Phi(a)$ space E which is not isomorphic to any $S_g(b, \infty)$ space. Here we would like to thank the referee for suggesting this proof which is considerably shorter than the original one. Let $\Phi = (\varphi_k)$ where $x^2 \leqslant \varphi_1(x) \leqslant \varphi_2(x) \leqslant \ldots$ and

$$\lim_{x\to\infty} \varphi_k^{(m)}(x)/\varphi_{k+1}(x) = 0 \quad \text{for all } k, m$$

where $\varphi^{(m)}$ denotes the m-fold composition. Let (a_n) satisfy $a_n \leqslant a_{n-1}^2$ (e.g. $a_n = 2^{2^n}$), and put $E = \lambda_\Phi(a)$. Suppose $E \cong S_g(b, \infty)$ for some b and g. In view of Theorem 3.2 this means $E \cong F = \lambda_\Psi(b)$ for some $\Psi = (\psi, \psi, \ldots)$. Now $E \cong F$ implies that the diametral dimensions $\Delta(E)$ and $\Delta(F)$ are equal, and hence E and F, both being G_∞ spaces (see [11]), coincide as sets. Therefore we may assume that $a_n \leqslant \psi(b_n)$ and $\psi^{(2)}(b_n) \leqslant \varphi(a_n)$ for large n (in particular $b_n \leqslant \varphi(a_n)$) where $\varphi = \varphi_{k_0} \circ \ldots \circ \varphi_1$ for some suitable k_0. We obtain:

1) Given x, choose n such that $a_{n-1} \leqslant x \leqslant a_n$; hence $x \leqslant a_n \leqslant a_{n-1}^2 \leqslant \varphi(a_{n-1}) \leqslant \varphi(x)$ and

$$\psi(x) \leqslant \psi(a_n) \leqslant \psi^{(2)}(b_n) \leqslant \varphi(a_n) \leqslant \varphi(a_{n-1}^2) \leqslant \varphi(x^2) \leqslant \varphi^{(2)}(x).$$

2) $$\varphi_{k_0+1}(b_n) \leqslant \varphi_{k_0+1}(\varphi(a_n)) \leqslant \psi^{(m)}(b_n) \leqslant \varphi^{(2m)}(b_n) \leqslant \varphi_{k_0}^{(2mk_0)}(b_n)$$

for some suitable m which contradicts the growth assumption on the φ_k's.

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