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Remarks concerning the paper "On a class of Hausdorff compacts and GSG Banach spaces"

by

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Abstract. The main result of the above paper by O. I. Reynov which appeared in Studia Math. 71 (1981) is incorrect.

We show, by counterexample, that the main result of the paper [7] is not correct. Moreover, it is clear that the mistake is not technical in nature, but fundamental: there is an improper use of interpolation techniques, not only those of [2] but of interpolation methods in general, e.g. [1]. We emphasize, however, that all the results of [7] that are true are easily obtained by applying the results and techniques of [2–6]; for example, Theorem 0.1 of [7] follows easily from [6].

EXAMPLE. We consider the Banach spaces l_1 and l_∞ in their natural duality.

Let $B = \{x \in l_\infty |\ |x_i| \leqslant i^{-2}\}$. This is a norm compact convex circled subset of l_∞.

Let $A = \bigcap_{n=1}^{\infty} (nB + n^{-1} D)$, where D is the unit ball of l_∞. Now, consider the sequence $a = (i^{-1})_i$. Since $1 \leqslant n/i + i/n$ for all $i, n \in N$, it follows that $1/i \leqslant n/i^2 + 1/n$ for all $i, n \in N$, and we have $a \in A$. We shall compute the distance (in the norm generated by A) from the element a to the linear span $\bigcup \{\lambda B |\ \lambda \geqslant 0\}$ of B in l_∞. Suppose $b = (b_i)_i \in B$, $\lambda \geqslant 0$, $\alpha \geqslant 0$ are such that $a - \lambda b \in \alpha A$. We must have $\alpha > 0$, because $a \notin \operatorname{span} B$. Then there exist $b_n = (b_{n,i})_i \in B$ and $x_n = (x_{n,i})_i \in D$ such that

$$\frac{1}{i} - \lambda b_i = \alpha \left(n b_{n,i} + \frac{1}{n} x_{n,i} \right) \quad \text{for all } i, n \in N.$$

Since, for $i \geqslant 2\lambda$,

$$\frac{1}{i} - \lambda b_i \geqslant \frac{\lambda}{i^2} > 0,$$

we conclude that for all $i \geqslant 2\lambda$ and all $n \in N$

$$\alpha = \frac{n}{i}(1-i\lambda b_i)(n^2 b_{n,i} + x_{n,i})^{-1}.$$

With $n = i$ for $i \geqslant 2\lambda$, we arrive at

$$\alpha = (1-i\lambda b_i)(i^2 b_{i,i} + x_{i,i})^{-1} \geqslant \tfrac{1}{2}(1-\lambda/i) \quad \text{for all } i \geqslant 2\lambda.$$

This shows that $\alpha \geqslant \frac{1}{4}$, and thus that $\operatorname{dist}_A(a, \operatorname{span} B) \geqslant \frac{1}{4}$.

We now denote, for a given bounded subset C of l_∞, by $(l_1)_C$ the completion of the quotient of l_1 by the nullspace of the Minkowski functional q_C of the polar $C^0 = \{x \in l_1 | \ |c(x)| \leqslant 1 \text{ for all } c \in C\}$ of C, normed by q_C. Furthermore, with the sets A and B as above, we denote by T the canonical operator from $(l_1)_A$ into $(l_1)_B$.

The considerations above then imply that $T^*((l_1)_B^*)$ is *not* dense in $(l_1)_A^*$. This contradicts (b) of Theorem 1.1 of [7] and all the results in [7] that require Theorem 1.1. (b).

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