By Banach's principle [5], Theorem IV. 11. 3

$$A(T, a)f(x) \to f(x) \text{ a.e.}, \quad f \in L_1,$$

as $a \searrow 0$ through Q^+. Since $A(T, a)f(x)$ depends continuously on a a.e. it follows that

$$\lim_{a \searrow 0} A(T, a)f(x) = f(x) \text{ a.e.}, \quad f \in L_1. \ ■$$

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Addendum to the paper "Weak-strong convolution operators on certain disconnected groups"

by

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Abstract. In [1] G.I. Gaudry and the author obtained several results concerning L^p convolution operators and multipliers on a totally disconnected group where the indices of sucessive subgroups remain bounded. More specifically, estimates were obtained for kernels (resp. multipliers) having a strong singularity at the origin (resp. at infinity). In this note we show how to extend the results of [1] to the case where the indices are unbounded, and in doing so answer a question implicit in the work of Peyrière and Spector [2].

1. Introduction. Let G denote a compact abelian group having the following properties:

(i) there exists a strictly decreasing sequence $\{G_n\}_{n=0}^{\infty}$ of open compact subgroups of G such that the index $G_{n+1}: G_n$ of G_{n+1} in G_n is finite;

(ii) $\bigcup G_n = G$ and $\bigcap G_n = \{0\}$;

(iii) $|G_0| = 1$ where $|S|$ denotes the Haar measure of a (measurable) set S;

(iv) $|G_n| \cdot |G_{n-1}|^{-1} \downarrow 0$.

Let Γ denote the dual group of G and Γ_n the annihilator of G_n in Γ. Then $\{\Gamma_n\}$ is an increasing sequence of open compact subgroups of Γ and $\Gamma_n : \Gamma_{n+1} = G_{n+1} : G_n$. Such groups divide naturally into two classes: (a) where $G_{n+1}: G_n \leqslant b$ for some positive integer $b \geqslant 2$, and (b) where $G_{n+1}: G_n \to \infty$. Groups satisfying (a) were treated in [1] and from now on we shall suppose that (b) holds.

We refer the reader to [1] for all the required definitions and notation.

2. Convolution estimates. The following result takes the place of Theorems 2.1 and 2.2 of [1]. (There is no real need to consider the case $\theta > 0$ of [1].)

THEOREM 1. *Suppose $k \in L^1$. If*

$$|\hat{k}(\gamma)| \leqslant B\{|G_{n+1}|/|G_n|\}^{1/2}, \quad \gamma \in \Gamma_{n+1} \setminus \Gamma_n \tag{1}$$

and

(2) $$\int_{G\setminus G_n} |k(x-y)-k(x)|\,dx \leqslant B, \quad \text{when} \quad y \in G_n,$$

then for all f in L^∞

(3) $$\|k*f\|_{\mathrm{B.M.O.}} \leqslant C\|f\|_{\mathrm{B.M.O.}},$$

where C depends on B only.

Proof. Fix an f in L^∞ and consider

$$I_n(f) = \left(\frac{1}{|G_n|}\int_{G_n} |k*f-D_{n-1}*k*f|^2\right)^{1/2},$$

where $D_n = \xi_{G_n}|G_n|^{-1}$. Split $f = f_1+f_2$ where $f_1 = f\xi_{G_{n-1}}$. Then, by (1) and the definition of f_1,

$$\begin{aligned} I_n(f_1) &\leqslant \frac{1}{|G_n|^{1/2}}\|k*f_1-D_{n-1}*k*f_1\|_2 \leqslant \frac{1}{|G_n|^{1/2}}\left(\int|\hat{k}\hat{f}_1|^2\right)^{1/2} \\ &\leqslant B\frac{1}{|G_n|^{1/2}}\frac{|G_n|^{1/2}}{|G_{n-1}|^{1/2}}\left(\int_{\Gamma\setminus\Gamma_{n-1}}|\hat{f}_1|^2\right)^{1/2} \leqslant \frac{B}{|G_{n-1}|^{1/2}}\|f_1-D_{n-1}*f_1\|_2 \\ &= B\left(\frac{1}{|G_{n-1}|}\int_{G_{n-1}}|f-D_{n-1}*f|^2\right)^{1/2} \\ &\leqslant B\left\{\left(\frac{1}{|G_{n-1}|}\int_{G_{n-1}}|f-D_{n-2}*f|^2\right)^{1/2}+\|D_{n-1}*f-D_{n-2}*f\|_\infty\right\} \\ &\leqslant 2B\|f\|_{\mathrm{B.M.O.}}. \end{aligned}$$

The argument of Theorem 2.2 of [1] shows that

$$I_n(f_2) \leqslant B\|f\|_{\mathrm{B.M.O.}}$$

so we obtain (3) with $C = 3B$.

3. Main results. The proofs of the following results are similar to those of the corresponding results of [1].

THEOREM 2. *Let $\theta(\gamma) = \{|G_{n+1}|\cdot|G_n|^{-1}\}^{1/2}$ $\gamma \in \Gamma_{n+1}\setminus\Gamma_n$. If k is a pseudo-measure equal to an integrable function away from 0 satisfying* (1) *and* (2), *then $\hat{k}(\gamma)\theta(\gamma)^{-\alpha}$ is an L^p Fourier multiplier when $p \in [2/(2-\alpha),\ 2/\alpha]$,* $0<\alpha<1$.

COROLLARY 1 (compare with [2]). *Suppose φ is a quasi-radial function on Γ, i.e. φ is constant on cosets of Γ_n in $\Gamma_{n+1}\setminus\Gamma_n$. If*

$$|\varphi(\gamma)| \leqslant B\{|\Gamma_n|\cdot|\Gamma_{n+1}|^{-1}\}^{(1-\alpha)/2}, \quad \gamma\in\Gamma_{n+1}\setminus\Gamma_n,$$

then φ is an L^p multiplier when $p \in [2/(2-\alpha), 2/\alpha]$, $0<\alpha<1$.

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