[7] T. Figiel, *An example of infinite dimensional reflexive Banach space non-isomorphic to its Cartesian square*, Studia Math. 42 (1972), pp. 295–306.
[8] — *Factorization of compact operators and applications to the approximation problem*, ibid. 45 (1973), pp. 191–210.
[9] C. W. Henson and L. C. Moore Jr., *The theory of nonstandard topological vector spaces*, Trans. Amer. Math. Soc. 172 (1972), pp. 405–435.
[10] — *Subspaces of the nonstandard hull of a normal space*, ibid. 197 (1974), pp. 131–143.
[11] — *Nonstandard hulls of the classical Banach spaces*, Duke Math. J. 41 (2) (1974), pp. 277–284.
[12] J. Horvath, *Topological vector spaces and distributions*, Vol. 1, Reading, Mass. 1966.
[13] H. Jarchow, *Die Universalität des Räumes c_0 für die Klasse der Schwartz-Räume*, Math. Ann. 203 (1973), pp. 211–214.
[14] J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces*, Berlin 1970.
[15] A. Pełczyński and H. P. Rosenthal, *Localization techniques in L^p spaces*, Studia Math. 52 (1975), pp. 263–289.
[16] D. J. Randtke, *A simple example of a universal Schwartz space*, Proc. Amer. Math. Soc. 37 (1973), pp. 185–188.
[17] — *On the embedding of Schwartz spaces into product spaces*,
[18] A. Robertson and W. d. Robertson, *Topological vector spaces*, 2nd ed. London 1973.
[19] A. Robinson, *Nonstandard analysis*, Amsterdam 1968.
[20] A. Robinson and E. Zakon, *A set-theoretical characterization of enlargements*, in *Applications of model theory*, New York 1969, pp. 109–122.
[21] H. H. Schaefer, *Topological vector spaces*, Berlin 1970.
[22] I. Singer, *Bases in Banach spaces I*, Berlin 1970.

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STUDIA MATHEMATICA, T. LXII. (1978)

Boundary limits of Green's potentials along curves II
Lipschitz domains

by

JANG-MEI G. WU (W. Lafayette, Ind.)*

Abstract. On a Lipschitz domain D in space, let μ be a mass distribution and u the Green potential of μ. Conditions on μ are given so that $u \not\equiv +\infty$; under the same condition we show that the boundary limits of u along curves with certain differentiability properties are zero almost everywhere.

Green's potential occurs in the study of subharmonic and superharmonic functions via Riesz decomposition theorem ([5], p. 116). Let D be an open subset of $\boldsymbol{R}^n$ having a Green's function G; Green's potential u given by a mass distribution μ is defined to be

$$u(x) = \int_D G(x, y)\, d\mu(y) \tag{0.1}$$

for every $x \in D$. When D is the unit disk in the plane, the necessary and sufficient condition for $u \not\equiv +\infty$ is

$$\int_D (1-|y|)\, d\mu(y) < +\infty;$$

under this condition u has radial limit zero at almost every point on the unit circle, see Littlewood [6]. Later in 1938, Privalov [7] proved the similar result for Green's potentials on the unit ball in $\boldsymbol{R}^n$. The nontangential limit of Green's potential need not exist at any point on the boundary, as pointed out by Zygmund, [9], pp. 644–645.

The purpose of this paper is to study the boundary limits of Green's potentials in a Lipschitz domain D in $\boldsymbol{R}^n$, $n \geqslant 3$ along curves with certain differentiability properties. The problem for $n = 2$ was studied in [11], where, with the aid of conformal mapping, we need only to study the limit of Green's potentials on $|z| < 1$ along curves with the same differentiability properties. When $n \geqslant 3$ the conformal mapping technique does not apply and it is not even obvious for which μ the Green's potential of μ is not identically $+\infty$. Our main tool is an estimate on a certain harmonic function in a cone derived from a series representation of that

* Current address: Purdue University, West Lafayette, Indiana.

function; the representation was used by Gariepy and Lewis, [4], pp. 261–262, to obtain a Phragmén–Lindelöf result for subharmonic functions in $\boldsymbol{R}^n$. In Theorem 1, we shall give a sufficient condition on μ in D for $u \not\equiv +\infty$ and in Theorems 2 and 3 we shall show that under this condition, the Green potential has the desired boundary property.

