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DEPARTMENT OF MATHEMATICS
UNIVERSITY OF GÖTEBORG AND
CHALMERS UNIVERSITY OF TECHNOLOGY

Received February 21, 1977 (1268)

STUDIA MATHEMATICA, T. LXVI. (1979)

On weakly* conditionally compact dynamical systems

by

W. SZLENK (Warszawa)

1. Let (X, φ) be a topological dynamical system, i.e. X is a compact metric space and $\varphi\colon X \to X$ is a continuous mapping. Denote by $C(X)$ the space of all continuous, real (or complex) valued functions on X, and let U_φ be an operator defined as follows: $U_\varphi f = f\circ\varphi$, $f \in C(X)$.

A sequence (f_n) of elements of a Banach space E is said to be *weakly* conditionally compact* if for every sequence of positive integers (n_k) there is a subsequence (n_{k_i}) such that for every linear continuous functional $\Phi \in E^*$ the sequence of scalars $\big(\Phi(f_{n_{k_i}})\big)$ is convergent. In the case of $E = C(X)$ it means that the sequence $\big(f_{n_{k_i}}(x)\big)$ is pointwise convergent (not necessarily to a continuous function).

If for every sequence (n_k) there exists a subsequence (n_{k_i}) and an element $f \in E$ such that $(f_{n_{k_i}})$ is weakly convergent to f, then the sequence (f_n) is said to be weakly conditionally compact.

DEFINITION. A system (X, φ) is said to be *weakly* [weakly] conditionally compact* if for every $f \in C(X)$ the sequence $(U^n f)$ is weakly* [weakly] conditionally compact. For brevity, we shall call these systems w*cc [wcc] systems.

The aim of the paper is to study some spectral properties, the strict ergodicity (under some additional assumptions) and the sequence entropy of w*cc systems.

In view of Rosenthal's theorem [8] for every $f \in C(X)$ there are two possibilities:

(1) The sequence $(U^n f)$ contains a subsequence $(U^{n_k} f)$ such that for some $c > 0$ and for every sequence of numbers (real or complex) $a_0, \ldots, a_{m-1}$ the following inequality holds:

$$\sup_{x\in X}\Big|\sum_{k=0}^{m-1} a_k U^{n_k} f(x)\Big| \geqslant c\sum_{k=0}^{m-1} |a_k|. \tag{1}$$

(2) The sequence $(U^n f)$ is w*cc.

If a system (X,φ) is a w*cc system, then the first possibility does not occur for any $f\in C(X)$. It means that φ "mixes" the points in X very slowly for every sequence of moments (n_k).

The wcc systems have been studied in [1] and in [10].

EXAMPLES. (i) Every homeomorphism of an interval is a w*cc system (and it is not a wcc system, except the identity) — this follows easily from Proposition 1.

(ii) Every homeomorphism of the circle S^1 is a w*cc system. The homeomorphism is a wcc system if and only if it is topologically equivalent to a rotation.

Proof. It is enough to consider the case where φ preserves orientation. If φ has a periodic point, then the problem can be reduced to the case (i). Suppose φ has no periodic points. Then either φ is conjugate to a rotation and evidently the system is a wcc system, or φ is a so called *Denjoy homeomorphism*. The set of all non-wandering points is then a Cantor set Δ in S^1. For every sequence (n_k) there exists a subsequence (n_{k_i}) such that $(\varphi^{n_{k_i}}(x))$ is convergent for every end-point of each interval of $S^1-\Delta$. Hence we easily conclude that $(\varphi^{n_{k_i}}(x))$ is convergent for every $x\in S^1$, which completes the proof (see Proposition 1).

(iii) Every Morse–Smale system (for the definition see [5] or [11]) is a w*cc system.

(iv) Let $X=T^2$ be the two-dimensional torus, let $\varphi_0\colon S^1\to S^1$ be a homeomorphism of the circle S^1 and let $k\colon S^1\to S^1$ be a continuous mapping. The map $\varphi(x^1,x^2)=(\varphi_0(x^1),x_2+f(x^1))$ is called the *skew rotation*. Let $\hat\varphi$ be a lift of φ to the plane R^2, and let Q be a unit square in $\boldsymbol{R}^2$. If $\sup_n \operatorname{diam}\hat\varphi^n(Q)$ is finite, then (T^2,φ) is a w*cc system. It easily follows from the Gottschalk–Heldlund lemma that for a continuous function f on S^1 if $f(x^1)+f(\varphi_0(x^1))\dots f(\varphi_0^{n-1}(x^1))$ is uniformly bounded, then there exists a continuous function g such that $g(\varphi_0(x^1))-g(x^1)=f(x^1)$.

