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Corrigendum and addendum to the paper

“A simple diophantine condition in harmonic analysis”

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by

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1. Lemma 2.3 in [1] is misstated and should be replaced by:

**Lemma 2.3.** Let $G$ be a discrete (not necessarily countable) abelian group. Let $(F_i)_{i=1}^r$ be a family of finite and mutually independent sets $(0 \neq F)$, i.e., $\text{g.p.}(F_i) \cap \text{g.p.}(F_j) = \emptyset$ whenever $i \neq j$. Then, $(F_i)$ is a sum norm partition for $G$.

Victimized by the misstatement of Lemma 2.3, the proof of Theorem C contains an error. We can conclude only that the $S_m$'s are independent in the sense that whenever $S_i \neq S_j$, $i = 1, \ldots, r$, then $F_i \neq F_j$. It follows that $(F_i)$ is an independent set. But, we cannot conclude that $\text{g.p.}(S_m) \cap \text{g.p.}(S_m) = \emptyset$ whenever $N \neq M$, and therefore we are unable to apply the (correctly stated) Lemma 2.3. We are unable to supply a correct proof of Theorem C. The above error does not affect the main results of the paper.

2. Our diophantine condition is necessarily satisfied by $E = \bigcup F_i$, where $(F_i)$ is as in Lemma 2.3: Without loss of generality, we assume that $\bigcup F_i = \bigoplus F_i$ where $F_i = \text{g.p.}(F_i)$ and $F_i = F_i$. Let $D_i$, as usual be a dense countable subgroup of $G$, and write $D = \bigoplus D_i$, which is then, a dense countable subgroup of $G$. The proof of the following proposition is a routine verification.

**Proposition.** $\text{g.p.}(E)$ accumulates precisely at 0 $(\text{g.p.}; \bigoplus F_i \mapsto \bigoplus D_i)$

Again, as at the end of [1], we note that the independence condition in the above proposition is sharp in the following sense: A sequence of disjoint and mutually lacunary blocks of integers, $(F_i)$, can be constructed so that $\text{g.p.}(E)$ is dense in $D_i$ for all $D_i \in I$. To see this, we mimic the construction at the end of [1], and add the requirement that $\|D_i\| = 1$. It then follows (see Lemma 1.2 in [2]) that $\bigcup \text{spec}(D_i)$ is dense in $\bar{D}$, the Bohr compactification of $D$. Our claim now follows from the observation that if $E \subset Z$ is dense in $\bar{Z}$, then $\text{g.p.}(E) = D_i$ for all $D_i \in I$.\[\]

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Finally, we remark that the examples $N \subset Z$ such that $L^2_n \to C^*_R \geq A^*_R$
constructed by Rosenthal in [3] follow from our Theorem B in [1]. It is
proved in [3], via the notion of sup-norm partitions, that $\bigcup_{n=0}^{\infty} E_n$,
where $E_n = \mathbb{F}^k \cap (2n)! \mathbb{Z}^k = \mathbb{F}^k$ are $R$-sets. Let $\mathcal{V}: \oplus_{n=0}^{\infty} (2n)! \mathbb{Z}^{2n} \to \mathbb{F}^k$ be the map that carries $a = (\{a_{n,k}\}_{n,k})$ into $\sum_{n=0}^{\infty} \frac{2\pi a_n}{(2n)!} (\text{mod} 2\pi)$. Set $D = \oplus_{n=0}^{\infty} \ker \mathcal{V}$. Since, for $N \subset \mathbb{Z}$, $F_{B_0}(N)/\mathbb{Z}$ is $N/(\text{mod}(2\pi))$, it follows that $F_{B_0}(\mathbb{F})$ and $F_{B_0}(\mathbb{F})$ accumulate only at 0 in $\mathcal{D}$ (a closed subgroup of $\oplus_{n=0}^{\infty} \mathbb{F}^k$). Now apply Theorem B.

References


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Karol Borsuk
THEORY OF SHAPE

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The original concepts of the theory of shape appear in 1967 in the papers of Professor Borsuk. At present there exists a wide literature on the subject — more than 200 original papers.

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