1. Preliminaries. We use $(x_1, x_2, \ldots, x_n)$ to denote a point x in $\boldsymbol{R}^n$, $n \geqslant 3$, $|x| = (\sum |x_k|^2)^{1/2}$, $x' = (x_2, \ldots, x_n)$ and $\cos\theta = x_1/|x|$ if $|x| \neq 0$. We denote the cone $\{x\colon |\theta| < t,\ |x| < h\}$ by $K(t, h)$. The symbol C will denote strictly positive constants that may vary from line to line.

From now on we shall let D be a bounded Lipschitz domain in $\boldsymbol{R}^n$, $n \geqslant 3$. That is, ∂D can be covered by a family of open right cylinders L; there is a local coordinate system $x = (x_1, x')$ corresponding to each L with $x_1 \in \boldsymbol{R}$, $x' \in \boldsymbol{R}^{n-1}$ and x_1-axis parallel to the axis of L so that $x_1 = f(x')$ is Lipschitz for x on $\partial D \cap L$ and $L \cap D = L \cap \{x\colon x_1 > f(x')\}$. And we shall let $\alpha > 0$, $a > 0$ be two number depending on D so that at every point x on ∂D there is a cone with vertex x of size $K(\alpha, a)$ completely exterior to $\bar{D}$.

We use $d(x)$ to denote the distance from a point $x \in D$ to ∂D; if $x \in L \cap D$, we use $\tilde{x}$ to denote the point on $\partial D \cap L$ with $\tilde{x}' = x'$ in the local coordinate system in L. It is clear that if x is in L and the point on ∂D closest to x is in $L \cap \partial D$, then $|x - \tilde{x}| \leqslant Cd(x)$. Let $G(x, y)$ be the Green's function on D, μ be a positive mass distribution on D and u be the Green's potential of μ given by (0.1).

For the properties of Lipschitz domain the reader is referred to [8]. For the properties of Green's potentials in general the reader is referred to [5].

Here we shall give an estimate of certain harmonic functions in cones. The two-dimensional version of Lemma 1 can be proved easily by conformal mapping.

LEMMA 1. *Let v, $0 < v < 1$, be a harmonic function on $K(t, h)$ symmetric about x_1-axis (that is, v can be regarded as a function of $|x|$ and θ alone) and with boundary value 0 on $\partial K(t, h) \cap \{|x| < h\}$. Then there is a positive number $\varrho = \varrho(n, t)$ so that*

$$v(x) \leqslant C\left|\frac{x}{h}\right|^{\varrho} \quad \text{on } K(t, h) \tag{1.1}$$

and, for any small $\varepsilon > 0$,

$$v(x) \geqslant c\left|\frac{x}{h}\right|^{\varrho} \quad \text{on } K(t-\varepsilon, h/2) \tag{1.2}$$

for some positive constant c depending on ε and v. Moreover, ϱ is a continuous strictly decreasing function of t and $\varrho(n, \pi/2) = 1$.

Proof. The representation (1.3) of v is adapted from Gariepy and Lewis ([4], p. 262). Let $0 < \gamma_1 < \gamma_2 \ldots$ be the eigenvalues of the boundary value problem

$$\delta\varphi + \gamma\varphi = 0 \text{ on } C(t),$$
$$\varphi = 0 \text{ on the boundary of } C(t),$$

where $C(t) = \{|x| = 1,\ |\theta| < t\}$ and δ is the operator defined in terms of the Laplacean Δ by

$$\Delta = r^{1-n}\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial}{\partial r}\right) + r^{-2}\delta.$$

Let $\{\varphi_k\}$ be the corresponding eigenfunctions normalized by

$$\int_{C(t)} \varphi_k^2\, dm = 1 \quad \text{for} \quad k = 1, 2, \ldots,$$

where m is the surface measure. Because φ_k are symmetric about x_1-axis, we may regard them as functions of θ alone whenever it is more convenient. Let ϱ_k be the positive root of $\varrho_k(\varrho_k + n - 2) = \gamma_k$ and $a_k = \int_{C(t)} \varphi_k(x) v(tx)\, dm(x)$. Then

$$v(x) = \sum_1^\infty a_k \left|\frac{x}{h}\right|^{\varrho_k} \varphi_k(\theta). \tag{1.3}$$

It is known ([1], VI, § 6) that φ_1 is either strictly positive or strictly negative in $C(t)$ and ([4], p. 262) that the series

$$\sum \left|\frac{x}{h}\right|^{\varrho_k - \varrho_1} |\varphi_k(\theta)|$$

converges uniformly in $K(t, h/2)$, in fact on $\overline{K(t, h/2)}$. From these facts, formula (1.3) and the definition of v, it is ready to see (1.1) and (1.2) if we let $\varrho = \varrho_1$. It is known ([1], VI, § 2) that γ_1 is a continuous, strictly decreasing function of t, and therefore so is ϱ. When $t = \pi/2$, it is easy to verify that $\varphi_1 = \cos\theta$ and $\gamma_1 = n - 1$. Thus $\varrho(n, \pi/2) = 1$. This proves Lemma 1.