Probably there are some w*cc skew rotations on the torus T^2 for which $\sup_n \operatorname{diam}\varphi^n(Q)=+\infty$.

2. PROPOSITION 1. *A system (X,φ) is* w*cc *iff for every sequence of positive integers (n_k) there exists a subsequence (n_{k_i}) such that the sequence $(\varphi^{n_{k_i}}(x))$ is convergent for every $x\in X$.*

A system (X,φ) is wcc *iff every cluster point ψ of the set $\{\varphi^n\}$ in pointwise topology is continuous.*

The proof is quite elementary, and so we omit it.

THEOREM 1. *Let μ be an invariant ergodic probabilistic measure for a* w*cc *system (X,φ). Then the metric system (X,B,μ,φ) (B σ-field of Borel sets) has a discrete spectrum. Moreover, the eigenfunctions of U_φ are of the first Baire class.*

Proof. Let $\|\cdot\|$ denote the norm in the space $C(X)$, and let $\|\cdot\|_2$ denote the norm in the space $L^2(X,B,\mu)$. By assumption every sequence $(U_\varphi^n f)$, $f\in C(X)$, is a bounded sequence of functions, conditionally compact in the topology of pointwise convergence. Therefore $(U_\varphi^n f)$ is conditionally compact in the space $L^2(X,B,\mu)$. Since $C(X)$ is dense in $L^2(X,B,\mu)$, the sequence $(U_\varphi^n f)$ is conditionally compact for every $f\in L^2(X,B,\mu)$. Thus by Kushnirenko's theorem [4] the spectrum of U_φ in $L^2(X,B,\mu)$ is discrete. Denote by (f_n) the sequence of all eigenfunctions of U_φ: $U_\varphi f_n=\lambda_n f_n$. For every $m=1,2,\dots$ there exists a continuous function g such that $(g,f_m)=a_m\neq 0$. In the space $L^2(X,B,\mu)$ we can write g in the form

$$g=\sum_{n=1}^{\infty}a_n f_n.$$

Applying U_φ^k to the last equality, where k is a fixed positive integer, we obtain

$$U_\varphi^k g=\sum_{n=1}^{\infty}a_n\lambda_n^k f_n,$$

and hence

$$\lambda_m^{-k}U_\varphi^k g=a_m f_m+\sum_{\substack{n=1\\ n\neq m}}^{\infty}a_n(\lambda_n\lambda_m^{-1})^k f_n.$$

By assumption the measure μ is ergodic and so $\lambda_n\neq\lambda_m$ for $n\neq m$. Therefore

$$(1)\qquad \frac{1}{p}\sum_{k=0}^{p-1}\lambda_m^{-k}U_\varphi^k g=a_m f_m+\sum_{\substack{n=1\\ n\neq m}}^{\infty}a_n\frac{1}{p}\,\frac{1-(\lambda_n\lambda_m^{-1})^p}{1-\lambda_n\lambda_m^{-1}}f_n.$$

Since $|\lambda_m|=1$ and the sequence $(U_\varphi^k g)$ is w*cc, the sequence $(\lambda_m^{-k}U_\varphi^k g)$ is also a w*cc sequence. We set $\lambda_m^{-1}U_\varphi=T$. In view of the remark in [2] on the p. 435 the arithmetic means

$$A_p g=\frac{1}{p}\sum_{k=0}^{p-1}T^k g$$

are also weakly* conditionally compact.

We shall prove that $(A_p g)$ is weakly* convergent for every $g\in C(X)$.

Let E be the space of all bounded functions $f\colon X\to\boldsymbol{R}$ of the first Baire class with norm $\|f\|=\sup_{x\in X}|f(x)|$.

From now on we proceed as in the proof of Theorem 3, Ch. 7.3 of [2]. Let $E_1=\overline{(I-T)E}$, $E_2=\{g\in E\colon (I-T)g=0\}$, and $E_3=\{\xi\in E^*\colon (I-T^*)\xi=0\}$.

We prove

(a) $E_1 \cap E_2 = \{0\}$.

Suppose now that $(A_p g)$ is not weakly* convergent for a certain $g \in C(X)$. Since $(A_p g)$ is weakly* conditionally compact, there exist two sequences of integers (n_i) and (m_i) such that $A_{n_i} g \underset{w^*}{\to} h_1$, $A_{m_i} g \underset{w^*}{\to} h_2$, $h_1 \neq h_2$. Of course, $h_1, h_2 \in E$. Next we prove

(b) $h_1, h_2 \in E$, which in view of (a) implies that $h_1 - h_2 \notin E_1$.

