convex topology on F^*. This means that if (F, t) is an infinite dimensional Banach space, then (F, t_w) cannot be a dual. On the other hand, in [4] we exhibit non-trivial examples of spaces of continuous functions which, when endowed with the weak topology of simple convergence, are duals.

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A multiplier counter-example for mixed-norm spaces

by

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Abstract. If $G = \Gamma H$ is a semi-direct product of an amenable locally compact group H with an arbitrary locally compact group Γ, and if T is an operator on $L^p(G) = L^p(\Gamma; L^p(H))$ (left-invariant Haar measure) of norm $\|T\|_{p,p}$ which commutes with all right translations, then Herz and Rivière have proved the theorem that for q between 2 and p, T is bounded on $L^p(\Gamma; L^q(H))$ with norm $\|T\|_{p,q} \leqslant \|T\|_{p,p}$. In this note, we show by example that in the simple case $G = \boldsymbol{R}^2 = \boldsymbol{R}^1 \times \boldsymbol{R}^1$ this theorem fails if q is not between 2 and p. One consequence is that certain spaces of multipliers do not interpolate by the method of Riesz.

1. Introduction. Suppose that the function $m(\xi)$ is known to be a multiplier of Fourier transforms of $L^p(\boldsymbol{R}^1)$ functions; that is, the transformation T_m defined by

$$(T_m f)\hat{}\,(\xi) = m(\xi)\hat{f}(\xi)$$

is a continuous mapping of $L^p(\boldsymbol{R}^1)$ into itself. We shall be concerned here with the extension of T_m to the spaces $L^p(L^q)$ consisting of those measurable functions $f(x, y)$ defined on $\boldsymbol{R}^2$ for which the norm

$$\|f\|_{p,q} = \Big[\int_{-\infty}^{\infty}\Big(\int_{-\infty}^{\infty}|f(x,y)|^q dy\Big)^{\frac{p}{q}} dx\Big]^{\frac{1}{p}}$$

is finite. The extension of T_m from an operator on L^p to the operator $\tilde{T}_m$ on $L^p(L^q)$ is that it should operate on the first variable:

$$(\tilde{T}_m f)\hat{}\,(\xi, \eta) = m(\xi)\hat{f}(\xi, \eta);$$

or, if T_m is given by convolution with the function $M(x)$:

$$(T_m f)(x) = \int_{-\infty}^{\infty} M(x-t)f(t)\,dt,$$

then the extension $\tilde{T}_m$ is given by

$$(\tilde{T}_m f)(x, y) = \int_{-\infty}^{\infty} M(x-t)f(t, y)\,dt.$$

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For $1 \leqslant p,\ q \leqslant \infty$, the spaces $L^p(L^q)$ are Banach spaces. It is immediate that they interpolate in the expected way with Riesz (or complex) interpolation: $\big(L^{p_0}(L^{q_0}), L^{p_1}(L^{q_1})\big)_\theta = L^{p_\theta}(L^{q_\theta})$, where $(p_\theta^{-1}, q_\theta^{-1}) = (1-\theta)\times(p_0^{-1}, q_0^{-1}) + \theta(p_1^{-1}, q_1^{-1})$, $0 \leqslant \theta \leqslant 1$. A simple application of Fubini's theorem shows that any bounded operator T on L^p, whether given by a multiplier or not, gives rise to an operator $\tilde{T}$ of the same norm on $L^p(L^p)$ when

$$(\tilde{T}f)(x, y) = \big(Tf(\cdot, y)\big)(x).$$

A less immediate, but well-known, theorem states that for a bounded operator T on L^p, the operator $\tilde{T}$ is bounded on $L^p(L^2)$ ([3], Corollary 2; [5], Théorème 1). It then follows by interpolation that if T is bounded on L^p, $\tilde{T}$ is bounded on $L^p(L^q)$ for q between 2 and p. For multipliers, this is also a special case of the theorem of Herz and Rivière [4], who have proved the interesting result that an $L^p(L^p) = L^p(\boldsymbol{R}^2)$ multiplier $m(\xi, \eta)$ is necessarily an $L^p(L^2)$ multiplier, even in a more general context of semidirect products of groups.

