Contents of volume LIV, number 2

STUDIA MATHEMATICA

Managing editors: **Z. Ciesielski, W. Orlicz** *(Editor-in-Chief)*, **A. Pełczyński, W. Żelazko**

The journal prints original papers in English, French, German and Russian, mainly on functional analysis, abstract methods of mathematical analysis and on the theory of probabilities. Usually 3 issues constitute a volume.

The papers submitted should be typed on one side only and they should be accompanied by abstracts, normally not exceeding 200 words in length. The authors are requested to send two copies, one of them being the typed, not Xerox copy. Authors are advised to retain a copy of the paper submitted for publication.

Manuscripts and the correspondence concerning the editorial work should be addressed to

STUDIA MATHEMATICA
ul. Śniadeckich 8
00-950 Warszawa, Poland

Correspondence concerning exchange should be addressed to

INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
ul. Śniadeckich 8
00-950 Warszawa, Poland

The journal is available at your bookseller or at

ARS POLONA
Krakowskie Przedmieście 7
00-068 Warszawa, Poland

PRINTED IN POLAND

WROCŁAWSKA DRUKARNIA NAUKOWA

STUDIA MATHEMATICA, T. LIV. (1975)

Multipliers on p-Fourier algebras*

by

MICHAEL J. FISHER

Abstract. The algebra $B_p(\Gamma)$ of multipliers on the p-Fourier algebra $A_p(\Gamma)$ is studied.

I. Preliminaries and basic properties of $A_p(\Gamma)$. Let G denote an LCA (Hausdorff) group and let Γ denote the dual group of G. If $1 \leqslant p < \infty$, let $L_p(G)$ (or $L_p(\Gamma)$) denote the Banach space of p-power integrable functions on G (or Γ) with respect to Haar measure on G (or Γ). Let q denote the index which is conjugate to p $(1/p+1/q = 1)$ and let

$$\langle f, g\rangle = \int_G f(x)g(x)\,dx$$

denote the dual pairing between $L_p(G)$ and $L_q(G)$ when $f\in L_p(G)$ and $g\in L_q(G)$. If $y\in\Gamma$, let t_y denote the translation operator on $L_p(\Gamma)$ which is given by $(t_yf)(x) = f(x+y)$.

For $f\in L_p(\Gamma)$ and $g\in L_q(\Gamma)$, define the convolution $(f*g)(y)$ of these functions by

$$(f*g)(y) = \int_\Gamma (t_yf)(x)g(x)\,dx = \langle t_yf, g\rangle;$$

$(f*g)(y)$ is a continuous function on Γ which vanishes at infinity.

For $1\leqslant p<\infty$, let $A_p(\Gamma)$ denote the space of continuous functions h on Γ of the form

$$h(y) = \sum_{j=1}^{\infty}(f_j*g_j)(y)$$

with all $f_j\in L_p(\Gamma)$, all $g_j\in L_q(\Gamma)$, and with

$$\sum_{j=1}^{\infty}\|f_j\|_p\|g_j\|_q < +\infty.$$

---

* The present version has been prepared by S. Kwapień.

Equip $A_p(\Gamma)$ with the norm

$$|h|_p = \inf\Big\{\sum_{j=1}^{\infty}\|f_j\|_p\|g_j\|_q \Big|\ h(y) = \sum_{j=1}^{\infty}(f_j * g_j)(y)\Big\}.$$

As a Banach space, $A_p(\Gamma)$ is isometrically isomorphic with $L_p(\Gamma)\hat{\otimes} L_q(\Gamma)/K$ where K is the kernel of the convolution operator,

$$C\colon L_p(\Gamma)\hat{\otimes}L_q(\Gamma)\to C(\Gamma), \qquad C(f\otimes g)(y) = (f*g)(y),$$

and where the quotient space has the quotient norm. $\hat{\otimes}$ denotes the projective tensor product.

$A_p(\Gamma)$ is the *p-Fourier algebra* introduced by Figa-Talamanaca [3]; he has shown that the conjugate space of $A_p(\Gamma)$ is isometrically isomorphic with the space $M_p(\Gamma)$ of translation invariant, bounded linear operators on $L_p(\Gamma)$.

