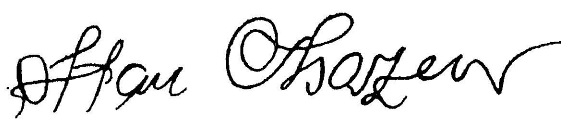

STANISŁAW MAZUR

1 January 1905 – 5 November 1981

Stanisław Mazur was born and educated in Lwów, where he left school in 1932. He then studied mathematics in Lwów and Paris, and in 1932 took his doctorate of philosophy at Jan Kazimierz University in Lwów, presenting a thesis on the theory of summability. His habilitation followed in 1935 on the basis of the dissertation "On convex sets and convex functionals". In 1932 he had begun work at the University as an assistant of Professor Hugo Steinhaus, but in 1935 he moved to the Technical University of Lwów. Subsequently, during the period 1939–1941 of Soviet rule in Lwów, he held the chair of geometry in the University of Lwów. During the German occupation he worked as a shop assistant. After the German withdrawal from Lwów, he returned to his former position, but for two years devoted himself to organizing the repatriation of the Polish population from the Soviet Union. After returning to Poland in 1946 he became a full professor of the newly-established University of Łódź. After two years there he transferred to the University of Warsaw, where he remained until his retirement. From 1946 to 1969 he was the director of the Institute of Mathematics of the University.

Stanisław Mazur was one of the organizers of the State Institute of Mathematics (later to become the Institute of Mathematics of the Polish Academy of Sciences) in which he was the head of the section of functional analysis until 1958. He was a full member of the Polish Academy of Sciences from its foundation in 1952, and its first Scientific Secretary; he was a foreign member of the Hungarian Academy of Sciences, and held an honorary doctorate of the University of Warsaw. During 1946–1954 he was a member of the Sejm (Parliament).

Stanisław Mazur had wide mathematical interest. His first publications, 1928–1929, concerned the theory of summability, and he returned to this field in later periods: using functional analytical techniques, he discovered, jointly with Władysław Orlicz, the theorem on consistency of Toeplitz summability methods.

Under the influence of Stefan Banach, he took up functional analysis. Many of the joint results of Banach and Mazur appeared in Banach's monograph "Théorie des opérations linéaires"; one of them is the theorem on universality of the space $C([0,1])$ for separable Banach spaces. Mazur was also the author of the "Remarks" at the end of the book. The problems formulated in these "Remarks" were the starting point of many researches, in particular on the geometry of Banach spaces and on infinite-dimensional topology. He was also the author of many problems in the Scottish Book.

Mazur is universally recognized as the inventor of geometrical methods in functional analysis. He is the author of the theorem on weak closedness of closed convex sets, and of the result on the character of the set of points of differentiability of convex functionals. He, independently of J. von Neumann, introduced the concept of a locally convex space, and, jointly with Orlicz, systematically studied completely metrizable locally convex spaces.

Jointly with Banach, he initiated research in the foundations of mathematics devoted to computable analysis, which he continued in the 1950's.

Stanisław Mazur was the teacher of many Polish mathematicians. With Orlicz, he reconstructed the Polish school of functional analysis after the losses of the Second World War. His outstanding personality and his talent as a teacher attracted many young mathematicians, and his seminars in the Institute of Mathematics of the Polish Academy of Sciences and at the University of Warsaw were for many years the inspiration of research in functional analysis in Warsaw.

Stanisław Mazur was closely associated with this journal from its foundation, and published in it 22 papers. From 1948 he was a member of the Editorial Board.

