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The Pełczyński property for some uniform algebras

by

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Abstract. Let A be a separable uniform algebra on its Shilov boundary X. If there are no singular orthogonal measures and if each element of the spectrum has a weakly compact set of representing measures, then A has the Pełczyński property. The theorem can be applied in the case of Banach algebras of analytic functions on suitable compact sets of the plane.

1. Introduction. In [11] Wojtaszczyk proved that some uniform algebras have the Pełczyński property. For the disc algebra this property was already known (see Kislakov [9] and Delbaen [2]). The present paper generalizes the results of [11] in two ways:

(i) the separability of the annihilator is dropped, and

(ii) the unicity of the representing measure is replaced by an assumption of weak compactness.

For any unexplained notion on uniform algebras we refer to Gamelin [5].

2. Wilken algebras. If A is a point separating subalgebra of $\mathscr{C}(X)$ (X a compact space) such that $1 \in A$, then we say that A is a *uniform algebra*. For simplicity we assume that X is the Shilov boundary of A. A positive measure m on X is *multiplicative* if $\int f\cdot g\,dm = \int f\,dm \cdot \int g\,dm$ for all f and g in A. From the Hahn–Banach theorem we learn that every nonzero multiplicative linear functional on A can be represented by such a measure. If μ is any measure on X, then μ is called *orthogonal* when $\int f\,d\mu = 0$ for all $f \in A$.

DEFINITION. An algebra is called a *Wilken algebra* if the only orthogonal measure which is singular to all multiplicative measures is the zero measure. (Wilken (see [5]) proved that $R(K)$ is such an algebra).

Notation. If X is a compact metric space which is the Shilov boundary for the uniform algebra $A \subset \mathscr{C}(X)$, then we denote by ∂X the set of peak points for A. This set is equal to the Choquet boundary of A.

THEOREM 1. *Let A be a Wilken algebra on the compact metric space X.*

If μ is a probability measure on X which is singular to all multiplicative measures, then $\mu(\partial X) = 1$.

Proof. Let $V = \{a^* \in A^*:\ a^*(1) = 1 = \|a^*\|\}$. We identify X with the subset of V. By Gamelin II. 11.5, ∂X can be identified with the set of extreme points of V.

The functional $a_0^*(f) = \int f d\mu$ is in V, so there exists a measure ξ on ∂X such that $a_0^*(f) = \int_{\partial x} f(t)\,d\xi(t)$. If we consider ξ as a measure on X we will have $\mu - \xi \in A^\perp$. Since μ is singular and this difference is absolutely continuous with respect to $\sum a_i m_i$, m_i multiplicative, we get $\xi = \mu + \beta$, $\mu \perp \beta$ for some β. But $\|\xi\| = \|\mu\|$; hence $\beta = 0$, so $\xi = \mu$ and, in particular, $\mu(\partial X) = 1$. ∎

The following theorem shows that a Wilken algebra has sufficiently many peak sets.

THEOREM 2. *Let A be a separable Wilken algebra on the compact metric space X. If μ is a singular probability measure (i.e. μ is singular to all multiplicative measures), then $\forall \varepsilon > 0\ \exists L' \subset \partial X$, a compact peak interpolating set, such that $\mu(L') > 1-\varepsilon$.*

Proof. The proof is divided into several lemmas.

Notation. If E is a Banach space, then $B(E)$ denotes the closed unit ball of E.

LEMMA 1. *If μ is singular, then $B(A)$ is $\sigma(L^\infty(\mu), L^1(\mu))$ dense in $B(L^\infty(\mu))$.*

Proof of Lemma 1. Suppose that $B(A)$ is not weak* dense in $B(L^\infty)$. Then $\exists g \in L^1(\mu)$ and $h \in B(L^\infty)$ such that

$$\alpha = \sup\left\{\left|\int gf\,d\mu\right| \mid f \in B(A)\right\} < \left|\int gh\,d\mu\right| = \beta.$$

Consider the linear map $t:\ A \to C$ defined as $t(f) = \int gf\,d\mu$. Clearly, $\|t\| = \alpha$. By the Hahn-Banach theorem, there is a norm preserving extension $\nu:\ \mathscr{C}(X) \to C$, $\|\nu\| = \alpha$.