The purpose of this note is to show, by example, that at least in the case of a direct product of $\boldsymbol{R}^1$ with itself, $\tilde{T}$ need not be bounded on $L^p(L^q)$ if q does not lie between 2 and p, even if T is given by an L^p multiplier. In particular, this shows that in general the range of q for which the Herz–Rivière theorem holds is at most from 2 to p. It remains, however, an open problem to characterize the values of q for which extensions are possible for various other products of groups. We understand from Rivière (private communication) that in the case of a direct product of Z_2 with itself, an $L^p(L^p)$ multiplier is necessarily an $L^p(L^{p'})$ multiplier also, where p' is the conjugate index to p. Our example should also be compared with the work of Benedek, Calderón, and Panzone [1] which gives sufficient conditions for certain operators to be bounded on spaces with mixed norms.

We wish to thank Professor N. M. Rivière who called our attention to the problem of determining the best range of q, and to whom the remark of Section 3 that $(\mathfrak{M}_1, \mathfrak{M}_p)_\theta \neq \mathfrak{M}_{[(1-\theta)+\theta p^{-1}]^{-1}}$ is due.

2. The L^p multiplier example. Our example will demonstrate the following

THEOREM. *Let $2 < p < \infty$. Suppose q is such that any $L^p(\boldsymbol{R}^1)$ multiplier is also an $L^p(L^q)$ multiplier. Then $q \geqslant 2$.*

The strategy of the proof of this theorem will be to construct a function $m(\xi)$ and obtain an upper bound for the norm of T_m on L^p. Then we will consider certain explicit functions $u(x, y)$ and obtain lower bounds for $\|\tilde{T}_m u\|_{p,q}$; these lower bounds will then show that $\|Tu\|_{p,q}/\|u\|_{p,q}$ can be bounded uniformly in u only if $2 \leqslant q$. The fact that we must have $q \leqslant p$ can be seen fairly easily either from general considerations, or by an example even simpler than that presented here. Consideration of adjoints leads to the analogous theorem when $1 < p < 2$: we must have $p \leqslant q \leqslant 2$.

We define the Fourier transform on $\boldsymbol{R}^1$ by $\hat{f}(\xi) = \int_{-\infty}^{\infty} e^{-2\pi i \xi x} f(x)\,dx$; the inversion formula then becomes $f(x) = \int_{-\infty}^{\infty} e^{2\pi i x\xi}\hat{f}(\xi)\,d\xi$.

The multiplier $m(\xi)$ which we shall use to prove our theorem is given by

$$m(\xi) = \hat{M}(\xi) = \begin{cases} \mu_n e^{2\pi i \lambda_n \xi}, & \text{if } |\xi - 2^{n+\frac{1}{2}}| < 1 \ (n = 1, 2, \ldots); \\ 0, & \text{otherwise.} \end{cases}$$

The weights μ and frequencies λ will be chosen later.

First, we obtain an upper bound for the norm of T_m on $L^p(\boldsymbol{R}^1)$ $(p > 2)$.

LEMMA. *For $2 < p < \infty$, there exists a constant B_p depending only upon p, and in particular independent of the μ's and λ's, such that the norm of m as an $L^p(\boldsymbol{R}^1)$ multiplier is bounded by $B_p\big(\sum_n |\mu_n|^{2p/(p-2)}\big)^{(p-2)/2p} = B_p\|\mu\|_{2p/(p-2)}$.*

Proof. Take $f(x) \in L^p(\boldsymbol{R}^1)$. Define

$$\begin{aligned} &f_n(x) && \text{by } \hat{f}_n(\xi) = \hat{f}(\xi)\cdot\chi_{[2^n < |\xi| < 2^{n+1}]},\ n = 0, \pm 1, \pm 2, \ldots; \\ &g(x) && \text{by } \hat{g}(\xi) = m(\xi)\hat{f}(\xi); \\ &g_n(x) && \text{by } \hat{g}_n(\xi) = m(\xi) f_n(\xi); \\ &h_n(x) && \text{by } \hat{h}_n(\xi) = \hat{f}_n(\xi)\cdot\chi_{[|\xi - 2^{n+\frac{1}{2}}| < 1]},\ n = 1, 2, \ldots \end{aligned}$$