Then there exists a linear functional $\xi \in E^*$ such that $\xi / E_1 = 0$ and $\xi(h_1 - h_2) \neq 0$. Thus $\xi\big((I-T)g\big) = 0$ for every $g \in E$, which gives $(I - T^*)\xi = 0$; i.e. $\xi \in E_3^*$. Therefore

$$\lim_p \xi(A_p g) = \lim_p (A_p^* \xi)(g) = \xi(g),$$

which implies that

$$\xi(h_1) = \lim_i \xi(A_{n_i} g) = \lim_i \xi(A_{m_i} g) = \xi(h_2).$$

This contradicts the choice of the functional ξ.

Letting $p \to +\infty$ in (1), we get

$$h_m = a_m f_m,$$

so f_m is equivalent to $a_m^{-1} h_m$, which is of the first Baire class.

Corollary 1. *For every continuous function g the limit*

$$\lim_p \frac{1}{p} \sum_{k=0}^{p-1} U_\varphi^k g(x) = h(x)$$

exists and the function h satisfies the following equation: $h \circ \varphi(x) = h(x)$ for every $x \in X$. Hence we can choose f_n in such way that $U_\varphi f_n(x) = \lambda_n f_n(x)$ for every $x \in X$.

The corollary follows immediately from the proof of Theorem 1.

Corollary 2. *Suppose now that $X_0 \subset X$ is a minimal set for a* w*cc *system (X, φ). Then the eigenfunctions of U are continuous on the set X_0.*

Proof. Suppose that an eigenfunction f_m has a point of discontinuity $x_0 \in X_0$. Then the oscillation of f at x_0 is positive: $\omega(f_n, x_0) > 0$. Since X_0 is minimal, the trajectory $\big(\varphi^n(x_0)\big)_{n=0}^{\infty}$ is dense in X_0, and we immediately conclude that $\omega(f_n, x) > 0$ for every $x \in X_0$. But in view of Theorem 1 the function f_n is of the first Baire class, and so it must have at least one point of continuity in every non-empty closed set, whence also in X_0.

In [9] it is proved that if a system (X, φ) is strictly ergodic and has a continuous spectrum, then the averages $\dfrac{1}{p} \sum_{k=0}^{p-1} \lambda^k U_\varphi^k f(x)$ are uniformly convergent to zero for every $\lambda \in S^1$, $\lambda \neq 1$, and for every $f \in C(X)$.

Corollary 3. *Every* w*cc, *minimal dynamical system (X_0, φ) has a factor (Y, ψ) of the following type: Y is a closed subset of an abelian compact group G, Y is invariant with respect to a shift τ on the group, and φ is the restriction of τ to Y.*

Proof. We set

$$Y = \{y = (y_n):\ y_n = f_n(x),\ n = 1, 2, \ldots,\ x \in X\},$$
$$\tau(y) = (\lambda_n y_n), \quad h(x) = \big(f_n(x)\big), \quad G = S^1 \times S^1 \times \ldots$$

The system (Y, τ) is the h-image of the system (X_0, φ).

Corollary 4. *Every minimal,* w*cc *dynamical system (X_0, φ) is strictly ergodic.*

Proof. Let $g \in C(X)$. In view of Corollaries 1 and 2 the function $h(x) = \lim_p \dfrac{1}{p} \sum_{k=0}^{p} U_\varphi^k g(x)$ is φ-invariant and continuous. Thus it has to be constant, which completes the proof.

Consider now the case where $X = S^1$ and φ is a homeomorphism of the circle (see Example (ii)). If φ has some periodic points or if φ is conjugate to a rotation, then it is easy to construct all the eigenfunctions of U_φ. Consider now the case where φ is a Denjoy homeomorphism. Let f be an eigenfunction of U_φ, and let $I = (a, b)$ be an arc in S^1 which is a component of the set of all wandering points $S^1 - \Delta$. Since there are no periodic points, the arcs $\varphi^n(I)$ are pairwise disjoint, and thus $\mathrm{dist}\big(\varphi^{-n}(a), \varphi^{-n}(b)\big) \to 0$ as $n \to +\infty$. Hence

$$|f(b) - f(a)| = \Big|f\big(\varphi^n(\varphi^n(b))\big) - f\big(\varphi^n(\varphi^n(a))\big)\Big|$$
$$= \Big|\lambda^n\big(f(\varphi^{-n}(b)\big) - \lambda^n\big(f(\varphi^{-n}(a)\big)\Big| \to 0$$

as $n \to +\infty$.