Let $d\nu = k\,d\mu + d\nu_s$ be the Lebesgue decomposition of ν with respect to μ. Since ν is an extension of t, we have that $\nu - t \perp A$. But $d(\nu - t) = (k-g)\,d\mu + d\nu_s$ and since A is a Wilken algebra, $\nu - t$ must be absolutely continuous with respect to a sequence of multiplicative measures hence $k = g$ μ-almost sure. This in turn implies $\alpha = \|\nu\| = \|k\|_1 + \|\nu_s\| = \|g\|_1 + \|\nu_s\|$. Hence $\beta \leqslant \|g\|_1 \leqslant \alpha$; a contradiction.

LEMMA 2. *$\forall \varepsilon > 0\ \exists L$, a compact set, such that $\mu(L) > 1-\varepsilon$ and the mapping*

$$A \to \mathscr{C}(L),$$
$$f \to f|_L$$

is a quotient mapping.

Proof of Lemma 2. Let h_n be a dense sequence in $B(\mathscr{C}(X))$. Let g_n be an infinite sequence of functions in $B(\mathscr{C}(X))$ such that every h_n occurs an infinite number of times. Let $\delta_n > 0$ be decreasing to zero and $\varepsilon_n > 0$ be such that $\sum \varepsilon_n < \varepsilon$. From Lemma 1 we know that $B(A)$ is dense in $B(L^\infty)$ for the weak* topology. Since it is absolutely convex, it is also dense for the Mackey topology $\tau(L^\infty, L^1)$. This topology, when restricted to $B(L^\infty)$, is nothing else but the convergence in μ-measure (see [6]).

So there is a sequence s_n in $B(A)$ tending to g_1 in μ-measure. By selecting a subsequence, we may suppose that this sequence s_n converges almost everywhere, and by Egorov's theorem, we can find a compact set L_1, $\mu(L_1) > 1-\varepsilon_1$ and n_0 such that for all $n \geqslant n_0$ $|s_n - g_1| < \delta_1$ (almost) everywhere on L_1. Continuing this procedure we find for all n a function $f_n \in B(A)$ and a compact set L_n such that

$$\|f_n - g_n\|_{L_n} < \delta_n,$$
$$\mu(L_n) > 1-\varepsilon_n.$$

If $L = \bigcap_{n\geqslant 1} L_n$, then $\mu(L) > 1-\varepsilon$. For every $h \in \mathscr{C}(L)$, $\|h\| \leqslant 1$, there is a function $h' \in B(\mathscr{C}(X))$ such that $h'|_L = h$. Let now $\delta > 0$ be arbitrary and let n_0 be such that $\delta_n < \delta/2$, for $n \geqslant n_0$. Let also h_k be such that $\|h_k - h'\| < \delta/2$. Since h_k occurs infinitely many times, there is $n_1 \geqslant n_0$ such that $g_{n_1} = h_k$. Also $\|g_{n_1} - f_{n_1}\|_{L_{n_1}} < \delta_{n_1} < \delta/2$.

Hence $\|h - f_{n_1}\|_L \leqslant \|h - h_k\|_L + \|h_k - f_{n_1}\|_L \leqslant \delta/2 + \delta/2 = \delta$. It follows that $B(A)|_L$ is dense in $B(\mathscr{C}(L))$ and from the open mapping principle it then follows that the restriction map $A \to \mathscr{C}(L)$ is a quotient mapping.

LEMMA 3. *If m is multiplicative on A and if L is not an atom for m, then $m(L) = 0$.*

Proof of Lemma 3. Suppose $m(L) > 0$ and suppose $C \subset L$ such that $0 < m(C) < m(L)$.