(χ_A denotes the characteristic function of the set A). Thus

$$g_n(x) = \begin{cases} \mu_n h_n(x + \lambda_n), & n = 1, 2, \ldots; \\ 0, & n = 0, -1, -2, \ldots \end{cases}$$

The theorem of Littlewood and Paley ([2], p. 1177) tells us that there is a constant A_p such that for any $f \in L^p(\boldsymbol{R}^1)$ we have

$$\int_{-\infty}^{\infty} |f(x)|^p\,dx \lessgtr A_p^{\pm p} \int_{-\infty}^{\infty} \Big(\sum_{n=-\infty}^{\infty} |f_n(x)|^2\Big)^{\frac{p}{2}} dx.$$

Thus

$$\begin{aligned} \|T_m f\|_{L^p}^p = \int_{-\infty}^{\infty} |g(x)|^p\,dx &\leqslant A_p^p \int_{-\infty}^{\infty} \Big(\sum_n |g_n(x)|^2\Big)^{\frac{p}{2}} dx \\ &= A_p^p \int_{-\infty}^{\infty} \Big(\sum_n |\mu_n|^2 |h_n(x + \lambda_n)|^2\Big)^{\frac{p}{2}} dx \\ &\leqslant A_p^p \int_{-\infty}^{\infty} \Big(\sum_n |\mu_n|^{2p/(p-2)}\Big)^{(p-2)/2} \Big(\sum_n |h_n(x + \lambda_n)|^p\Big) dx \end{aligned}$$

(by the use of Hölder's inequality with exponents $p/(p-2)$ and $p/2$)

$$= A_p^p \|\mu\|_{2p/(p-2)}^p \Big(\sum_n \int_{-\infty}^{\infty} |h_n(x)|^p\,dx\Big)$$

$$\leqslant A_p^p \|\mu\|_{2p/(p-2)}^p \Big(\sum_n M_p^p \int_{-\infty}^{\infty} |f_n(x)|^p\,dx\Big)$$

(here M_p is the norm on L^p of the multiplier $\chi_{(\text{Interval})}$, which is independent of the particular interval in $\boldsymbol{R}^1$ chosen)

$$\leqslant A_p^p M_p^p \|\mu\|_{2p/(p-2)}^p \int_{-\infty}^{\infty} \Big(\sum_n |f_n(x)|^2\Big)^{\frac{p}{2}} dx$$

$$\leqslant A_p^p M_p^p \|\mu\|_{2p/(p-2)}^p \cdot A_p^p \|f\|_{L^p}^p .$$

We may thus take $B_p = A_p^2 M_p$. With a little more care, one can show that $B_p = O(p)$ as $p \to \infty$, but we shall not need this fact.

Now we shall construct our functions $u(x, y)$. We take λ so that $\lambda_n + 1 < \lambda_{n+1}$ (later, we shall choose $\lambda_n \to \infty$ much more rapidly), and define

$$u(x, y) = \begin{cases} a_n \exp(2^{n+\frac{3}{2}} \pi i x) \\ \quad \text{in the rectangle } [|x-\lambda_n| < .01] \times [|y-n| < \tfrac{1}{2}],\ n = 1, 2, \ldots, \\ 0 \quad \text{otherwise.} \end{cases}$$

Direct calculation shows that $\|u\|_{p,q} = (50)^{-\frac{1}{p}} \big(\sum_n |a_n|^p\big)^{\frac{1}{p}}$, independently of q.

For the sake of definiteness, it is probably best at this point to take explicitly

$$a_n = n^{-\frac{1}{p}} [\log(n+3)]^{-\frac{1}{2}}, \qquad \text{so } \|u\|_{p,q} < \infty;$$

and

$$\mu_n = n^{-(p-2)/2p} [\log(n+3)]^{-\frac{1}{2}}, \qquad \text{so that by the lemma,}$$

$$m \text{ is an } L^p \text{ multiplier}.$$

In general, an appropriate choice of the λ's depends slightly upon the particular μ's used.

Our aim is to insure, by choosing the λ's appropriately, that the function

$$(M * u)(x, y) = \int_{-\infty}^{\infty} M(x-t)\, u(t, y)\, dt$$

has $L^p(L^p)$ norm exceeding $c\big(\sum_n |\mu_n a_n|^q\big)^{\frac{1}{q}}$, for some fixed positive constant c.