Mathematical publications of Stanisław Mazur*

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[3] *Une remarque sur l'homéomorphie des champs fonctionnels*, Studia Math. 1 (1929), 83–85.
[4] *Über die kleinste konvexe Menge, die eine gegebene kompakte Menge enthält*, Studia Math. 2 (1930), 7–9.
[5] *Über die Nullstellen linearer Operationen*, Studia Math. 2 (1930), 11–20.
[6] *Eine Anwendung der Theorie der Operationen bei der Untersuchung der Toeplitzschen Limitierungsverfahren*, Erste Miteilung, Studia Math. 2 (1930), 40–50.
[7] *Bemerkung zu meiner Arbeit: Über die Nullstellen linearer Operationen*, Studia Math. 2 (1930), 249–250.
[8] (with S. Ulam) *Sur les transformations isométriques d'espaces vectoriels normés*, C.R. Acad.Sci. Paris 194 (1932), 946–948.
[9] (with L. Sternbach) *Über die Borelschen Typen von linearen Mengen*, Studia Math. 4 (1933), 48–53.
[10] (with L. Sternbach) *Über Konvergenzmengen von Folgen linearer Operationen*, Studia Math. 4 (1933), 54-65.
[11] *Über konvexe Mengen in linearen normierten Räumen*, Studia Math. 4 (1933), 70–84.
[12] (with S. Banach) *Eine Bemerkung über die Konvergenzmengen von Folgen linearer Operationen*, Studia Math. 4 (1933), 90–94.
[13] (with S. Banach) *Zur Theorie der linearen Dimension*, Studia Math. 4 (1933), 100–112.
[14] *Über schwache Konvergenz in den Räumen* (L^p), Studia Math. 4 (1933), 128–133.
[15] (with W. Orlicz) *Über Folgen linearer Operationen*, Studia Math. 4 (1933), 152–157.
[16] (with W. Orlicz) *Sur les méthodes linéaires de sommation*, C. R. Acad. Sci. Paris 196 (1933), 32–34.
[17] (with W. Orlicz) *Grundlegende Eigenschaften der polynomischen Operationen*, I, Studia Math. 5 (1934), 50–68.
[18] (with W. Orlicz) *Grundlegende Eigenschaften der polynomischen Operationen*, II, Studia Math. 5 (1934), 179–189.
[19] (with S. Banach) *Über mehrdeutige stetige Abbildungen*, Studia Math. 5 (1934), 174–178.
[20] (with W. Auerbach and S. Ulam) *Sur une propriété caractéristique de l'ellipsoide*, Mh. Math. Phys. 42 (1935) 45–48.
[21] *On convex sets and convex functionals in linear spaces*, Lwów 1936 [in Polish]

* This is a copy of the list published in vol. 22.2 of Wiadomości Matematyczne; it is not certain whether the list is complete.

[22] (with W. Orlicz) *Polynomische Operationen in abstrakten Räumen*, Comptes Rendus du Congrès International des Mathématiciens à Oslo (1936), vol. 2, 107–108.

[23] (with J. P. Schauder) *Über ein Prinzip in der Variationsrechnung*, Comptes Rendus du Congrès International des Mathématiciens à Oslo (1936), vol. 2, 65.

[24] (with W. Orlicz) *Sur la divisibilité des polynômes abstraits*, C. R. Acad. Sci. Paris 202 (1936), 621–623.

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[35] *On continuous mappings on Cartesian products*, Fund. Math. 39 (1952), 229–238.

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STUDIA MATHEMATICA, T. LXXI. (1982)

On bases and unconditional bases in the spaces $L^p(d\mu)$, $1 \leqslant p < \infty$

by

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Abstract. Necessary and sufficient conditions are found for a Borel measure μ in order that the system of functions $\{\chi_{n_i}(x)\}_{i=1}^{\infty}$ resulting from the Haar system by removing finitely many members be a basis in the space $L^p(d\mu)$, $1 \leqslant p < \infty$. It is also shown that if such a cofinite subsystem of the Haar system constitutes a basis in $L^p(d\mu)$, $1 < p < \infty$, then it actually constitutes an unconditional basis in that space.

1. Introduction. Let E be a Borel set on the real line and μ be a finite positive Borel measure on E. The symbol $L_E^p(d\mu)$, $1 \leqslant p < \infty$, will denote the Banach space of all functions f such that

$$\|f\|_{L_E^p(d\mu)} = \Big(\int_E |f(x)|^p d\mu\Big)^{1/p} < \infty. \tag{1}$$

Further, by definition,

$$\|f\|_{L_E^\infty(d\mu)} = \operatorname*{sup\,ess}_{x\in E} |f(x)| \quad (\text{relative to } \mu). \tag{2}$$

In case where μ is the Lebesgue measure, we write just L_E^p instead of $L_E^p(d\mu)$.

In most problems concerning the Haar system and the Walsh system, the functions which enter those systems can be given quite arbitrary values at the discontinuity points. In this paper, however, we are dealing with general Borel measures, and so these values gain significance. Thus, we define the exact values of Haar and Walsh functions in such a way as to obtain closed systems in $C[0, 1]$. Namely, by convention, the value at an interior discontinuity point is to be equal to the arithmetic mean of the one-sided limits at this point and the value at an endpoint is to be just the appropriate one-sided limit value.

A system of functions $\{f_n(x)\}$ is said to be *closed* in the space $L^p(d\mu)$, $1 \leqslant p < \infty$, iff every function in $L^p(d\mu)$ can be norm-approximated by finite linear combinations of the f_n's. A system $\{f_n(x)\}$ in $L^q(d\mu)$, q denoting the conjugate exponent to p, $1 \leqslant p < \infty$, is said to be *total* with respect to $L^p(d\mu)$ iff the only function in $L^p(d\mu)$ orthogonal to all the f_n's is the