Therefore every eigenfunction of U_φ has the same values at the end-points of the components of the set $S^1 - \Delta$. Let h be a Cantor function of Δ, i.e. h maps S^1 onto S^1, and h is continuous and constant on every component of the set $S^1 - \Delta$. It is easy to see that h also maps Δ onto S^1. The map φ induces a map $\tilde{\varphi}$ on $h(\Delta) = S^1$: $\varphi\big(h(x)\big) = h\big(\varphi(x)\big)$, $x \in \Delta$. It is easy to see that $\tilde{\varphi}$ is conjugate to a rotation of S^1, and so all the eigenfunctions $\tilde{f}_n$ of $U_{\tilde{\varphi}}$ are of the form $\tilde{f}_n(z) = \big(k(z)\big)^n$, where $z \in S^1$, k conjugates $\tilde{\varphi}$ and the rotation. Since every eigenfunction f_n of U_φ takes the same values at the end-points of the components of $S^1 - \Delta$, there exists a function g_n on $h(\Delta)$ such that $f_n(x) = g_n \circ h(x)$, $x \in S^1$. Hence f_n are of the form $f_n(x) = \big(k \circ h(x)\big)^n$. Denote by E the subspace of $C(\Delta)$ spanned by all the eigenfunctions of f_n restricted to Δ. In view of Kadec's result (see [7], Corrolaries 9.12 and 2.3) the subspace E has no closed complement in $C(\Delta)$.

K. Deleeuw and I. Glicksberg [1] have shown that every wcc dynamical system (X, φ) has the following property: the space $C(X)$ can be decomposed into the direct sum of two closed subspaces E and F, where E is the subspace spanned by all the eigenfunctions of U_φ. So we have the following

Remark 1. The Deleeuw and Glicksberg result cannot be extended onto the class of w*cc mappings.

3. The w*cc dynamical systems seem to be so close to isometries and so similar to homeomorphisms of the circle, that one could expect that every sequence entropy (for the definition see [3]) of a w*cc mapping is equal to zero. But this is not true.

In this section we shall present an example of a w*cc dynamical system for which a sequence entropy is positive. The space X will be chosen as a closed subset of the space of all zero-one sequences, invariant with respect to the left-side shift; the map φ will be the shift.

Let $n \geqslant 0$ be a fixed, positive integer and let r_n and l_n be two non-negative integers such that $r_n = 2^n l_n$. Consider the family of all sequences $v_n = (v_1, v_2, \ldots, v_{r_n})$ where $v_k = 0$ for $k \neq il_n$ and v_k is arbitrary for $k = il_n$ (i.e. v_k is either 0 or 1). Obviously there are 2^{2^n} sequences v_n. Denote them by $v_n^{(1)}, v_n^{(2)}, \ldots, v_n^{(2^{2^n})}$. Let V_n be the sequence formed by all $v_n^{(i)}$ written in one row:

$$V_n = v_n^{(1)} v_n^{(2)} \ldots v_n^{(2^{2^n})}.$$

Now we set

$$x_0 = V_0 V_1 V_2 \ldots V_n \ldots \in \{0, 1\}^Z.$$

We have to define the integers l_n. Let $v_n^{(1)} = (0)$, $v_0^{(2)} = (1)$, i.e. $l_0 = 1$, $r_0 = 1$. We require that l_n should be greater that the length of the sequence $V_0 V_1 V_2 \ldots V_{n-1}$. Since the length of V_i is equal to $2^{2^i} r_i$, the number l_n has to be greater than $\sum_{i=0}^{n-1} 2^{2^i} r_i$.

So we set

$$l_n = n 2^{2^n} r_{n-1} = n 2^{2^n + n} l_{n-1}.$$

Denote the coordinates of x_0 by $\xi_0, \xi_1, \ldots$ Let p_n be an integer such that

$$(\xi_0, \ldots, \xi_{p_n}) = V_0 V_1 \ldots V_n \quad \text{for} \quad n = 0, 1, \ldots$$

In other words, $p_n - p_{n-1}$ = length of V_n.

Denote $x_n = \varphi^n(x_0)$ and set X = closure $\{x_n, n = 0, 1, \ldots\}$.

PROPOSITION 3. *The space X contains only points of the form* (i) x_n, $n = 0, 1, \ldots$, (ii) $e_n = (0, 0, \ldots, 1, 0, \ldots)$, $n = 0, 1, \ldots$, (iii) $(0, 0, \ldots)$.

Proof. Suppose $x = (\eta_0, \eta_1, \ldots)$ is a cluster point of the set $\{x_n\}$. Then there exists a sequence (x_{n_k}), $n_k < n_{k+1}$, such that $x_{n_k} \to x$. Suppose that $\eta_m = 1$ for an index m. Thus for k large enough the mth coordinates of x_{n_k} have to be 1. Let $i \neq m$ be fixed. Since $l_n \to +\infty$, all the coordinates of x_{n_k} have to be zero for k large enough. Hence $\eta_i = 0$ for arbitrary $i \neq m$ and $e_m = x \in X$. Since $e_m \to (0, 0, \ldots)$, the point $(0, 0, \ldots)$ also belongs to X.