(We may take $c = .005$.) Once this is done, a necessary condition on q that our L^p multiplier m be also an $L^p(L^q)$ multiplier is simply that

$$\infty > c^{-1} \|M * u\|_{p,q} \geqslant \sum_n \big(n^{-(p-2)/2p} n^{-\frac{1}{p}} [\log(n+3)]^{-1}\big)^q$$

$$= \sum_n n^{-\frac{q}{2}} [\log(n+3)]^{-q};$$

that is, we must have $q \geqslant 2$.

To estimate $(M * u)(x, y)$, we have

$$M(x) = \sum_n \mu_n \exp[2\pi i (x+\lambda_n) \cdot 2^{n+\frac{1}{2}}] \cdot \frac{\sin 2\pi (x+\lambda_n)}{\pi (x+\lambda_n)} .$$

For fixed $y > \frac{1}{2}$ let m be that integer for which $m - \frac{1}{2} < y < m + \frac{1}{2}$. Then $(M * u)(x, y) = 0$ for $y < \frac{1}{2}$, and otherwise

$$(M * u)(x, y) = \int_{\lambda_m - .01}^{\lambda_m + .01} M(x-t)\, u(t, y)\, dt$$

$$= a_m \sum_n \int_{\lambda_m - .01}^{\lambda_m + .01} \mu_n \exp[2\pi i (x - t + \lambda_n) \cdot 2^{n+\frac{1}{2}}] \times$$

$$\times \frac{\sin 2\pi (x - t + \lambda_n)}{\pi (x - t + \lambda_n)} \cdot \exp(2^{n+\frac{3}{2}} \pi i t)\, dt$$

$$(*) \qquad = a_m \Big\{ \mu_m \exp[2\pi i (x+\lambda_m) \cdot 2^{m+\frac{1}{2}}] \int_{\lambda_m - .01}^{\lambda_m + .01} \frac{\sin 2\pi (x - t + \lambda_m)}{\pi (x - t + \lambda_m)}\, dt +$$

$$+ \sum_{n \neq m} \mu_n \exp[2\pi i (x+\lambda_n) \cdot 2^{n+\frac{1}{2}}] \times$$

$$\times \int_{\lambda_m - .01}^{\lambda_m + .01} \exp[-2\pi i t \cdot 2^{n+\frac{1}{2}}] \frac{\sin 2\pi (x - t + \lambda_n)}{\pi (x - t + \lambda_n)}\, dt \Big\}.$$

We show that for $|x| \leqslant .01$, the leading term of $(*)$ dominates.

For $|x| \leqslant .01$, $\int_{\lambda_m - .01}^{\lambda_m + .01} \frac{\sin 2\pi (x - t + \lambda_m)}{\pi (x - t + \lambda_m)}\, dt \geqslant .039$. (Use $(\sin u)/u \geqslant 1 - \frac{u^2}{6}$; the worst case is when $|x| = .01$.) Thus the leading term of $(*)$ has absolute value at least $.039\, |a_m \mu_m|$ for $|x| \leqslant .01$.

Now choose $\lambda_n = 100^n$ $(n = 1, 2, \ldots)$.
For $|x| < .01$ and $n \neq m$, we have

$$\left|\int_{\lambda_m-.01}^{\lambda_m+.01} \exp[-2\pi it\cdot 2^{n+\frac{1}{2}}]\frac{\sin 2\pi(x-t-\lambda_n)}{\pi(x-t+\lambda_n)}\,dt\right|$$

$$\leqslant \int_{\lambda_m-.01}^{\lambda_m+.01}\left|\frac{1}{\pi(x-t+\lambda_n)}\right|dt$$

$$\leqslant \frac{.02}{\pi}\,\frac{1}{|100^n-100^m|-.02} < 10^{-4}\,100^{-|m-n|},$$

since $|100^n-100^m| \geqslant .99\cdot 100^{\max(m,n)} \geqslant .99\cdot 100^{|m-n|} \geqslant .98\cdot 100^{|m-n|}+.02$. To show that the leading term of (*) dominates, we compare the relative size of $.039\mu_m$ with $10^{-4}\sum_{n\neq m}\mu_n\cdot 100^{-|m-n|}$. Consider

$$\frac{1}{\mu_m}\sum_{n\neq m}\mu_n\cdot 100^{-|m-n|} = 100^{-m}\sum_{n<m}\frac{\mu_n}{\mu_m}100^n + \sum_{n>m}\frac{\mu_n}{\mu_m}100^{-n+m}.$$