2. Main result. We recall that there is an exterior cone of size $K(\alpha, a)$ at every point on ∂D and let $\varrho = \varrho(n, \pi - \alpha)$ as defined in Lemma 1. We shall give a sufficient condition on μ for $u \not\equiv +\infty$.

THEOREM 1. *If*

$$\int_D d(y)^{\varrho}\, d\mu(y) < +\infty, \tag{2.1}$$

then $u \not\equiv +\infty$.

Theorem 1 is a simple consequence of the following lemma and the fact that, under the assumption (2.1), μ is finite on every compact subset of D.

LEMMA 2. *For each $x \in D$, there is a constant C depending on x such that*

$$G(x, y) \leqslant C d(y)^{\varrho}$$

whenever $y \in D$ and $2d(y) < d(x)$.

Proof. We let $\lambda = d(x)$, $y \in D$ and $2d(y) < \lambda$, w a point on ∂D closest to y and K a cone at w of size $K(\pi - \alpha, \lambda/2)$ whose complementary cone is exterior to $\bar{D}$. Let v be the harmonic function in K with boundary value 1 on the spherical piece of ∂K, with boundary value 0 on the remaining part of ∂K. When z is in $D \cap$ the spherical piece of ∂K, we have $|z - x| > \lambda/2$, and thus

$$G(z, x) \leqslant |z - x|^{2-n} < C\lambda^{2-n} = C. \tag{2.2}$$

From (2.2) and the maximum principle for harmonic functions, we have

$$G(z, x) \leqslant Cv(z) \tag{2.3}$$

for $z \in D \cap K$. Therefore, from (2.3) and Lemma 1, we have

$$G(y, x) \leqslant Cd(y)^{\varrho}.$$

THEOREM 2. *Suppose that μ satisfies condition (2.1) in Theorem 1 and L is a right cylinder intersecting ∂D, on which there is a local coordinate system with properties described in the definition of D. Then for all points P in $L \cap \partial D$ except a set of $n-2+\varrho$-dimensional Hausdorff measure zero, u has limit zero along the line segment parallel to the axis of L ending at P.*

Proof. By covering $L \cap \partial D$ with small cylinders and using the Lebesgue number argument, we may assume that the diameter of L is less than a and that μ is concentrated on a subset S of L, so that for each $x \in S$ the point on ∂D closest to x is in L.

From now on we use s, σ to denote $|x - \tilde{x}|$ and $|y - \tilde{y}|$, respectively, whenever x and y are points in L. Fix x in L; if y is in L, we use γ to denote $|y' - x'|$. (We recall that $x' = (x_2, x_3, \ldots, x_n)$ and $y' = (y_2, \ldots, y_n)$.) We divide S into three sets in terms of y as follows:

$$S_1\colon\ y \in S,\ \gamma \leqslant s,\ |\sigma - s| \leqslant s/2,$$

$$S_2\colon\ y \in S,\ \gamma \leqslant s,\ |\sigma - s| > s/2,$$

$$S_3\colon\ y \in S,\ \gamma > s.$$

We want to show

$$G(x, y) \leqslant C\gamma^{2-n} \text{ in } S_1, \tag{2.4}$$

$$G(x, y) \leqslant C\sigma^{\varrho} s^{2-n-\varrho} \text{ in } S_2, \tag{2.5}$$

$$G(x, y) \leqslant Cs^{\varrho}\sigma^{\varrho}\gamma^{2-n-2\varrho} \text{ in } S_3. \tag{2.6}$$

