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Addendum and corrigendum to the paper

"Some applications of Zygmund's lemma to non-linear differential equations in Banach and Hilbert spaces"

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by

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1. By using an idea of Diaz and Weinacht [1], Theorem 2 of the above paper can be strengthened by the replacement of the condition (1.4), viz

$$\operatorname{re}\langle f(t, y)-f(t, z), y-z\rangle \leqslant \tfrac{1}{2}g(t, \|y-z\|^2), \tag{1}$$

by the condition

$$\operatorname{re}\langle f(t, y)-f(t, z), y-z\rangle \leqslant \|y-z\|g(t, \|y-z\|), \tag{2}$$

where g satisfies the (usual Kamke) condition (A) of §1. In particular, when $g(t, x) = x/(t-t_0)$ (which gives Nagumo's condition), the replacement of (1) by (2) removes the factor $\frac{1}{2}$ on the right of (1).

The proof of the new version of Theorem 2 follows similar lines to that of the original version, but we now take $\sigma_{m,n}(t) = \|\varphi_m(t)-\varphi_n(t)\|$, where, for each n, φ_n is an ε_n-approximate solution of the equation $y' = f(t, y)$ such that $\varphi_n(t_0) = y_0$. If $\varphi_m(t) \neq \varphi_n(t)$, then

$$\begin{aligned}\sigma'_{m,n}(t) &= \frac{d}{dt}\{\|\varphi_m(t)-\varphi_n(t)\|^2\}^{\frac{1}{2}} \\ &= \frac{\operatorname{re}\langle \varphi'_m(t)-\varphi'_n(t), \varphi_m(t)-\varphi_n(t)\rangle}{\|\varphi_m(t)-\varphi_n(t)\|} \\ &= \frac{\operatorname{re}\langle f(t, \varphi_m(t))-f(t, \varphi_n(t)), \varphi_m(t)-\varphi_n(t)\rangle}{\|\varphi_m(t)-\varphi_n(t)\|} + \\ &\quad + \frac{\operatorname{re}\langle \varphi'_m(t)-f(t, \varphi_m(t))-\varphi'_n(t)+f(t, \varphi_n(t)), \varphi_m(t)-\varphi_n(t)\rangle}{\|\varphi_m(t)-\varphi_n(t)\|} \\ &\leqslant g(t, \sigma_{m,n}(t))+\varepsilon_m+\varepsilon_n.\end{aligned} \tag{3}$$

On the other hand, if $\psi_m(t) = \psi_n(t)$, then

$$(4) \quad D_+\sigma_{m,n}(t) \leqslant \|\psi_m'(t)-\psi_n'(t)\| = \big\|\psi_m'(t)-f\big(t,\psi_m(t)\big)-\psi_n'(t)+f\big(t,\psi_n(t)\big)\big\| \leqslant \varepsilon_m+\varepsilon_n,$$

and therefore the final inequality in (3) holds for all $t \in I$.

If now $\omega_n = \sup\limits_{m>n} \sigma_{m,n}$, then exactly as before we see that there exists a subsequence (ω_{n_r}) of (ω_n) converging uniformly on I to a function ω, and that $\omega(t_0) = 0$ and $D_+\omega(t) \leqslant g\big(t, \omega(t)\big)$ for all $t \in I^\circ$. The third identity in (3) and the final inequality in (4) show also (again as before) that $\omega'(t_0) = 0$, and therefore $\omega = 0$, as required.

2. A similar improvement can be effected in Theorem 4 by the substitution of (2) for (1); moreover, the proof is simplified, and Lemma 6 is no longer necessary.

In my paper it is incorrectly stated that the special case of Theorem 4 where $g(t,x) = x/(t-t_0)$ is a result of Murakami, but in fact Murakami's result is the same special case of the revised version of Theorem 4.

Reference

[1] J. B. Diaz and R. J. Weinacht, *On nonlinear differential equations in Hilbert spaces*, Applicable Analysis, 1 (1971), pp. 31-41.

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