We list the most basic properties of the spaces $A_p(\Gamma)$.

1.1. *With pointwise multiplication, $A_p(\Gamma)$ is a Banach algebra* (cf. [7]). *As a Banach algebra it is commutative, semi-simple, self-adjoint, and regular. Its maximal ideal space is Γ* (cf. [1]).

1.2. *For $1 < p < \infty$, the functions in $A_p(\Gamma)$ with compact support are dense in $A_p(\Gamma)$.*

1.3. *$A_p(\Gamma)$ contains the same approximate identities of norm* 1, *for all* $1 \leqslant p < \infty$.

1.4. *If q is the conjugate index to p then $A_p(\Gamma) = A_q(\Gamma)$.*

1.5. *If $1 \leqslant r \leqslant p \leqslant 2$ then $A_p(\Gamma) \subset A_r(\Gamma)$, and if $h \in A_p(\Gamma)$ then* $|h|_r \leqslant |h|_p$.

1.6. *$A_1(\Gamma)$ coincides with the algebra of all bounded, uniformly continuous functions on Γ.*

1.7. *$A_2(\Gamma)$ is isometrically isomorphic to $A(\Gamma)$, the algebra of Fourier transforms in $L_1(G)$ with the norm*

$$|\hat{H}|_A = \int_G |H(x)|\,dx \qquad \text{(cf. [2], [3])}.$$

II. Multipliers on A_p. A bounded linear operator T on $A_p(\Gamma)$ is a *multiplier* if $T(hk) = hT(k) = T(h)k$ for every pair of functions h and k in $A_p(\Gamma)$. By 1.1, $A_p(\Gamma)$ has a maximal ideal space Γ, and this implies that

$$\frac{T(k)(y)}{k(y)} = \frac{T(h)(y)}{h(y)} = t(y)$$

for every pair of functions h, $k \in A_p(\Gamma)$; $t(y)$ is a bounded continuous function on Γ which satisfies $T(h)(y) = t(y)h(y)$ for every $h \in A_p(\Gamma)$. Let $B_p(\Gamma)$ denote the algebra of bounded continuous functions on Γ such that $t(y)h(y) \in A_p(\Gamma)$ for every $h \in A_p(\Gamma)$.

It follows from the closed graph theorem that each of these functions defines a multiplier on $A_p(\Gamma)$. We define the norm of a function t in $B_p(\Gamma)$ as the operator norm

$$|||t|||_p = \sup\{|th|_p \,|\, |h|_p \leqslant 1\}.$$

Since $A_p(\Gamma)$ contains an approximate identity of norm 1 (by 1.3), we may regard $A_p(\Gamma)$ as an isometric subalgebra of $B_p(\Gamma)$ by identifying $h \in A_p(\Gamma)$ with the operator $(T_h k)(y) = h(y)k(y)$.

THEOREM 1. *If $1 \leqslant p \leqslant r \leqslant 2$, the inclusions $B_2(\Gamma) \subset B_r(\Gamma) \subset B_p(\Gamma) \subset B_1(\Gamma)$ are continuous and $A_r(\Gamma)$ is dense in $B_p(\Gamma)$ in the strong operator topology.*

Proof. If $h \in A_r(\Gamma)$ then $|h|_p \leqslant |h|_r$. Let $\{e_n\}$ be an approximate identity of norm 1 in $A_2(\Gamma)$. Then if $f \in B_r(\Gamma)$,

$$|fe_n|_p \leqslant |fe_n|_r \leqslant |||f|||_r,$$

and for $h \in A_p(\Gamma)$,

$$|(fe_n)h|_p \leqslant |||f|||_r |h|_p.$$

This implies that $|fh|_p \leqslant |||f|||_r|h|_p$ and that $|||f|||_p \leqslant |||f|||_r$.

One can use the approximate identity $\{e_n\}$ to show that $A_p(\Gamma)$ is strongly dense in $B_p(\Gamma)$.