For any $y \in S_1$

$$G(x, y) \leqslant |x - y|^{2-n} \leqslant \gamma^{2-n}.$$

We observe that there is a constant $c > 0$ depending only on the Lipschitz condition on L so that $|y - x| > cs$ whenever $y \in S_2$. Fix $y \in S_2$ and assume that the origin 0 of the local coordinate system is at $\tilde{y}$. Let F be $\{z \in D\colon |z| < cs/2\}$ and observe that

$$G(z, x) \leqslant Cs^{2-n}$$

for $z \in \partial F$. Let K be a cone at $\tilde{y} = 0$ of size $K(\pi - \alpha, cs/2)$ whose complementary cone is exterior to $\bar{D}$ and v be the harmonic function in K with boundary value 1 on the spherical piece of ∂K, value 0 on the remaining part of ∂K. From Lemma 1, we see that

$$v(z) \leqslant C|z|^{\varrho}(cs/2)^{-\varrho}$$

for $z \in \bar{F}$. By the maximum principle, on F we have

$$G(z, x) \leqslant C|z|^{\varrho}s^{2-n-\varrho}.$$

If y is in F, then

$$G(y, x) \leqslant C|y|^{\varrho}s^{2-n-\varrho} = C\sigma^{\varrho}s^{2-n-\varrho};$$

if y is not in F, then $\sigma \geqslant cs/2$ and

$$G(y, x) \leqslant |x - y|^{2-n} \leqslant (cs)^{2-n} \leqslant C\sigma^{\varrho}s^{2-n-\varrho}.$$

We have proved (2.5).

Now fix y in L with $\gamma > s$ and assume $\tilde{y}$ is the origin. Let T be $\{z \in D\colon |z| \leqslant \gamma/4\}$ and observe that $|x| = |x - \tilde{y}| \geqslant \gamma$. Therefore, for $z \in \partial T$

$$G(z, x) \leqslant |z - x|^{2-n} \leqslant C\gamma^{2-n}. \tag{2.7}$$

Let K be a cone at $\tilde{y} = 0$ of size $K(\pi - \alpha, \gamma/4)$ whose complementary cone is exterior to $\bar{D}$ and v be the harmonic function on K defined as in the last paragraph. From Lemma 1, we see that

$$v(z) \leqslant C|z|^{\varrho}(\gamma/4)^{-\varrho}$$

for $z \in \bar{T}$. By the maximum principle, on T we have

$$G(z, x) \leqslant C|z|^{\varrho}\gamma^{2-n-\varrho}.$$

If y is in T, then

$$G(y, x) \leqslant C|y|^{\varrho}\gamma^{2-n-\varrho} = C\sigma^{\varrho}\gamma^{2-n-\varrho};$$

if y is not in T, then $\sigma \geqslant \gamma/4$ and

$$G(y, x) \leqslant |x-y|^{2-n} \leqslant C\gamma^{2-n} \leqslant C\sigma^{\varrho}\gamma^{2-n-\varrho}.$$

We have, in fact, proved that for any two points x, y in L if $|x'-y'| > |x-\tilde{x}|$ then

$$G(y, x) \leqslant C\sigma^{\varrho}\gamma^{2-n-\varrho}. \tag{2.8}$$

Under the notation in the last paragraph, if z is on ∂T, then $|z'-x'| > \gamma/2 \geqslant |z-\tilde{z}|$. Thus, from (2.8),

$$G(z, x) \leqslant Cs^{\varrho}\gamma^{2-n-\varrho} \tag{2.9}$$

for $z \in \partial T$. Following the argument in the last paragraph with (2.7) replaced by (2.9) if $y \in T$, or switching the roles of x and y, then following the proof of (2.5) if $y \notin T$, we may obtain

$$G(y, x) \leqslant Cs^{\varrho}\sigma^{\varrho}\gamma^{2-n-2\varrho}$$

if $y \in L$ and $\gamma > s$. We have proved (2.6).

The following part of the proof is a slight variant of Littlewood's ([6], pp. 392–394); we shall not give too much detail. Let $L(q) = \{y \in L: |y-\tilde{y}| < q\}$,

$$\varepsilon(q) = \int\limits_{L(q)} d(y)^{\varrho} d\mu(y),$$

and for $\tilde{x} \in L \cap \partial D$, let $\Phi(\tilde{x}, t)$ be the integral $\int d(y)^{\varrho} d\mu(y)$ extended over $L(q) \cap \{y: |y'-\tilde{x}'| < t\}$. It can be shown by a lemma in [3], p. 210, that

$$\limsup_{t\to 0} \frac{\Phi(\tilde{x}, t)}{t^{n-2+\varrho}} \leqslant \sqrt{\varepsilon(q)} \tag{2.10}$$

on a set $E(q)$ whose complement in $L \cap \partial D$ is of $(n-2+\varrho)$-dimensional Hausdorff measure $\leqslant C\sqrt{\varepsilon(q)}$. In order to prove the theorem, it is enough ([6], p. 392) to show that for each small $q > 0$ and for each $\tilde{x}$ in $E(q)$,

$$\limsup \int\limits_{L(q)} G(x, y) d\mu(y) \leqslant C\sqrt{\varepsilon(q)} \tag{2.11}$$

as $x \to \tilde{x}$ along the segment $x' = \tilde{x}'$.