Since $B(A)|_L$ is dense in $B(\mathscr{C}(L))$, there is $f_n \in B(A)$ such that on L, $f_n \to 1_C$ in m-measure or even better almost everywhere. Since $H^\infty(m)$ is weak* closed, we have (take a subsequence) that $f_n \to f$ for $\sigma(L^\infty(m), L^1(m))$ and for $f \in B(H^\infty)$. By general arguments on duality and since the Mackey topology on $B(L^\infty(m))$ is the convergence in m-measure, we obtain that $f|_L = 1_C$ m-almost everywhere. Since m is multiplicative, we obtain $m(f^n) = m(f)^n$ and since $|m(f)| \leqslant \int |f|\,dm \leqslant 1 - m(L\setminus C) < 1$, we deduce $m(f^n) \to 0$. Let now $g_n = \sum_{k=1}^{n} f^k/n$ then $g_n \to 1_{\{f=1\}}$. Also $m(g_n) = \frac{1}{n}\big(m(f) + \ldots + m(f^n)\big) \to 0$ and so $m(C) \leqslant m(f=1) = 0$; a contradiction.

Let now $L' \subset L \cap \partial X$ be such that $\mu(L') > 1-\varepsilon$. By regularity and by Theorem 1, this is possible.

LEMMA 4. *$m(L') = 0$ or 1 for all multiplicative measures m.*

Proof of Lemma 4. If L' is not an atom for m, then by Lemma 3 $m(L') = 0$. If, on the other hand, L' is an atom for m', then there is $x \in L'$ and $1 \geqslant a > 0$ such that $m|_{L'} = a\delta_x$. It follows that $m = a\delta_x + (1-a)m'$. Since m and δ_x are not mutually singular, they are in the same Gleason part. But x is a peak point; hence is a trivial Gleason part and so $m = \delta_x$, proving that $m(L') = 1$.

Proof of the theorem. Since L' is clearly interpolating, we only have to show that it is a peak set. We use Glicksberg's criterion ([5]). Let $\lambda \perp A$. Since A is a Wilken algebra, there is a sequence of mutually singular multiplicative measures m_n such that $\lambda = \sum \lambda_n$ and λ_n is absolutely continuous with respect to m_n. Also by Riesz' theorem $\lambda_n \perp A$. Now $\lambda|_{L'} = \sum_{m_n(L')=1} \lambda_n$ is clearly orthogonal to A. ■

3. The Havin lemma. To prove the Pełczyński property (see definition below) one needs functions which look like humps. For analytic functions this is not possible and so a substitute is needed. Havin [7] therefore proved an ingenious lemma. Kislakov [9] and Wojtaszczyk used this idea to prove the Pełczyński property. (In Delbaen [2] another but related approach is taken). The purpose of this chapter is to generalize Havin's lemma.

A will be a uniform algebra on its Shilov boundary X. For each $\Phi: A \to \mathbf{C}$ in the spectrum of A (i.e. Φ is a non-zero multiplicative linear functional) we denote by M_Φ the set of positive representing measures, i.e.:

$M_\Phi = \{m \mid m$ positive measure on X such that $m(f) = \Phi(f)$ for all $f \in A\}$.

We suppose throughout this section that M_Φ is weakly compact, i.e. $\sigma(M(X), M(X)^*)$ compact where $M(X)$ is the Banach space of Radon measures on X.

It follows from [4], p. 307 that if M_Φ is weakly compact, then there is a measure $m \in M_\Phi$ such that $M_\Phi \subset L^1(m)$. (Of course, a measure absolutely continuous with respect to m is identified with its Radon Nikodym derivative.)

HAVIN'S LEMMA. *Under the above weak compactness conditions we have: For all $\varepsilon > 0$ there is $\eta > 0$ with the property: For all closed set E, $m(E) < \eta$, there are k_E and $K_E \in A$ such that*

(a) $|k_E(t)| + |K_E(t)| \leqslant 1$ *for all* $t \in X$,

(b) $\sup_{t\in E} |K_E(t) - 1| < \varepsilon$ *and* $\int |K_E|\,dm < \varepsilon$,

(c) $\sup_{t\in E} |k_E(t)| < \varepsilon$ *and* $\int |1-k_E|\,dm < \varepsilon$.

Proof. We shall use following result due to Kolmogorov: there is a constant a such that for all $f \in A$ with $f = u + i\tilde{u}$, $u \geqslant 0$, $\int \tilde{u}\,dm = 0$ we have

$$\|\tilde{u}\|_{1/2} = \Big(\int |\tilde{u}|^{1/2}\,dm\Big)^2 \leqslant a\|u\|_1 = a\int u\,dm.$$

For the proof see [5], p. 99.