Several facts regarding the algebras $B_p(\Gamma)$ should be noticed. Since $A_1(\Gamma)$ coincides with the algebra of all bounded uniformly continuous functions $B_1(\Gamma) = A_1(\Gamma)$, it follows from the Helson–Wendel theorem (cf. [6], [12]) that $B_2(\Gamma)$ consists of the Fourier transforms of bounded Borel measures on G; $B_2(\Gamma)$ is equipped with the inherited total variation norm. It follows from the Bochner representation theorem that every positive definite continuous function f is in $B_p(\Gamma)$ and

$$|||f|||_1 = |||f|||_p = |||f|||_2.$$

If Γ is a compact group, then, since $A_p(\Gamma)$ contains all constant functions, $A_p(\Gamma) = B_p(\Gamma)$.

Conversely, if for some $1 < p \leqslant 2$ for an LCA group Γ we have $A_p(\Gamma) = B_p(\Gamma)$, then the constant functions are in $A_p(\Gamma)$. Since $A_p(\Gamma) \subset C_0(\Gamma)$, this implies that Γ is a compact group.

We shall need the following result, which we proved in [5] (cf. also [10]).

THEOREM 2. *Let $1 \leqslant p \leqslant 2$ and suppose that*

$$\left|\frac{1}{r} - \frac{1}{2}\right| \leqslant \left|\frac{1}{p} - 1\right| \quad \text{while} \quad 1 \leqslant r < \infty.$$

Then $f \in B_p(\Gamma)$ is the Fourier transform of an operator T_f in $M_r(G)$ with $\|T_f\|_r \leqslant |||f|||_p$. The map $f \to T_f$ is a faithful representation of $B_p(\Gamma)$ as an algebra of $L_r(G)$ multipliers.

We shall say that a complex valued function F defined on the complex plane *operates on* $B_p(\Gamma)$ if for every function $u \in B_p(\Gamma)$ there exists a function v in $B_p(\Gamma)$ such that $\alpha(v) = F(\alpha(u))$ for every continuous complex homomorphism α of $B_p(\Gamma)$.

THEOREM 3. *Let $1 < p \leqslant 2$ and let Γ be non-compact. Let F be a complex function defined on $[-1, 1]$. Then $F(\hat{\mu}) \in B_p(\Gamma)$ for every function $\hat{\mu} \in B_2(\Gamma)$ with range in $[-1, 1]$ if and only if F is the restriction of an entire function to $[-1, 1]$.*

Only the entire functions operate on $B_p(\Gamma)$.

Proof. It is clear that entire functions operate on $B_p(\Gamma)$. It was proved by Igari (cf. Theorem 1 of [8]) that if for some $r \neq 2$ $F(\hat{\mu})$ is the Fourier transform of an operator in $M_r(G)$ for every $\mu \in M_1(G)$ with $\hat{\mu}(\gamma) \in [-1, 1]$ for every $\gamma \in \Gamma$ then F is the restriction to $[-1, 1]$ of an entire function. Since $B_2(\Gamma)$ consists of the Fourier transforms of the operators in $M_1(G)$, this and Theorem 2 proves the first statement of Theorem 3.

Now assume that H is a complex function which operates on $B_p(\Gamma)$. By the already proved, there exists an entire function H_1 such that $(H - H_1) = 0$ on $[-1, 1]$. Given any $z_0 \notin [-1, 1]$, the function $(z_0 - z)^{-1}$ is not entire, therefore, by the above result, there exists $\hat{\mu} \in B_2(\Gamma)$ with range in $[-1, 1]$ such that the function $z_0 - \hat{\mu}$ is not invertible in $B_p(\Gamma)$. This implies that for some complex continuous homomorphism α of $B_p(\Gamma)$ we have $z_0 = \alpha(\hat{\mu})$. Since $H_1(\alpha(\hat{\mu})) = \alpha(H_1(\hat{\mu})) = \alpha(H(\hat{\mu})) = H(\alpha(\hat{\mu}))$, we have $H(z_0) = H_1(z_0)$. So H is an entire function.

COROLLARY 1. *If $1 < p \leqslant 2$, Γ is non-compact, then the algebra $B_p(\Gamma)$ is not self-adjoint and not regular, and Γ is not dense in the maximal ideal space of $B_p(\Gamma)$.*

Proof. The function $F(z) = \bar{z}$ is not analytic, therefore it does not operate on $B_p(\Gamma)$. Thus $B_p(\Gamma)$ is not self-adjoint. If a function f is in $B_p(\Gamma)$, then $\bar{f}$ is also. Since $B_p(\Gamma)$ is not self-adjoint, this shows that Γ is not dense in Δ_p, the space of maximal ideals of $B_p(\Gamma)$.