We recall that μ is concentrated on $S \subseteq L$ and, for each $y \in S$, $\sigma = |y-\tilde{y}| \leqslant Cd(y)$. From (2.4), (2.5), and (2.6) we may obtain that

$$\int\limits_{L(q)} G(x, y) d\mu(y) \leqslant s^{-\varrho}\int\limits_0^s C\gamma^{2-n} d\Phi(\gamma) + \int\limits_0^s Cs^{2-n-\varrho} d\Phi(\gamma) + s^{\varrho}\int\limits_0^C C\gamma^{2-n-2\varrho} d\Phi(\gamma),$$

where $\Phi(\gamma) = \Phi(\tilde{x}, \gamma)$. (2.11) may be obtained ([6], p. 386) by applying (2.10) and integration by parts to the above inequality. We have proved Theorem 2.

A similar proof gives the radial limit of u in a starlike Lipschitz domain whose boundary is given by $r = f(\xi)$ where $|\xi| = 1$ and f is Lipschitz.

It should be emphasized that when $a < \pi/2$, the exceptional set in Theorem 2 is smaller than the expected set of $(n-1)$-dimensional Hausdorff measure zero, which is the same as the harmonic measure zero for Lipschitz domains [2]. Although condition (2.1) is too strong in general, the exponent ϱ in (2.1) cannot be improved. This can be seen in the case $D = K(\pi-a, 1)$ and μ is on the negative x_1-axis, with the aid of (1.2).

It is not known (1) at this point if a nontrivial Green potential on a starlike Lipschitz domain has radial limit zero almost everywhere.

We may also consider the limit of u along curves instead of line segments, especially curves which nearly preserve the x_1 and x' distances between points in L.

THEOREM 3. *Let D, L, μ and u be the same as in Theorem 1. Let $f = (f_1, f_2, \ldots, f_{n-1})$ be a C^1 function from $L \cap D$ to $\mathbf{R}^{n-1}$ and E be a subset of $L \cap \partial D$ of positive $(n-1)$-dimensional Hausdorff measure (or positive surface measure). Suppose for each point x in E, f and $\nabla f_1, \nabla f_2, \ldots, \nabla f_{n-1}$ can be extended continuously to x through some interior cone at x; moreover the normal of ∂D at x, if it exists, is not on $\{\sum c_k \nabla f_k: c_k \in \mathbf{R}\}$. Then for all $x \in E$ except a set of $(n-1)$-dimensional Hausdorff measure zero, there is a unique level curve of f ending at x, nontangential to ∂D and u has boundary limit zero along that level curve at x. Moreover, the exceptional set can be reduced to $(n-2+\varrho)$-dimensional Hausdorff measure zero if the set on E where no nontangential level curves end is small enough.*

For the proof, we first construct a saw-toothed region Ω in D with teeth at points of density of E, next use Whitney's extension theorem to modify f outside Ω so that f is C^1 on $\bar{D}$, finally prove the theorem when f is C^1 on $\bar{D}$. Details are similar to those in [11]; we refer the reader to [11].

3. Regions with smooth boundaries. If ∂D is C^1, then (2.1) can be replaced by

$$\int\limits_D d(y)^{\varrho_0} d\mu(y) < \infty \quad \text{for some } \varrho_0 \in (0, 1).$$

(1) *Added in proof.* The answer is now known to be positive.

This is true because for every $\beta \in (0, \pi)$ there exists some $b > 0$ depending on β so that there is an exterior cone of size $K(\beta, b)$ at every point on ∂D.

If ∂D is C^2, then at every point x on ∂D there is a ball of a fixed size exterior to $\bar{D}$ and tangent to ∂D at x. If v is a positive harmonic function outside a ball and vanishes on the sphere, then the value of v at a point near the sphere is proportional to the distance from that point to the ball. Using this fact instead of Lemma 1 we may replace ϱ by 1 in Theorems 1, 2, and 3. Thus condition (2.1) is weakened but the corresponding exceptional set is enlarged.

