This follows from results of [6] (cf. the remark at the end of § 4) and because φ is order-preserving.

It follows from the definitions of L_i, P and Y, that L can be determined if one knows the Ω_i, which are determined by the Markov diagram, and if one knows $\bar{C}$ or $\mathfrak{J}$. In [1] $\mathfrak{J}$ is determined for certain transformations. One can find examples for all possible cases described in the paper. For example, T defined by

$$T(x) = \begin{cases} \frac{1}{8}x+\frac{21}{80} & \text{for} \quad x\in[0, \frac{27}{260}], \\ \frac{27}{8}x-\frac{3}{40} & \text{for} \quad x\in[\frac{27}{260}, \frac{1}{5}], \\ 2x+\frac{1}{5} & \text{for} \quad x\in[\frac{1}{5}, \frac{2}{5}), \\ x-\frac{2}{5} & \text{for} \quad x\in[\frac{2}{5}, 1) \end{cases}$$

belongs to case (a), but $Y = \{\frac{1}{4}, \frac{7}{10}\}$.

By the methods of [3] one can transmit $x \to ax(1-x)$, where $2 \leqslant a \leqslant 4$, into a piecewise monotonic transformation. If one blows up each of the points in $\bigcup_{j=0}^{\infty} T^{-j}\{T^i(0), i \geqslant 0\}$ to an interval, one gets again a nonempty Y which has infinitely many elements for certain values of a.

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On the Wiener–Eberlein theorem

by

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Abstract. A counterexample is presented to the main theorem of a paper by J.-M. Belley and P. Morales that appeared in Studia Mathematica 72 (1982), pp. 27–36.

Given a locally compact Abelian group G, let μ be a bounded complex-valued countably additive measure defined on the Borel sets of the character group G^*. Then the Fourier transform $\hat{\mu}$,

$$\hat{\mu}(x) = \int_{G^*} (x, -y)\, d\mu(y) \qquad (x \in G)$$

is a weakly almost periodic (w.a.p.) function on G [3]. The following result is due to Norbert Wiener ([6], Vol. 2, pp. 259–261, and Vol. 1, p. 108; [5]) in the special cases $G = R$ and $G = Z$ and to the author in the general case [4].

THEOREM. $M[|\hat{\mu}|^2] = \sum_{y \in G^*}' |\mu\{y\}|^2$.

Here the mean value $M(f)$ of a w.a.p. function f may be defined as the (necessarily unique) constant that is the uniform limit of convex combinations of translates of f. When $G = R$, the additive group of the reals, M has the representation

$$M(f) = \lim_{L \to \infty} (2L)^{-1} \int_{-L}^{L} f(x)\, dx.$$

In a recent paper in this journal, Belley and Morales [1] purport to generalize this theorem to the case of *finitely* additive μ. Here is a counterexample to the extended theorem when G^* is non-compact (= G non-discrete) – say $G = R = G^*$: Pick any point y_0 in $\beta G^* - G^*$, where βG^* is the Čech–Stone compactification of G^*, and let ν be a unit measure concentrated at y_0. If f is any bounded continuous function on G^*, denote its extension to βG^* by F. Then ν induces a finitely additive bounded regular measure μ on the Borel subsets of G^* such that $\int_{G^*} f\,d\mu = \int_{\beta G^*} F\,d\nu$ ([2], p. 262). Clearly, $\mu\{y\} = 0$ for any y in G^*. But when $f(y) = (x, -y)$, $|f| = 1$ on G^* and $|F| = 1$ on βG^*, whence $|\hat{\mu}(x)| = |F(y_0)| = 1$. Hence $M[|\hat{\mu}|^2] = M(1) = 1 \neq 0 = \sum_{y \in G^*} |\mu\{y\}|^2$.

When G^* is compact, the Belley–Morales extension is, of course, vacuous since by the Riesz representation theorem ([2], p. 265) a finitely additive μ is automatically countably additive.

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Corrigendum and addendum to "A generalization of Wiener's criteria for the continuity of a Borel measure"

Studia Math. 72 (1982), 27–36

by

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Let G be a locally compact abelian group with dual group $G^\wedge$ and let $\langle z, \hat{z}\rangle$ denote $\hat{z} \in G^\wedge$ evaluated at $z \in G$. Proposition 3.3 in [1] claims that the function $f\colon G \times G^\wedge \to \boldsymbol{C}$ given by $f(z, \hat{z}) = \langle z, \hat{z}\rangle$ satisfies the double limit condition. This is false, the source of the error being Lemma 3.2. Consequently, the proof of Theorem 4.1 is wrong and Corollaries 4.2 and 4.4 are possibly incorrectly stated. Here, we present the corrections along with some supplementary results.

We adopt the same terminology and notation as in [1]. Given a charge μ on $\mathscr{B}(G)$, $\hat{\mu}$ is a finite linear combination of positive definite functions which are continuous on the set $(G^\wedge)_d$ of characters with discrete topology. By Bochner's theorem, there exists a regular Borel measure μ^* on the Bohr compactification G^* of G; such that $\hat{\mu} = \hat{\mu}^*$. Now, by a result of W. F. Eberlein [3], $\mu^*(\{z\}) = \mathrm{MV}(\hat{\mu}(\cdot)\langle z, \cdot\rangle)$ for all $z \in G^*$, and $\sum_{z\in G^*} |\mu^*(\{z\})|^2 = \mathrm{MV}(|\hat{\mu}|^2)$, where MV denotes the mean value of the weakly almost periodic function $\hat{z} \to \hat{\mu}(\hat{z})$ on $(G^\wedge)_d$. Our results in [1] provide a measure theoretic approach to the operator MV, by means of a charge (on an algebra of sets in $G^\wedge$) which is invariant under a large class of operators (which includes the translations).

Theorem 4.1 in [1] is contained in the following:

THEOREM 4.1′. *There exists an algebra M of subsets of $G^\wedge$ and a non-negative charge ν on M with the following properties*:

(1) *For all charges μ on $\mathscr{B}(G)$, $\hat{\mu}$ is M-continuous.*

(2) *For all $z \in G^*$,*

$$\mu^*(\{z\}) = \int_{G^\wedge} \hat{\mu}(\hat{z})\overline{\langle z, \hat{z}\rangle}\, d\nu(\hat{z}).$$