$B_p(\Gamma)$ cannot be regular for, by Theorem 1.4.3 of [9], Γ is dense in Δ_p where Δ_p is considered with the hull-kernel topology. Thus the Gelfand topology on Δ_p does not coincide with the hull-kernel topology and $B_p(\Gamma)$ is not regular.

Remark 1. The maximal ideal space Δ_p contains Γ as an open subset (cf. [9], p. 39).

COROLLARY 2. *If $p \neq 1, \infty$ and Γ is not finite, then $B_p(\Gamma)$ is a proper subspace of $C_0(\Gamma)$.*

Proof. If Γ is not compact this is an immediate consequence of Theorem 3. If Γ is compact this follows from the following:

Remark 2 (cf. [8]). If $p \neq 1, \infty$, Γ is an infinite compact group, then each function F, complex valued defined on $[-1, 1]$, for which $F(\hat{\mu}) \in B_p(\Gamma)$ whenever $\hat{\mu} \in B_2(\Gamma)$ has range in $[-1, 1]$, is the restriction to $[-1, 1]$ of a function which is analytic in a neighborhood of $[-1, 1]$.

Only the analytic functions operate on $B_p(\Gamma)$.

In the sequel we shall study isometries between $A_p(\Gamma)$ spaces. The following theorem, which is known in a big part (cf. [10], [12]), will be useful.

THEOREM 4. *A function f in $B_p(\Gamma)$ is an isometric multiplier on $A_p(\Gamma)$ if and only if f is a complex unit multiple of a continuous character on Γ.*

$\tilde{G}$, the character group of Γ regarded as a group of multipliers on $A_p(\Gamma)$ and equipped with the strong operator topology, is topologically isomorphic to G.

Proof. If $g \in G$ and if ω is a complex number with $|\omega| = 1$, then $\omega g(\gamma)$ is an isometric multiplier on $A_p(\Gamma)$ since $g(\gamma)$ and $g^{-1}(\gamma)$ are positive definite functions with B_p-norm 1.

Suppose that $f \in B_p(\Gamma)$ is an isometric multiplier. By Theorem 2, f and f^{-1} are Fourier transforms of operators T and T^{-1} in $M_r(G)$ for some $r \neq 2$. Furthermore, $\|T\|_r \leqslant 1$ and $\|T^{-1}\|_r \leqslant 1$ since $|||f|||_p = 1$ and $|||f^{-1}|||_p = 1$. Thus T is an isometric operator in $M_r(G)$. Strichartz [11] has shown that T must be a complex unit multiple of a translation operator over G. Thus $f(\gamma) = \hat{T}(\gamma) = \omega g(\gamma)$ for some $|\omega| = 1$ and some $g \in G$.

If a net (g_α) converges to g in G, then $g_\alpha(\gamma)h(\gamma)$ converges to $g(\gamma)h(\gamma)$ in $A_2(\Gamma)$ for every h in $A_2(\Gamma)$ since translation by $g \in G$ is strongly continuous on $L_1(G)$. Since $A_2(\Gamma)$ is continuously and densely included in $A_p(\Gamma)$, an $\varepsilon/3$-argument shows that (g_α) converges strongly to g over $A_p(\Gamma)$. Conversely, if (g_α) converges strongly to g over $A_p(\Gamma)$, then (g_α) converges uniformly on compact subsets of Γ. This is due to the presence of local units in $A_p(\Gamma)$ and to the fact that $|f(\gamma)| \leqslant |f|_p$ for every $f \in A_p(\Gamma)$ and for every γ.

Helson ([6]) and Wendel ([12], [13]) have proved that $A_2(\Gamma)$ determines Γ. Our objective now is to prove that $A_p(\Gamma)$ *determines* Γ. We shall do this by proving that if $A_p(\Gamma)$ and $A_p(\Lambda)$ are isometrically isomorphic, then Γ and Λ are topologically isomorphic (isomorphic and homeomorphic). To prove the main theorem we need several preliminary results.

