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Correction to the paper "On weighted H^p spaces" (Studia Math., 40 (1971), pp. 109-159)

by

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The author's above-mentioned paper contains the false assertion that $L^{pq} = (L^{p'q'})'$ for $1 < p < \infty$, $q \geqslant 1$ (p. 104, line 22). This is well known to be valid only if $q > 1$ (see R. A. Hunt, *On $L(p, q)$ spaces*, Enseignement math. 12 (1966), pp. 249–276 and M. Cwikel and Y. Sagher, *Continuous linear functionals on weak L^p spaces*, Notices Amer. Math. Soc. 18 (1971) p. 960). Since some arguments in the author's paper are based on this false assertion they require modification. What saves the situation is the fact that the restriction of any element of the dual of $L^{p'\infty}$ to the subspace $\mathscr{K}$ of continuous functions of compact support still is given by a function in L^{p1} $(1 < p < \infty)$ (since $\mathscr{K}(R^n)$ is not dense in $L^{p'\infty}(R^n)$ the Hahn–Banach theorem therefore implies $L^{p1} \neq (L^{p'\infty})'$).

First note that Proposition 1 on p. 112 obviously should require $(1+|\cdot|)^{-n-1}C(R^{n*})$ to be dense in B. The conclusion of the corollary immediately following Proposition 1 will hold if w^{-1} is required to be continuous (or equivalently w, w^{-1} to be locally bounded) whenever $q = 1$ (instead of $p = 1$). For, since by inequality (5) on p. 112 $w^{-1}L^{p1} = L^{p1}_w \subset (1+|\cdot|)^{n+1}\mathscr{M}^1$ there are a measure μ and a complex number δ such that $F(x, y) = P(\cdot, y) * \mu(x) + \delta y$. Suppose $g \in \mathscr{K}$ then

$$\left|\int \mu(x) g(x) w(x)\, dx\right| = \left|\lim_{y\to 0}\int F(x, y) g(x) w(x)\, dx\right| \leqslant CM\|g\|_{p'\infty}.$$

This implies first that μw hence μ is absolutely continuous with respect to Lebesgue measure, i.e., there is a locally integrable function f such that $d_\mu(x) = f(x)\,dx$ and second that $\|wf\|_{p1} \leqslant CM$.

Proposition 2 on p. 127 requires no change. For, in the case of a radial function $\omega \in S_1^{*1}$ (see p. 110) the above argument still shows that

$$F(x, y) = P(\cdot, y) * f(x) + \gamma P(\cdot, y) * \varepsilon_0(x) + \delta y \tag{1}$$

where $\|\omega f\|_{pq} + |\gamma| \leqslant CM$ and ε_0 is the measure defined by $(\varepsilon_0, g) = g(0)$.

(It is understood that ω does not vanish identically). To conclude that $\gamma = 0$ note that if $s^*_{\omega y}$ denotes the decreasing rearrangement of $\omega F(\cdot, y)$ on $(0, \infty)$ then as $y \to 0$

$$(2) \qquad y^{-n} \int_{|x| \leqslant y} |F(x, y)| \omega(x)\, dx \leqslant C y^{-n} \int_0^{Cy^n} s^*_{\omega,y}(t)\, dt$$

$$\leqslant C y_y^{-n/p} \int_0^{Cy^n} s^*_{\omega,y}(t)\, t^{-1/p'}\, dt = o(y^{-n/p})$$

As a consequence of Lemma 3 (p. 119) it follows likewise that

$$(3) \qquad y^{-n} \int_{|x| \leqslant y} |P(\cdot, y) * f(x)| \omega(x)\, dx = o(y^{-n/p}).$$

Furthermore for some $C > 0$

$$(4) \qquad y^{-n} \int_{|x| \leqslant y} P(x, y) \omega(x)\, dx \geqslant C y^{-n} \inf_{y/2 \leqslant s \leqslant y} \omega(s) \geqslant C y^{-n} \omega(y).$$

(1)–(4) imply $|\gamma| \omega(t) = o(t^{n/p'})$ as $t \to 0$. Since by assumption $\omega(t) t^{-n/p'} \downarrow$ and $\omega > 0$ it follows that $\gamma = 0$.

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