COROLLARY 5. *The system (X, φ) is* w*cc.

Indeed, by Proposition 3 the space X is denumerable, and so every map of X is w*cc.

PROPOSITION 4. *The sequence entropy of the system (X, φ) is positive.*

Proof. Let $A_n = \bigcup_{i=0}^{n} \{p_i + sl_i\}_{s=1}^{i}$. Then $\operatorname{Card} A_n = \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$. We set $A = \bigcup_{n=0}^{\infty} A_n$ and we choose an open cover of X in the standard way:

$$a = (\{x = (\zeta_i) \colon \zeta_0 = 0\}, \{x = (\zeta_i) \colon \zeta_0 = 1\}).$$

(It is also a partition of X.) In view of the definition of v_n, V_n and l_n it is easy to check that the cover $a_n = \bigvee_{k \in A_n} \varphi^{-k}(a)$ contains at least as many sets as the cardinality of the set of all v_n, i.e.

$$\operatorname{Card} \bigvee_{k \in A_n} \varphi^{-k}(a) \geqslant 2^{2^n}.$$

Thus

$$\frac{1}{\operatorname{Card} A_n} \log N(a_n) \geqslant \frac{1}{2^{n+1}-1} \log 2^{2^n} = \frac{2^n}{2^{n+1}-1} \log 2.$$

Hence

$$h_A(\varphi, a) = \limsup_n \frac{1}{\operatorname{Card} A_n} \log N(a_n) \geqslant \frac{1}{2} \log 2.$$

The example presented above gives us the following

COROLLARY 6. *The sequence entropy cannot be attained on the set of non-wandering points.*

Remark 2. The example presented shows also that the supremum of all measure sequence entropies is not equal to the topological sequence entropy. This result has been observed by Goodman [3]. This example is different from that of Goodman.

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Received April 19, 1977 (1298)

STUDIA MATHEMATICA, T. LXVI. (1979)

The algebra of compact operators does not have any finite-codimensional ideal

by

PIERRE DE LA HARPE (Genéve)

Abstract. The Lie algebra of compact operators on an infinite dimensional Hilbert space has no non-trivial finite-dimensional quotient. It follows that, as a Banach–Lie algebra, all its extensions are trivial.

Let H be an infinite-dimensional real or complex Hilbert space and let $C(H)$ be the algebra of all compact operators on H. It is well known that any non-trivial two-sided ideal of $C(H)$ contains the ideal of all finite rank operators (Theorem 1.7 in [2]); the standard examples are the von Neumann–Schatten ideals [5]. The purpose of this note is to show that any such ideal has infinite codimension. We shall indeed prove the following stonger statement, which is phrased in the Lie algebra setting; as a corollary we obtain that any extension (in a suitable sense) of the Lie algebra considered is trivial.

THEOREM. *Let $\boldsymbol{gl}(H, C)$ be the Lie algebra defined by the commutator product on $C(H)$ and let $\boldsymbol{a}$ be a non-trivial ideal in $\boldsymbol{gl}(H, C)$. Then $\boldsymbol{a}$ contains the space $\boldsymbol{sl}(H, C_0)$ of all finite rank operators with zero trace and $\boldsymbol{a}$ has infinite codimension.*

That $\boldsymbol{a}$ contains $\boldsymbol{sl}(H, C_0)$ is an easy corollary of Schur's lemma and of the simplicity of the Lie algebra $\boldsymbol{sl}(H, C_0)$; see for example [3], page I.2. For the second statement, consider a finite-dimensional Lie algebra $\boldsymbol{g}$, say, of dimension $d > 0$, and a morphism $\pi\colon \boldsymbol{gl}(H, C) \to \boldsymbol{g}$. We have to show that the kernel of π is $\boldsymbol{gl}(H, C)$ itself. We do this below in the complex case; the real case will follow from a standard complexification argument.

We denote by $\boldsymbol{N}$ the set of natural integers including zero and by c_0 the space of sequences $(x_n)_{n\in \boldsymbol{N}}$ of complex numbers converging towards zero.

LEMMA 1. *Let H_0 be a closed subspace of H of infinite dimension and infinite codimension. Let $(e_n)_{n\in \boldsymbol{N}}$ be an orthonormal basis in H_0 and let $(x_n)_{n\in \boldsymbol{N}}$*