LEMMA 1. *A bounded linear operator T on $A_p(\Gamma)$ is a multiplier if and only if T commutes with the operations of multiplication by continuous characters on Γ.*

Proof. It is clear that multipliers commute with multiplication by characters. If, on the other hand, T is a bounded linear operator on $A_p(\Gamma)$ which commutes with multiplication by characters, then $T(gh)(\gamma)$

$= g(\gamma)T(h)(\gamma)$ is a continuous function of $g \in G$. Let $h \in A_2(\Gamma)$. Then if $K \in L_1(G)$, we have

$$\hat{K}(\gamma)T(h)(\gamma) = \int_G g(\gamma)T(h)(\gamma)K(g)\,dg = \int_G T(gh)(\gamma)K(g)\,dg$$
$$= T\Big(\int_G g(\gamma)h(\gamma)K(g)\,dg\Big) = T(\hat{K}h)$$

since

$$\int_G g(\gamma)h(\gamma)K(g)\,dg = (H*K)\hat{}\,(\gamma) \quad \text{where} \quad \hat{H} = h.$$

Since $A_2(\Gamma)$ is dense in $A_p(\Gamma)$, this equality implies that T is a multiplier.

LEMMA 2. *Let Γ and Λ be LCA groups. If $\gamma_0 \in \Lambda$, $a\colon \Gamma \to \Lambda$ is a topological isomorphism, then $a_0\colon A_p(\Lambda) \to A_p(\Gamma)$ defined by $a_0(h)(\gamma) = h(\gamma_0 + a(\gamma))$ is an isometric isomorphism.*

Proof. In m and n denote the respective Haar measures on Γ and Λ, then $m \cdot a_0^{-1}$ is a Haar measure on Λ which must satisfy $m \cdot a_0^{-1} = cn$ for some positive constant c.

Define $a_p\colon L_p(\Lambda) \to L_p(\Gamma)$ by

$$a_p(f)(\gamma) = c^{-1/p} f(a_0(\gamma)).$$

Then a_p is an isometric isomorphism of Banach spaces. If $h \in A_p(\Lambda)$ and if $\varepsilon > 0$, choose sequences $(f_k) \in L_p(\Lambda)$ and $(g_k) \in L_q(\Lambda)$ such that

$$h(\lambda) = \sum_{k=1}^{\infty} \langle t_\lambda f_k, g_k\rangle$$

and

$$\sum_{k=1}^{\infty} \|f_k\|_p \|g_k\|_q \leqslant |h|_p + \varepsilon.$$

Then

$$a_0(h)(\gamma) = h(\gamma_0 + a(\gamma)) = \sum_{k=1}^{\infty} \langle t_{\gamma_0 + a(\gamma)} a_p(f_k), a_q(g_k)\rangle$$

satisfies

$$|a_0(h)|_p \leqslant |h|_p + \varepsilon.$$

Since $\varepsilon > 0$ is arbitrary, $|a_0(h)|_p \leqslant |h|_p$. Since a_0^{-1} is of the same type, given by $a_0^{-1}(-\gamma_0) \in \Gamma$ and $a^{-1}\colon \Lambda \to \Gamma$, a_0 is an isometric isomorphism of $A_p(\Gamma)$ onto $A_p(\Gamma)$.

THEOREM 5. *If Φ is an isometric isomorphism of $A_p(\Gamma)$ onto $A_p(\Lambda)$, then there is a topological isomorphism $a\colon \Lambda \to \Gamma$ and an element $\gamma_0 \in \Gamma$ such that*

$$\Phi(h)(\lambda) = h(\gamma_0 + a(\lambda)) \quad \textit{for every } h \in A_p(\Gamma).$$