Part I. *Construction of K_E.* Let $\varepsilon > 0$ be given. Let $M < \infty$ be such that $|e^z - 1| < \varepsilon/2$ for z complex and $|z| \leqslant 1/M$. Let $\delta_1 > 0$ be such that (i) $a^{1/2}\delta_1^{1/8} < \varepsilon/3$, (ii) $\exp(-\frac{1}{3}\delta_1^{-1/2}) < \varepsilon/6$. Let $\delta > 0$ be such that $M\delta + \frac{1}{2}\delta_1 < \delta_1$ (or $M\delta < \frac{1}{2}\delta_1$). Also let $\varepsilon/2 > \eta_1 > 0$ be such that for all Borel sets $C \subset X$, $m(C) < \eta_1$ implies $\sup_{\mu\in M_\Phi} \mu(C) < \delta$. The existence of η_1 follows from weak compactness. Let now E be a closed set such that $m(E) < \eta_1$. By regularity of the measure m, there is $G \supset E$, G open and $m(G) < \eta_1$. Since $m(G) < \eta_1$, we have $\sup_{\mu\in M_\Phi} \mu(G) < \delta$. Let now f be a function in $C_{\mathbf{R}}(X)$ such that $f = -\frac{1}{2}\delta_1$ on G^c, $f = -M$ on E and $-\frac{1}{2}\delta_1 \geqslant f \geqslant -M$ elsewhere. For all $\mu \in M_\Phi$ we then have

$$\int f\,d\mu \geqslant -M\mu(G) - \delta_{1/2} \geqslant -M\delta - \tfrac{1}{2}\delta_1 > -\delta_1.$$

Hence $\inf_{\mu\in M_\Phi} \int f\,d\mu > -\delta_1$. From [5], p. 32 it then follows that there is $h \in A$ such that

$$\omega = \operatorname{Re} h \leqslant f \quad \text{and} \quad \int h\,dm = \int \operatorname{Re} h\,dm > -\delta_1.$$

Let $h = \omega + i\tilde{\omega}$. By the holomorphic functional calculus ([5]), we obtain that $h^{-1} \in A$ (remark that $\operatorname{Re} h < -\frac{1}{2}\delta_1$). Let $K = \exp h^{-1}$. Since $\operatorname{Re} h^{-1} < 0$, is it clear that $|K| \leqslant 1$. On E we have $|h^{-1}(t)| = \left|\dfrac{\omega - i\tilde{\omega}}{\omega^2 + \tilde{\omega}^2}\right| \leqslant \dfrac{|\omega|}{\omega^2+\tilde{\omega}^2} \leqslant \dfrac{1}{|\omega|} \leqslant \dfrac{1}{M}$, hence $|K(t) - 1| < \varepsilon/2$.

We now estimate $\int |K(t)|\,m(dt)$:

(α) On G we have $\int_G |K(t)|\,dm \leqslant m(G) < \eta_1 \leqslant \varepsilon/2$.

(β) On G^c and on $\{|\tilde{\omega}| \leqslant \delta_1^{3/4}\}$

$$\frac{\omega}{\omega^2+\tilde{\omega}^2} \leqslant \frac{\omega}{\omega^2 + \delta_1^{3/2}} = \frac{-\frac{1}{2}\delta_1}{\frac{1}{4}\delta_1^2 + \delta_1^{3/2}} = \frac{-\delta_1}{\frac{1}{2}\delta_1^2 + 2\delta_1^{3/2}} \leqslant \frac{-\delta_1}{3\delta_1^{3/2}} = -\frac{1}{3\delta_1^{1/2}}.$$

Hence $|K(t)| = \exp\dfrac{\omega}{\omega^2+\tilde{\omega}^2} \leqslant \exp\left(-\dfrac{1}{3\delta_1^{1/2}}\right) < \dfrac{\varepsilon}{6}$.