Since our choice of μ is a decreasing sequence, the second sum above is dominated by $\sum_{n>m}100^{-n+m} = .01(1-.01)^{-1} < .02$. In the first sum above, $n \leqslant m-1$, so that

$$\frac{\mu_n}{\mu_m} = \left(\frac{m}{n}\right)^{\frac{1}{2}-\frac{1}{p}}\left(\frac{\log m+3}{\log n+3}\right)^{\frac{1}{2}} \leqslant \left(\frac{m}{n}\right)^{\frac{1}{2}-\frac{1}{p}}\left(\frac{m}{n}\right)^{\frac{1}{2}} \leqslant \left(\frac{m}{n}\right) \leqslant 4^{m-n};$$

hence

$$100^{-m}\sum_{n<m}\frac{\mu_n}{\mu_m}100^n \leqslant 25^{-m}\sum_{n<m}25^n = 25^{-m}\frac{25^m-1}{25-1} \leqslant \frac{1}{24} \leqslant .05.$$

Combining these estimates gives $\frac{1}{\mu_m}\sum_{n\neq m}\mu_n\,100^{-|m-n|} \leqslant .07$; therefore for $|x| \leqslant .01$, the absolute value of (*) is at least

$$a_m\Big\{.039\,\mu_m - 10^{-4}\sum_{n\neq m}\mu_n\cdot 100^{-|m-n|}\Big\} \geqslant a_m\mu_m[.039-10^{-4}\cdot .07] \geqslant .038\,a_m\mu_m.$$

A lower bound for the $L^p(L^q)$ norm of $M*u$ may now be obtained. For fixed x, $|x| \leqslant .01$, we have

$$\Big(\int_{-\infty}^{\infty}|(M*u)(x,y)|^q\,dy\Big)^{\frac{1}{q}} \geqslant \Big(\sum_{m=1}^{\infty}\int_{m-\frac{1}{2}}^{m+\frac{1}{2}}|.038\,\mu_m a_m|^q\,dy\Big)^{\frac{1}{q}}$$

$$= .038\Big(\sum_m|\mu_m a_m|^q\Big)^{\frac{1}{q}},$$

and so

$$\|M*u\|_{p,q} \geqslant \Big(\int_{-.01}^{.01}\Big[.038\Big(\sum_m|\mu_m a_m|^q\Big)^{\frac{1}{q}}\Big]^p\Big)^{\frac{1}{p}}$$

$$= (50)^{-\frac{1}{p}}\,.038\Big(\sum_m|\mu_m a_m|^q\Big)^{\frac{1}{q}} \geqslant .005\Big(\sum_m|\mu_m a_m|^q\Big)^{\frac{1}{q}}.$$

3. The interpolation of spaces of multipliers. Denote by $\mathfrak{M}_p$ the Banach space of multipliers of $L^p(\boldsymbol{R}^1)$, given their operator norm, and by $\mathfrak{M}_{p,q}$ the multipliers on $L^p(L^q)$. It is well known that $\mathfrak{M}_1$ corresponds to convolution with measures of finite total variation; thus $m\in\mathfrak{M}_1$ if and only if $m(\xi) = \int_{-\infty}^{\infty}e^{-2\pi ix\xi}\,dM(x)$ for some function $M(x)$ of finite total variation and $(T_m f)(x) = \int_{-\infty}^{\infty}f(x-t)\,dM(t)$. From this it can be seen immediately that $\tilde{T}_m\in\mathfrak{M}_{1,q}$ for all q, $1\leqslant q\leqslant\infty$, and indeed that $\|\tilde{T}_m\|_{1,q} = \|T_m\|_1 = \int_{-\infty}^{\infty}|dM|$.