Proof. The isometric isomorphism $\Phi\colon A_p(\Gamma) \to A_p(\Lambda)$ induces an isometric isomorphism $\Phi_1\colon B_p(\Gamma) \to B_p(\Gamma)$ by $\Phi_1(f) = \Phi f \Phi^{-1}$; notice that Φ_1 is bicontinuous in the strong operator topologies on $B_p(\Gamma)$ and $B_p(\Lambda)$. If $g \in \tilde{G}$, then $\Phi_1(g)$ is an isometric multiplier on $A_p(\Lambda)$. By Theorem 4, there is a complex unit ω_g and a character $\bar{a}(g)$ on Λ such that $\Phi_1(g) = \omega_g \bar{a}(g)$. In addition, $\omega_{g_1}\omega_{g_2}\bar{a}(g_1)\bar{a}(g_2) = \Phi_1(g_1 g_2) = \omega_{g_1 g_2}\bar{a}(g_1 g_2)$. Since $\Phi_1(g)$ is a continuous function of g, $g \to \omega_g$ is a character on $\tilde{G}$ and $\bar{a}$ is a continuous homeomorphism. Set $\gamma_0(g) = \omega_g$. Since Φ_1 is an isometric isomorphism, $\bar{a}$ is an isomorphism of $\tilde{G}$ onto $\tilde{H} = \tilde{\Lambda}$; $\bar{a}$ is a homeomorphism of $\tilde{G}$ onto $\tilde{H}$ since Φ_1 is bicontinuous in the strong operator topologies.

Thus $\bar{a}$ induces a topological isomorphism, which is also called $\bar{a}$, from G to H; see Theorem 4. Let $a = \bar{a}^*\colon \Lambda \to \Gamma$ be defined by

$$\langle a(\lambda), g\rangle = \langle \lambda, a(g)\rangle.$$

Define $\Phi_0\colon A_p(\Gamma) \to A_p(\Lambda)$ by

$$\Phi_0(h)(\lambda) = h(\gamma_0 + a(\lambda))$$

and set

$$\Phi_2 = \Phi^{-1}\Phi_0;$$

then $\Phi_2\colon A_p(\Gamma) \to A_p(\Gamma)$ is an isometric isomorphism. Let $g \in G$ and let T_g denote the operation on $A_p(\Gamma)$ of multiplication by $g(\gamma)$; we shall show that $\Phi_2 T_g = T_g \Phi_2$, so that Φ_2 will be an isometric multiplier on $A_p(\Gamma)$ by Lemma 1. Then Φ_2 must have the form $\Phi_2(h)(\gamma) = \omega g(\gamma) h(\gamma)$ for some $g \in G$ and $|\omega| = 1$ by Theorem 4. This will imply that

$$\omega g(\gamma)h(\gamma)k(\gamma) = \Phi_2(hk)(\gamma) = \Phi_2(h)(\gamma)\Phi_2(k)(\gamma) = \omega^2 g(\gamma)^2 h(\gamma)k(\gamma),$$

so that $\omega = 1$ and $g(\gamma) = 1$; Φ_2 is the identity operator. So the proof will be complete if $T_g\Phi_2 = \Phi_2 T_g$ for every $g \in G$.

Since

$$\Phi_0(h)(\lambda) = h(\gamma_0 + a(\lambda)) = h(a(\lambda_0 + \lambda)),$$

we have

$$\Phi_0^{-1}(k)(\gamma) = k(a^{-1}(-\gamma_0 + \gamma)) \quad \text{where} \quad a(\lambda_0) = \gamma_0.$$

Thus

$$\Phi_0 T_g \Phi_0^{-1} = \Phi T_g \Phi^{-1}$$

and this implies that

$$\Phi_2 T_g = T_g \Phi_2$$

and completes the proof of the theorem.

Remark 2. The assumption on the isomorphism Φ in the preceding theorem can be relaxed to the extent that one only need suppose that $\|\Phi\| \leqslant 1$. As in [13] it follows that $\Phi(h)(\lambda) = h(\gamma_0 + a(\lambda))$ where a is a topological isomorphism of Λ onto Γ; thus Φ is an isometry by Lemma 2. Wendel's Lemma 1 (cf. [19], p. 257) can be used that Φ_1 is an isometric multiplier on $A_p(\Lambda)$. When Γ is connected and Φ is an isomorphism with $\|\Phi\| \leqslant 2$, the analogue of Helson's theorem [6] should hold for $A_p(\Gamma)$. It seems best to approach this theorem within the study of almost periodic multipliers. We hope to do this in a sequel to the present paper.

