When G^* is compact, the Belley–Morales extension is, of course, vacuous since by the Riesz representation theorem ([2], p. 265) a finitely additive μ is automatically countably additive.

The author is indebted for conversations to Arthur and Dorothy Maharam Stone.

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Corrigendum and addendum to "A generalization of Wiener's criteria for the continuity of a Borel measure"

Studia Math. 72 (1982), 27–36

by

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Let G be a locally compact abelian group with dual group $G\hat{}$ and let $\langle z, \hat{z}\rangle$ denote $\hat{z} \in G\hat{}$ evaluated at $z \in G$. Proposition 3.3 in [1] claims that the function $f\colon G \times G\hat{} \to \boldsymbol{C}$ given by $f(z, \hat{z}) = \langle z, \hat{z}\rangle$ satisfies the double limit condition. This is false, the source of the error being Lemma 3.2. Consequently, the proof of Theorem 4.1 is wrong and Corollaries 4.2 and 4.4 are possibly incorrectly stated. Here, we present the corrections along with some supplementary results.

We adopt the same terminology and notation as in [1]. Given a charge μ on $\mathscr{B}(G)$, $\hat{\mu}$ is a finite linear combination of positive definite functions which are continuous on the set $(G\hat{})_d$ of characters with discrete topology. By Bochner's theorem, there exists a regular Borel measure μ^* on the Bohr compactification G^* of G; such that $\hat{\mu} = \hat{\mu}^*$. Now, by a result of W. F. Eberlein [3], $\mu^*(\{z\}) = \mathrm{MV}(\hat{\mu}(\cdot)\langle z, \cdot\rangle)$ for all $z \in G^*$, and $\sum_{z \in G^*} |\mu^*(\{z\})|^2 = \mathrm{MV}(|\hat{\mu}|^2)$, where MV denotes the mean value of the weakly almost periodic function $\hat{z} \to \hat{\mu}(\hat{z})$ on $(G\hat{})_d$. Our results in [1] provide a measure theoretic approach to the operator MV, by means of a charge (on an algebra of sets in $G\hat{}$) which is invariant under a large class of operators (which includes the translations).

Theorem 4.1 in [1] is contained in the following:

THEOREM 4.1′. *There exists an algebra M of subsets of $G\hat{}$ and a non-negative charge ν on M with the following properties:*

(1) *For all charges μ on $\mathscr{B}(G)$, $\hat{\mu}$ is M-continuous.*

(2) *For all $z \in G^*$,*

$$\mu^*(\{z\}) = \int_{G\hat{}} \hat{\mu}(\hat{z})\overline{\langle z, \hat{z}\rangle}\, d\nu(\hat{z}).$$

(3) *For any translation or continuous surjective endomorphism* $T\colon G^\wedge \to G^\wedge$, *we have* $TE\in M$ *and* $\nu(TE)=\nu(E)$ *for all* $E\in M$.

Proof. We give an outline of the proof. Let $\mathrm{FA}(G)$ designate the class of all charges on the Borel subsets $\mathscr{B}(G)$ of G. On the space $\{\hat\mu^*\colon \mu\in\mathrm{FA}(G)\}$ we introduce the linear functional L given by $L(\hat\mu^*)=\mu^*(\{0\})$ where 0 is the identity in G. On page 118 in [5] it is shown that $\mu^*(\{0\})=\lim_\alpha\int_{G^\wedge}\hat f_\alpha\hat\mu\,d\hat z$ where $d\hat z$ is a Haar measure on $(G^\wedge)_d$ and $\hat f_\alpha$ is a positive function on $G^\wedge$. Hence, L is positive on $\{\hat\mu^*\colon \mu\in\mathrm{FA}(G)\}$. So, an application of a representation theorem of Bochner ([2], pp. 773-775) yields the following:

(1) The class M of all sets $E\subset G^\wedge$ for which

$$\inf\{\mu_2^*(\{0\})-\mu_1^*(\{0\})\colon \hat\mu_1^*\leqslant\chi_E\leqslant\hat\mu_2^*;\ \mu_1,\mu_2\in\mathrm{FA}(G)\}=0,$$

is an algebra of sets such that, when $\hat\mu^*$ is real valued,

$$\{\hat z\in G^\wedge\colon \hat\mu^*(\hat z)\geqslant\alpha\}\in M$$

for a dense set of α's in $\boldsymbol{R}$.

