STUDIA MATHEMATICA, T. LXXXIV (1986)

Addendum to the paper "On isomorphism of anisotropic Sobolev spaces with «classical Banach spaces» and a Sobolev type embedding theorem"

by

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After paper [1] had been accepted for publication we observed that the Sobolev type embedding theorem 4.1 can be improved by removing the additional hypothesis that one of the numbers n and m is even. First, we have

PROPOSITION 3.1a. *Let $n \geq 2$ and $m \geq 2$ be integers and let α and β be nonnegative numbers such that*

$$\alpha n^{-1} + \beta m^{-1} = 1 - n^{-1} - m^{-1}. \tag{3.1}$$

Then for each of the four derivatives $D(\alpha, \beta)$ there exist a locally absolutely integrable function $E\colon \mathbf{R}^2 \to \mathbf{C}$, an $f \in L^\infty(\mathbf{R}^2)$ and an absolute positive constant $C_1 = C_1(\alpha, \beta, n, m)$ such that

$$E * (D_x^n + \varepsilon D_y^m) u = D(\alpha, \beta) u \quad \text{for every } u \in C_0^\infty(\mathbf{R}^2), \tag{3.4}$$

$$E = C_1 \log(|x|^m + |y|^n) + f \tag{3.5}$$

with $\varepsilon = \varepsilon(n-m) = i^{n-m+1}$.

The proof of Proposition 3.1a is exactly the same as the proof of Proposition 3.1. The only change is in the definition of the function $r \to \varepsilon(r)$. Note that with the new definition $\varepsilon(r) = i^{r+1}$ the denominator in (3.10) is different from zero away from the origin also in the case where both integers n and m are odd.

Applying Proposition 3.1a one can improve Theorem 3.1 by removing from the hypothesis the assumption that one of the integers $n \geq 2$ and $m \geq 2$ is even.

Now Theorem 4.1 can be strengthened to the following

THEOREM 4.1a. *Let $n \geq 2$, $m \geq 2$ be integers and let $a \geq 0$, $b \geq 0$ satisfy*

$$an^{-1} + bm^{-1} = 1 - (2n)^{-1} - (2m)^{-1}.$$

Then there exists $C = C(a, b, n, m)$ *such that for every* $u \in C_0^\infty(\mathbf{R}^2)$

$$\| |D_x^a D_y^b| u \|_2 \leqslant C(\|D_x^n u\|_1 + \|D_y^m u\|_1).$$

The proof does not require a separate argument for $n = m$.

Reference

[1] A. Pełczyński and K. Senator, *On isomorphisms of anisotropic Sobolev spaces with "classical Banach spaces" and a Sobolev type embedding theorem*, this volume, 169–215.

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Received January 21, 1986 (2131)