Suppose that D is a Liapunov or a Liapunov–Dini region [10]; by an estimate of a harmonic function obtained in [10], we may also replace ϱ by 1 in the above theorems.

References

[1] R. Courant and D. Hilbert, *Methods of mathematical physics*, *I*, Wiley–Interscience, 1966.

[2] B. Dahlberg, *On estimates of harmonic measure*, Arch. Rational Mech. Anal. 65, No. 3 (1977), pp. 275–288.

[3] N. Dunford and J. T. Schwartz, *Linear Operators*, *I*, Wiley–Interscience, 1957.

[4] R. Gariepy and J. L. Lewis, *A maximal principle with applications to subharmonic functions in n-space*, Arkiv f. Mat. 12 (1974), pp. 253–266.

[5] L. L. Helms, *Introduction to potential theory*, Wiley–Interscience, 1969.

[6] J. E. Littlewood, *On functions subharmonic in a circle* (*II*), Proc. London Math. Soc. (2) 28 (1928), pp. 383–394.

[7] N. Privalov, *Boundary problems of the theory of harmonic and subharmonic functions in space* (Russian), Mat. Sbornik 45 (1938), pp. 3–25.

[8] E. M. Stein, *Singular integrals and differentiability properties of functions*, Princeton 1970.

[9] E. B. Tolsted, *Limiting values of subharmonic functions*, Proc. Amer. Math. Soc. 1 (1950), pp. 636–647.

[10] K.-O. Widman, *Inequalities for the Green function and boundary continuity of the gradient of solutions of elliptic differential equations*, Math. Scand. 21 (1967), pp. 17–37.

[11] J. M. Wu, *Boundary limits of Green's potentials along curves*, Studia Math. 60 (1977), pp. 137–144.

UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

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On extending and lifting continuous linear mappings in topological vector spaces

W. GEJLER (Saransk, U.S.S.R.)

Abstract. (1) Let $0 < p < 1$. Then there is no non-zero topological vector space which has the extension property for the class of all p-Banach spaces with separating continuous duals.

(2) If $\mathscr{K}$ is the class of all Fréchet spaces (or of all separable Fréchet spaces, or of all nuclear Fréchet spaces, or of all metric vector spaces) and a space P ($P \in \mathscr{K}$) has the lifting property for $\mathscr{K}$, then P is finite-dimensional.

Let $\mathscr{K}$ be any class of topological vector spaces([1]) (briefly TVS's), and let E be any TVS. The space E is said to have the *extension property for* $\mathscr{K}$ if for every $X \in \mathscr{K}$ and for every subspace $Y \subset X$, each mapping (= linear continuous mapping) $f\colon Y \to E$ has an extension to a mapping $g\colon X \to E$. Dually, E is said to have the *lifting property for* $\mathscr{K}$ if for every $X \in \mathscr{K}$ and for every closed subspace $N \subset X$, each mapping $f\colon E \to X/N$ has a lifting to a mapping $g\colon E \to X$ (i.e. $f = p \circ g$, where p is the quotient mapping from X onto X/N). If $E \in \mathscr{K}$ and E has the extension property for $\mathscr{K}$ [E has the lifting property for $\mathscr{K}$], then E is called an *injective* [*projective*] *space in* $\mathscr{K}$.

Let $\mathscr{K}$ be the class of all Banach spaces. Then (a) E is an injective space in $\mathscr{K}$ iff E is a P_λ-space for some $\lambda \geqslant 1$; (b) E is a projective space in $\mathscr{K}$ iff E is isomorphic to $l_1(\Gamma)$ for a certain set Γ ([2], [10], [11], [13]). Any product [countable product] of injective Banach spaces is an injective space in the class of all locally convex spaces [of all Fréchet spaces] (see [11]). From an argument of G. Köthe ([10], p. 182; see also S. Rolewicz [12], p. 65) it follows that for each $p \in (0, 1]$ the spaces $l_p(\Gamma)$ are projective in the class of all p-Banach spaces. The author proved in [3] that in the class of all locally convex spaces a space E is projective iff E is a direct sum of one-dimensional spaces. This result is also true for the class of all complete locally convex spaces [4]. Using the method of [3], one can show that in the class of all TVS's a space E is projective iff the topology of E is the finest vector topology for the vector space E.

([1]) we include the Hausdorff condition in the definition of TVS.