(2) There exists an additive function $\nu\colon M\to[0,1]$ for which

(a) $$\mu^*(\{0\})=\int_{G^\wedge}\hat\mu^*\,d\nu\qquad(\mu^*\in M(G^*))$$

and

(b) $$\nu(E)=\sup\{\mu^*(\{0\})\colon \hat\mu^*\leqslant\chi_E;\ \mu^*\in M(G^*)\}$$
$$=\inf\{\mu^*(\{0\})\colon \chi_E\leqslant\hat\mu^*;\ \mu^*\in M(G^*)\}.$$

Now, substituting the measure $E\to\mu^*(E-z)$ for $E\to\mu^*(E)$ we get, for all $z\in G^*$

$$\mu^*(\{z\})=\int_{G^\wedge}\overline{\langle z,\hat z\rangle}\,\hat\mu^*(\hat z)\,d\nu(\hat z).$$

When $T\colon G^\wedge\to G^\wedge$ is a translation $T\hat z=\hat z+\hat z_0$ (where $\hat z_0$ is fixed in $G^\wedge$), write μ_T^* for the measure given by

$$\mu_T^*(E)=\int_E\langle z,\hat z_0\rangle\,d\mu^*(z)$$

for all Borel sets $E\subset G^*$. Then $\{\hat\mu_T^*\colon \mu\in\mathrm{FA}(G)\}=\{\hat\mu^*\colon \mu\in\mathrm{FA}(G)\}$ and $\mu_T^*(\{0\})=\mu^*(\{0\})$.

Similarly, when $T\colon (G^\wedge)_d\to(G^\wedge)_d$ is a (continuous) surjective homomorphism, there exists a unique continuous surjective homomorphism $T^\wedge\colon G^*\to G^*$ such that $\langle T^\wedge z,\hat z\rangle=\langle z,T\hat z\rangle$ $(z\in G^*,\ \hat z\in G^\wedge)$ (see [4], p. 242). Let μ_T^* be the measure given by $\mu_T^*(E)=\mu^*(T^{\wedge-1}E)$ for all Borel sets $E\subset G^*$ and all $\mu\in\mathrm{FA}(G)$. Then

$$\{\hat\mu_T^*\colon \mu\in\mathrm{FA}(G)\}=\{\hat\mu^*\colon \mu\in\mathrm{FA}(G)\}\quad\text{and}\quad\mu_T^*(\{0\})=\mu^*(\{0\}).$$

In either case, it follows that

$$\inf\{\mu^*(\{0\})-\lambda^*(\{0\})\colon \hat\lambda^*\leqslant\chi_{TE}\leqslant\hat\mu^*;\ \lambda,\mu\in\mathrm{FA}(G)\}$$
$$=\inf\{\mu_T^*(\{0\})-\lambda_T^*(\{0\})\colon \hat\lambda_T^*\leqslant\chi_E\leqslant\hat\mu_T^*;\ \lambda,\mu\in\mathrm{FA}(G)\}=0$$

and

$$\nu(TE)=\sup\{\mu^*(\{0\})\colon \hat\mu^*\leqslant\chi_{TE},\ \mu\in\mathrm{FA}(G)\}$$
$$=\sup\{\mu_T^*(\{0\})\colon \hat\mu_T^*\in\chi_E;\ \mu\in\mathrm{FA}(G)\}=\nu(E).$$

So $T\colon M\to M$ and $\nu\circ T=\nu$. Theorem 4.1′ now follows.

Corollaries 4.2 and 4.4 in [1] should now be stated as follows.