(γ) On G^c and on $\{|\tilde{\omega}| > \delta_1^{3/4}\}$

$$\int_{G^c \cap \{|\tilde{\omega}| > \delta_1^{3/4}\}} |K(t)|\,dm \leqslant m(\{|\tilde{\omega}| > \delta_1^{3/4}\}).$$

Since $\omega \leqslant 0$, we can apply Kolomogorov's result and hence

$$\left(\int |\tilde{\omega}|^{1/2}\,dm\right)^2 \leqslant a\|\omega\|_1 .$$

So $m(|\tilde{\omega}| > \delta_1^{3/4})^2\delta_1^{3/4} \leqslant a\delta_1$ and hence $m(|\tilde{\omega}| > \delta_1^{3/4}) < \delta^{1/2}\delta_1^{1/8}$, summing (α), (β) and (γ) we have

$$\int |K(t)|\,dm < \varepsilon/2+\varepsilon/3+\varepsilon/6 = \varepsilon .$$

Put now $K_E = (1-\varepsilon/2)K$; then

(i) $\int |K_E|\,dm < \varepsilon$.

(ii) On E: $|K_E(t)-1| = |(1-\varepsilon/2)K(t)-1| \leqslant \varepsilon/2+|K(t)-1| < \varepsilon$.

Part II. *Construction of* k_E. Let $\beta = \log(1-|K_E|) \geqslant \log(\varepsilon/2)$. Since $K_E \to 0$ in measure if $m(E)\to 0$ and since the family $\{\log(1-|K_E|)\,|\,E$ closed$\}$ is uniformly bounded by $-\log(\varepsilon/2)$, we have that the weak compactness of M_Φ implies

$$\inf_{\mu\in M_\Phi} \int \log(1-|K_E|)\,dm \to 0 .$$

From [5], p. 32 it follows that there are elements $v_E \in \operatorname{Re} A$, $v_E \leqslant \beta$ such that $\int v_E\,dm \to 0$. If $k_E = \exp(v_E+i\tilde{v}_E)$, then $|k_E| = \exp v_E \leqslant \exp\beta = 1-|K_E|$. Hence $|k_E|+|K_E| \leqslant 1$ and hence on E $|k_E| \leqslant \varepsilon$. Also $\int k_E\,dm = \int \exp(v_E+i\tilde{v}_E)\,dm = \exp(\int v_E\,dm) \to 1$ as $m(E)\to 0$. Since $|k_E| \leqslant 1$, this implies $k_E \to 1$ in measure. Hence, for $m(E)$ small enough (say, $m(E) < \eta \leqslant \eta_1$), we have $\int |1-k_E|\,dm < \varepsilon$.

Remark. By taking η small enough, one can even obtain

$$\sup_{\mu\in M_\Phi} \int |1-k_E|\,d\mu < \varepsilon \quad \text{and} \quad \sup_{\mu\in M_\Phi} \int |K_E|\,d\mu < \varepsilon .$$

4. The Pełczyński property.

DEFINITION. A sequence f_n in a Banach space B is *weakly unconditionally converging* (WUC) if for every $b \in B^*$

$$\sum_{n=1}^{\infty} |b^*(f_n)| < \infty .$$

DEFINITION. A Banach space B has the *Pełczyński property* if every set $V \subset B^*$ such that

$$\lim_{n\to\infty} \sup_{b^*\in V} |b^*(f_n)| = 0 \quad \text{for every } (f_n) \text{ WUC}$$

is relatively weakly compact $(\sigma(B^*, B^{**}))$.

From [1] it follows that the Pełczyński property is equivalent with the following: If $b_n^* \in B^*$ is a bounded sequence such that $b_n^*(b_n) \to 0$ for all sequences b_n, equivalent with the unit vector base in c_0, then b_n^* is relatively weakly compact. Suppose now that B has the Pełczyński property and that $V \subset B^*$ is a bounded set which is not relatively weakly compact. By the Eberlein theorem there is a sequence $b_n^* \in V$ with no weakly convergent subsequence and this implies that no subsequence is relatively weakly compact. This remark will be used whenever a subsequence is needed. We now state the main theorem.