References

[1] N. Bourbaki, Seminaire, 1969, L'expose n° 367.
[2] P. Eymard, *L'algèbre de Fourier d'un groupe localement compact*, Bull. Soc. Math. France 92 (1964), pp. 181–236.
[3] A. Figa-Talamanaca, *Translation invariant operators on L_p*, Duke Math. Jour. 32 (1965), pp. 495–501.
[4] A. Figa-Talamanaca and G. I. Gaurdy, *Density and representation theorems of type (p, q)*, Jour. Austr. Math. Soc. 7 (1967), pp. 1–6.
[5] M. J. Fisher, *Recognition and limit theorems for L_p-multipliers*, Studia Math. 50 (1974), pp. 31–41.
[6] H. Helson, *Isomorphism of abelian group algebras*, Ark. Math. 2 (1953), pp. 475–485.
[7] C. Herz, *The theory of p-spaces with an application to convolution operators*, Trans. Amer. Math. Soc. 154 (1971), pp. 69–82.
[8] S. Igari, *Functions of L_p-multipliers*, Tôhoku Math. Jour. 21 (1969), pp. 304–320.
[9] R. Larsen, *The multiplier problem*, Springer-Verlag lecture notes 105, Berlin 1969.
[10] N. Lohoue, *Sur certains propriétés remarquables des algèbres $A_p(G)$*, C. R. Acad. Sc. de Paris 273 (1971), pp. 893–896.
[11] R. Strichartz, *Isomorphism of group algebras*, Proc. Amer. Math. Soc. 17 (1966), pp. 858–862.
[12] J. G. Wendel, *On isometric isomorphism of group algebras*, Pac. Jour. Math. 1 (1951), pp. 305–311.
[13] — *Left centralizers and isomorphisms of group algebras*, ibid. 2 (1952), pp. 251–261.

Received May 4, 1974 (637)

STUDIA MATHEMATICA, T. LIV. (1975)

О поведении коэффициентов Фурье равноизмеримых функций

А. Б. ГУЛИСАШВИЛИ (Тбилиси, СССР)

Резюме. В работе показано, что для любой функции $f \in \mathscr{L}_1(T^m)$ имеет место равенство

$$\inf_{\omega \in \Omega_m} \|\widehat{f \circ \omega}\|_{2,\varrho} = (2\pi)^{-m} \sqrt{\varrho(0)} \left| \int_{T^m} f(x)\, dx \right|,$$

где Ω_m — множество сохраняющих меру Лебега обратимых преобразований T^m на себя, $\widehat{f \circ \omega}$ — преобразование Фурье $f \circ \omega$, а норма берется в пространстве $l_2(Z^m)$ с весом ϱ, где ϱ положительно на Z^m и стремится к нулю на бесконечности.

1. Введение. Основным результатом настоящей работы является теорема, согласно которой любую интегрируемую функцию можно так „переставить", что у полученной функции коэффициенты Фурье будут вести себя, в некотором смысле, как коэффициенты функции из $\mathscr{L}_2$. Работа состоит из четырех пунктов. В первом приводятся необходимые обозначения и формулируется основная теорема. Следующие два пункта посвящены её доказательству. В четвертом пункте дается следствие, касающееся абсолютной сходимости рядов Фурье интегралов дробного порядка от преставленных функций.

Приведем список используемых обозначений:

а) Обозначения для различных пространств и множеств: R^m, $m \geqslant 1$, — евклидово пространство размерности m; Z^m — целочисленная решетка в R^m; $T^m = R^m/2\pi Z^m$ — тор размерности m; R^+ — множество $\{y \in R^1\colon y \geqslant 0\}$; Z^+ — множество $\{n \in Z^1;\ n > 0\}$.

б) Обозначения, связанные с измеримостью и мерой: S_m — кольцо борелевских подмножеств T^m; μ — m-мерная нормированная лебеговская мера $(2\pi)^{-m} dx$; Ω_m — множество сохраняющих меру μ преобразований $\omega\colon T^m \to T^m$, для которых существует $A_\omega \in S_m$, $\mu A_\omega = 0$, такое, что ω отображает $T^m \setminus A_\omega$ на себя взаимно однозначно.

в) Обозначения для функциональных пространств и коэффициентов Фурье: $\mathscr{L}_1(T^m)$ — пространство интегрируемых по Лебегу на T^m функций; $l_2(\varrho)(Z^m)$ — пространство, состоящее из комплексных