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Addendum to: "Linear operations, tensor products, and contractive projections in function spaces" (Studia Math. 38 (1970), pp. 131–186)

by

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0. In this paper, an attempt was made to present a collection of results under a global assumption (cf. the italicized statement early on page 134), but to write them up in a (possibly) more general framework with appropriate side conditions. In a condensation, the above aim was unfortunately submerged and even a distorsion seems to have resulted. Using the page numbers and references of the paper, the following clarifications and corrections will now be indicated.

1. For all the work, it is assumed without further comment that:

GLOBAL ASSUMPTION. *The L^ϱ spaces on a nontrivial (Ω, Σ, μ) are such that the function norm ϱ satisfies both the conditions* (I) *and* (J) *and that the L^ϱ are complete.*

Under this assumption, Theorem 1.13 was already established in ([13], Corollary 16 on p. 52) when μ is σ-finite and ϱ has the weak Fatou property. The present case is a minor modification when μ, ϱ are slightly more general. Only (I) is crucial here. Thus one may assume the result of Theorem 1.13 in place of the above assumption and the further representation theory can be read, for more general spaces L^ϱ, for which Theorem 1.13 holds. Such spaces may be called *generalized MT-spaces* of M. Morse and W. Transue, J. Analyse Math. 4 (1955).

2. The part played by (J) appears in Theorem 4.4 on p. 151. It follows almost from definition of (J) (cf. [13], p. 7) that $\varrho(U_\pi f) \leqslant \varrho(f)$ and M^ϱ has m.a.p. If ϱ is such that M^ϱ has m.a.p. (and L^ϱ is a generalized MT-space) then the next two results may be read without the global assumption. These are some of the intended "generalizations". This m.a.p. is a.p. in [14].

3. A clause is missing from Theorem 5.8 on p. 158, and it should read as follows:

"THEOREM 5.8. *With the above notation, $L^\varrho \otimes_\lambda \mathscr{X} \cong \tilde{W}^\varrho_{\mathscr{X}}$. Thus $\tilde{W}^\varrho_{\mathscr{X}}$ can be identified isometrically as the space of all compact linear operators of $(L^\varrho)^*$ into $\mathscr{X}$, where $\tilde{W}^\varrho_{\mathscr{X}} = \overline{\mathrm{sp}}\{fx\colon f \in L^\varrho, x \in \mathscr{X}\} \subset W^\varrho_{\mathscr{X}}$.*"

Then the following (18) should read:

$$\text{“(18)}\qquad L^{\varrho}\otimes_{\gamma}\mathscr{X}\subset \mathscr{M}^{\varrho}_{\mathscr{X}}\subset L^{\varrho}_{\mathscr{X}}\subset W^{\varrho}_{\mathscr{X}}\supset \tilde{W}^{\varrho}_{\mathscr{X}}\cong L^{\varrho}\otimes_{\lambda}\mathscr{X},\text{”}$$

4. On pages 158–9, the two $W^{\varrho}_{\mathscr{X}}$ should be $\tilde{W}^{\varrho}_{\mathscr{X}}$, and in eq. (22), $(L^{\varrho}\otimes_{\lambda}\mathscr{X})$ should be $(L^{\varrho}\otimes_{\lambda}\mathscr{X})^*$, where $\mathscr{X}$ is such that every $T\in B(L^{\varrho},\mathscr{X})$ is weakly compact.

5. It may be noted that, in Theorem II.1.5 on p. 165, the conditional expectation $E^{\mathscr{B}}\colon L^{\varrho}(\Sigma)\to L^{\varrho}(\mathscr{B})$, is a contractive operator as an element of the Banach space $B\big(L^{\varrho}(\Sigma), L^{\varrho}(\mathscr{B})\big)$ if only ϱ is a Fatou norm. However, this does *not* imply that $E^{\mathscr{B}}\in B\big(L^{\varrho}(\Sigma), L^{\varrho}(\Sigma)\big)$. The latter is true only under an additional hypothesis, such as (J). This is discussed in some detail in [29], and hence Chapter II was even more severely curtailed. The latter paper will soon appear in Sankhyā, Ser A., 35. The unforseen delay is regrettable. Also, in most of the discussion of Chapter II, the case that $L^{\varrho} = L^2$ is excluded, this being a trivial case needing an easy but separate discussion. Professor W. Żelazko has kindly informed me that, if L^{ϱ} is a symmetric space, the results of Theorems II.1.5 and II.2.3 are immediate consequences of the interpolation theorems of E. M. Semenov and B. S. Mitjagin.

6. The fact that $E^{\mathscr{B}}$ is a contractive endomorphism in $L^{\varrho}(\Sigma)$ for any σ-field $\mathscr{B}\subset\Sigma$ (with $\mu_{\mathscr{B}}$ localizable and ϱ a Fatou norm) iff ϱ has the (J)-property has been shown by N. Gretsky (unpublished). But if ϱ is only a Fatou norm, then also it is true that $E^{\mathscr{B}}$ is idempotent (algebraic property) and has norm at most one as an element of $B\big(L^{\varrho}(\Sigma), L^{\varrho}(\mathscr{B})\big)$. This generalization has useful implications and it is this that I tried to indicate in the paper with scant explanation.

7. On p. 180, (lines 10 and 11 from bottom) Σ should be $\mathscr{B}$. Also, for simplicity, assume that $0\leqslant f\leqslant g$ and $f<g$ on a set of positive measure, $\varrho(f)<\infty$, implies $\varrho(f)<\varrho(g)$, in Chapter II.

8. The sketches and alternate arguments of Chapter II, are originally considered in detail for the Orlicz spaces $L^{\varrho} = L^{\Phi}$, in [29], and the details may therefore be constructed from it since it will soon appear. The conditions (I) and (J), (cf. the global assumption above) are naturally present for the Orlicz spaces L^{Φ}, and thus are also natural for the work of this paper (as well as for [13]) since these spaces are natural generalizations (together with the results) of the L^{Φ}-case.

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Correction to the paper "H^2 spaces of generalized half-planes" (Studia Math. 44 (1972), pp. 379–388)

by

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It has been pointed out to us by M. Vergne that the definition of $\hat{B}$ on p. 382 requires modification, since as it is defined there, it is not necessarily real-valued on $\boldsymbol{R}^{n_2}\times\boldsymbol{R}^{n_2}$.

One may proceed as follows. For fixed $\lambda\in\Omega^*$, let U_λ be a unitary matrix diagonalizing the Hermitian form $(z_2, w_2)\mapsto\langle\lambda, \Phi(z_2, w_2)\rangle$ on $\boldsymbol{C}^{m_2}$. It is easy to see that U_λ can be chosen so as to depend measurably on λ.

Let $E_\lambda = U_\lambda^{-1}(\boldsymbol{R}^{n_2})$; this is a real form of $\boldsymbol{C}^{n_2}$. In the definition of B_λ and throughout Section 4 let $\overline{w}_2$ (resp. $\overline{z}_2$) denote the complex conjugate of w_2 (resp. z_2) *with respect to* E_λ.

In the proof of Theorem 4.1, whenever we write $z_2 = x_2+iy_2$, it should be replaced by $z_2 = x_2^{(\lambda)}+iy_2^{(\lambda)}$, with $x_2^{(\lambda)}, y_2^{(\lambda)}\in E_\lambda$. Integration on $\boldsymbol{R}^{n_2}$ should everywhere be replaced by integration on E_λ; in particular α should always be a point in E_λ. With these modifications the proof remains valid.

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