THEOREM. *Let A be a uniform algebra on a compact metric space X (X is supposed to be the Shilov boundary for A). We suppose that A is a Wilken algebra and for each multiplicative linear functional Φ we suppose the set of positive representing measures to be weakly compact. Under these conditions we have that A has the Pełczyński property.*

Proof. Using the same ideas (an application of Zorn's lemma) as in [3], we see that

$$A^* = \Big(\sum_{i\in I} \oplus L^1(m_i)/H^\infty(m_i)^\perp\Big)_{l_1} \oplus_1 \Big(\sum_{j\in J} \oplus L^1(m_j)\Big)_{l_1} .$$

All the measures $(m_s)_s \in I\cup J$ are mutually singular. Let now a_n^* be a sequence in A^* with no weakly converging subsequence. As in [11] we suppose that $\|a_n^*\| = 1 = a_n^*(g_n)$ where $g_n \in A$ and $\|g_n\| = 1$. The measure ν_n is then defined as a norm preserving Hahn–Banach extension of a_n^* to $\mathscr{C}(X)$. The probability measure η_n is defined as $d\eta_n = g_n\,d\nu_n$. Each η_n can be decomposed as

$$d\eta_n = \sum_{i\in I} \varphi_{i,n}\,dm_i + d\eta_n^s = d\eta_n^a + d\eta_n^s ,$$

where η_n^s are singular measures and where only a countable number of the functions $\varphi_{i,n}$ are nonzero. Since taking subsequences does not do any harm, we may suppose that $\eta_n^a \to \tilde{\mu}$ (weak*).

Furthermore, $\tilde{u} = \mu_a+\mu_s$, where μ_s is a singular measure and $\mu_a \in \big(\sum_{i\in I} \oplus L^1(m_i)\big)_{l_1}$. Only a countable number of measures m_i and m_j occur in the decompositions, so we may suppose that there is subsequence m_k in $(m_i)_I$ such that

$$\mu_a \ll \sum_{k=1}^{\infty} \frac{1}{2^k} m_k ;$$

for all n:

$$\eta_n^a \ll \sum_{k=1}^{\infty} \frac{1}{2^k} m_k .$$

We consider different cases.

1. *The sequence η_n^s is not weakly relatively compact.* Since only a countable number of measures are involved, there is (by the Lebesgue decomposition theorem) a Borel set $E \subset X$ such that $m_k(E) = 0$ for all k and $\eta_n^s(E^c) = 0$ for all n. Since the sequence η_n^s is not relatively weakly compact, there is a sequence O_n of disjoint open sets as well as $\varepsilon > 0$ such that $\eta_n^s(O_n) > \varepsilon$ (see [6]). Hence $\eta_n^s(O_n \cap E) > \varepsilon > 0$.

By Theorem 2, for each n there is a peak interpolating set L_n such that $\eta_n^s(L_n) > (1-\varepsilon/2)\eta_n^s(X)$. It follows that $\eta_n^s(L_n \cap O_n \cap E) > \varepsilon/2$. By regularity, there is a compact set $K_n \subset O_n \cap L_n \cap E$ such that $\eta_n^s(K_n) > \varepsilon/2$. Since a compact set in a metrizable peak interpolating set is necessarily itself a peak set, we see that there are functions $f_n \in A$, $f_n|_{K_n} = 1$ and $|f_n| < 1$ on K_n^c. Replacing f_n by $f_n^{s_n}$, where s_n is sufficiently large gives a sequence equivalent with the unit vector base of c_0 such that $|\eta_n^a(f_n^{s_n})| < \varepsilon/4$ for all n and such that $|\eta_n^s(f_n^{s_n})| > \varepsilon/2$. Hence $|\eta_n(f_n^{s_n})| > \varepsilon/4$. Let now $h_n = f_n^{s_n} \cdot g_n$; then $a_n^*(h_n) = v_n(g_n f_n^{s_n}) = \eta_n(f_n^{s_n})$ and so $|a_n^*(h_n)| > \varepsilon/4$. Since $f_n^{s_n}$ is WUC we also have that h_n is WUC. The end of part I.

2. *μ_n^s is relatively weakly compact but $\mu_s \neq 0$.*

3. *μ_n^s is relatively weakly compact and $\mu_s = 0$.*

The proof in the above cases is an exact copy of Wojtaszczyk's proof [11], Theorem 2.4.

5. Concluding remarks. A proof along the one used in [2] can also be given. For some consequences of the Pełczyński property we refer to Pełczyński's text [10].

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