COROLLARY 4.2′. *For any charge* μ *on* $\mathscr{B}(G)$, *we have*

$$\sum_{z\in G^*}|\mu^*(\{z\})|^2=\int_{G^\wedge}|\hat\mu(\hat z)|^2\,d\nu(\hat z).$$

COROLLARY 4.4′. *Let* μ *be a charge on* $\mathscr{B}(G)$. *If* $|\hat\mu|=1$ *then*

$$\sum_{z\in G^*}|\mu^*(\{z\})|^2=1.$$

To prove Corollary 4.2′ we write $\tilde\mu^*(E)$ for $\bar\mu^*(-E)$ and obtain

$$\sum_{z\in G^*}|\mu^*(\{z\})|^2=(\mu^*\times\tilde\mu^*)(\{0\})=\int_{G^\wedge}(\mu^*\times\tilde\mu^*)^\wedge\,d\nu=\int_{G^\wedge}|\hat\mu|^2\,d\nu$$

where $\times$ denotes convolution.

Remarks. (1) When μ is countably additive, the equations in Theorem 4.1′ and Corollary 4.2′ reduce to

$$\mu(\{z\})=\int_{G^\wedge}\hat\mu(\hat z)\langle z,\hat z\rangle\,d\nu(\hat z)\quad(z\in G)\quad\text{and}\quad\sum_{z\in G}|\mu(\{z\})|^2=\int_{G^\wedge}|\hat\mu|^2\,d\nu,$$

respectively. These are the classical results of N. Wiener when $G=\boldsymbol{R}$ or $G=\boldsymbol{Z}$ and of W. F. Eberlein for the general case, written in our measure theoretic form.

(2) The algebra M in Theorem 4.1′ contains the M in [1]. Also, $\mu^*(\{z\})=\mu(\{z\})$ for all $z\in G$. Hence, Theorem 4.1′ contains Theorem 4.1 in [1].

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Über Umkehrbedingungen bei gewöhnlicher und absoluter Limitierung

von

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Abstract. It is known that if one of the conditions (1) to (3) [(5) to (7_∞)] is a Tauberian condition [absolute Tauberian condition] for a summability method V of a very general class, then each of these conditions is a Tauberian condition [absolute Tauberian condition] for V. It is the purpose of this paper to show that in this context (1) to (3) [(5) to (7_∞)] can be replaced by (1) to (4) [(5) to (8_∞)].

1. Einleitung. Bei gegebener Reihe $\sum a_n$ mit komplexen Gliedern sei

$$\delta_n := \frac{1}{n+1} \sum_{k=0}^{n} k a_k \quad \text{für} \quad n = 0, 1, \ldots$$

Wir betrachten die vier Aussagen

(1) $$n a_n = o(1) \quad \text{für} \quad n \to \infty,$$

(2) $$\delta_n = o(1) \quad \text{für} \quad n \to \infty,$$

(3) $$\sum (-1)^n n a_n \text{ konvergiert},$$

(4) $$\sum (-1)^n \delta_n \text{ konvergiert}.$$

Mit den Bezeichnungen aus Nr. 2 gilt: Ist V ein permanentes Verfahren zur Summierung von Reihen $\sum a_n$ und ist (1) eine TB für V, so ist auch (2) eine TB für V. Dies wurde zum Beispiel in [9] gezeigt. (Wegen Verallgemeinerungen und Varianten vergleiche man [9], [10], [16] sowie Stieglitz [12], Leviatan [7], Kangro [5] und Sõrmus [14].) Umgekehrt ist wegen der Permanenz des Cesàro-Verfahrens (der Ordnung 1) mit (2) stets auch (1) eine TB für V. Nun zeigte Goes [3], 4.6, durch Anwendung eines Darstellungssatzes für Zahlenfolgen, die einer gewissen Wachstumsbedingung genügen, dass (2) auch eine TB für V ist, wenn nur (3) eine solche ist, und natürlich ebenso umgekehrt. Damit liegt die Frage nahe, ob (3) genau dann eine TB für V ist, wenn auch (4) eine TB für V ist. Dies ist der Fall (Satz. 2.1).

Wir verwenden jetzt auch die Bezeichnungen aus Nr. 3 und betrachten vier andere Aussagen.