Now suppose that for some r, $1<r<2$, and some s, $r<s\leqslant 2$, we had $\mathfrak{M}_r = (\mathfrak{M}_1, \mathfrak{M}_s)$ with $r^{-1} = (1-\theta)+\theta s^{-1}$. Then given $m\in\mathfrak{M}_r$ of norm 1, we would be able to find a function $m(\xi, z)$ on $\boldsymbol{R}^1\times[0<\operatorname{Re}z<1]$, analytic in z, with the properties $\|m(\cdot, iy)\|_{\mathfrak{M}_1}\leqslant 1$, $\|m(\cdot, 1+iy)\|_{\mathfrak{M}_s}\leqslant 1$ uniformly in y. This would yield $\|m(\cdot, iy)\|_{\mathfrak{M}_{1,\infty}}\leqslant 1$ and $\|m(\cdot, 1+iy)\|_{\mathfrak{M}_{s,2}}\leqslant 1$ uniformly in y. Using that fact, and that the spaces $L^p(L^q)$ interpolate in the expected way, we would then have $\|m\|_{\mathfrak{M}_{r,t}}\leqslant 1$ with $t^{-1} = \theta/2$. Since the dual of $L^p(L^q)$ is $L^{p'}(L^{q'})$, we would also have $\|m\|_{\mathfrak{M}_{r',t'}}\leqslant 1$ for every $m\in\mathfrak{M}_r = \mathfrak{M}_{r'}$ of norm 1. But $r'>2$ and $t'<2$; our example shows that $\|m\|_{\mathfrak{M}_{r',t'}}\leqslant\|m\|_{\mathfrak{M}_{r'}}$ cannot be true. Thus we see that $\mathfrak{M}_r$ cannot be $(\mathfrak{M}_1, \mathfrak{M}_s)_\theta$. Further, since no estimate of the form $\|m\|_{\mathfrak{M}_{r',t'}}\leqslant C\|m\|_{\mathfrak{M}_{r'}}$ can hold, we see that $\mathfrak{M}_r$ properly contains $(\mathfrak{M}_1, \mathfrak{M}_s)_\theta$.

Whether the spaces $\mathfrak{M}_p$ interpolate in the range $1<p<2$ seems to be an open problem.

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Weak type estimates for the Hardy–Littlewood maximal functions

by

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Abstract. In this paper we give sharp estimates for the weak type constants for the Maximal Operator of Differentiation. This is done in the case of one parameter m-dimensional parallelopipeds as a differentiation basis. The dependence on the parameter is asked to be more general than the usual monotonic one.

Introduction. The purpose of this paper is to improve and to extend results which have been obtained by Cotlar in [3] and [4]. These results are going to be used in [1].

1. Statement of results.

1.1. $R(x,t)$ will denote an m-dimensional rectangle having edges parallel to the coordinate axes, centered at the point x and edges given by $h_j(t)$, $j = 1, 2, \ldots, m$. Here $h_j(t)$ will denote the edge length corresponding to the x_j axis. The functions $h_j(t)$ are assumed to be continuous and non-negative and sytisfying the following conditions:

(1.1.1) $$t_1 \geqslant t_2 \Rightarrow k_j \cdot h_j(t_1) \geqslant h_j(t_2), \quad j = 1, 2, \ldots, m,$$

here k_j depends only on j and $k_j > 0$, $j = 1, 2, \ldots, m$,

(1.1.2) $$h_j(t) > 0, \quad t > 0; \quad h_j(0) = 0, \quad j = 1, 2, \ldots, m,$$

(1.1.3) $$h_j(t) \to \infty, \text{ as } t \to \infty \quad \text{for } j = 1, 2, \ldots, m.$$

1.2. By $f^*(x)$ we denote the maximal function

(1.2.1) $$\sup_{t>0} \left| \frac{1}{\mu(R(x,t))} \int_{R(x,t)} f\,d\mu \right|$$

where $R(x,t)$ are rectangles under the conditions of $(1,1)$, μ is a non-negative σ-additive measure defined on the Borel subsets of $\boldsymbol{R}^m$ and f is any μ-measurable and μ-locally integrable function. In the same way we define $\nu^*(x)$ for any σ-additive measure defined on the Borel subsets