Choose $n(k)$ satisfying $m(k) \leqslant n(k)$ and so that $J_N \subset J_{n(k)}$, $J_n \subset J_{n(k)}$. Then if $B_k = \Pi_{J_{n(k)}} R_{m(k)} \Pi_{J_{n(k)}}$, we have $\|B_k\| > \varepsilon$. To see that $\|D_k\| \leqslant 2\|M\|$ apply (i) and (ii) of the induction hypothesis above.

THEOREM 13. *Let $M = (c_{ij})$: $\lambda_s \to l_p$ be a matrix map. Then M is compact.*

Proof. If M is not compact, then by Proposition 12 there is an $\varepsilon > 0$ and indices $1 = m(1) \leqslant n(1) < \ldots < m(k) \leqslant n(k) < \ldots$ so that $\|B_k\| > \varepsilon$ and $\|D_k\| \leqslant 2\|M\|$ for all k. Let $a^{[1]}, a^{[2]}, \ldots, a^{[k]}$ be unit vectors in λ satisfying $\|B_k(a^{[k]})\|_p = \|B_k\|$. This choice is possible because each B_k is a finite matrix. Let $\xi = (1, 1, \ldots, 1, \ldots)$. Clearly ξ is a unit vector of λ_s. By f.m.p. we may choose $\beta^{[t]} \epsilon \lambda$, where $\beta^{[t]} = (b_1^{[t]}, \ldots, b_j^{[t]}, \ldots)$ satisfies:

$$b_i^{[t]} = \begin{cases} a_i^{[k]} & \text{if } m(k) \leqslant i \leqslant n(k), \quad \text{where } k = 1, 2, \ldots, t, \\ 0 & \text{if } n(k) < i < m(k+1), \quad \text{where } k = 1, 2, \ldots, t-1, \end{cases}$$

and $\|\beta^{[t]}\| \leqslant \|\xi\| = 1$.

Thus, $\|D_t(\beta^{[t]})\|_p = \big(\sum_{k=1}^{t} \|B_k\|^p\big)^{1/p} = t^{1/p}\varepsilon$.

A contradiction now results because $2\|M\| \geqslant \|D_t\| \geqslant \|D_t(\beta^{[t]})\|_p = t^{1/p}\varepsilon$, where t is arbitrary.

Remark. It is not hard to check that the l_q spaces satisfy f.m.p. if $1 < q < \infty$ and that c_0 also satisfies f.m.p. Arguing as in Theorem 13 we may apply Theorem 12 to show that there are no non-compact operators T: $l_q \to l_p$ or T: $c_0 \to l_p$, if $1 \leqslant p < q < \infty$. This is the Littlewood-Pitt Theorem [6].

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Factorable and strictly singular operators, I

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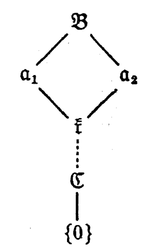
Let X and Y be Banach spaces; a linear bounded operator T: $X \to Y$ is *strictly singular* [5] if the subspaces $Z \subset X$ for which the restriction $T|_Z$: $Z \to T(Z)$ has a bounded inverse $T(Z) \to Z$ are necessarily finite dimensional. Assume X is the cartesian product $X = E_1 \times E_2$ of the Banach spaces E_1 and E_2. An operator T: $X \to X$ is *factorable* through E_i (for $i = 1, 2$) if it is of the form $T = SQ$ for suitable Q : $X \to E_i$ and S: $E_i \to X$. Denote by $\mathfrak{a}_i$ the (uniform) closure of the set of operators factorable through E_i. We prove below that under special conditions, an operator T is strictly singular if and only if $T \epsilon \mathfrak{a}_1 \cap \mathfrak{a}_2$. This is applied notably to the case $E_1 = \ell^p$ and $E_2 = \ell^q$ for $p \neq q$ (and more particularly, when $p > q = 2$) to obtain various relations between these types of operators. As a consequence, it can be seen ((3.1. e) below) that in this case, if T_1 and T_2 are strictly singular, then $T_1 T_2$ is compact.

Finally, the lattice of all closed two-sided ideals of the algebra of all bounded linear operators on $X = \ell^p \times \ell^2$, for $p > 2$, is studied in (3.4): it consists of six elements 0, $\mathfrak{C}$, $\mathfrak{k}$, $\mathfrak{a}_1$, $\mathfrak{a}_2$, $\mathfrak{B}$ ordered as follows:



where solid lines mean that there are no ideals strictly between the ideals joined by them.

1. Terminology. We observe the standard terminology concerning Banach spaces, operators, sequence spaces ℓ^p and Lebesgue spaces $L^p = L^p[0,1]$.

For any Banach spaces X and Y, $\mathfrak{B}(X, Y)$, $\mathfrak{F}(X, Y)$, $\mathfrak{C}(X, Y)$ and $\mathfrak{k}(X, Y)$ shall denote, respectively, the Banach space of all bounded linear operators $T\colon X \to Y$, and the subspaces consisting of operators of finite rank, compact operators and strictly singular operators; we set $\mathfrak{B}(X) = \mathfrak{B}(X, X)$, $\mathfrak{F}(X) = \mathfrak{F}(X, X)$, $\mathfrak{C}(X) = \mathfrak{C}(X, X)$ and $\mathfrak{k}(X) = \mathfrak{k}(X, X)$. $\mathfrak{F}(X) \subset \mathfrak{C}(X) \subset \mathfrak{k}(X)$ are two sided ideals of $\mathfrak{B}(X)$ and, moreover, $\mathfrak{C}(X)$ and $\mathfrak{k}(X)$ are closed in $\mathfrak{B}(X)$.

All throughout, when we consider the product $X = E_1 \times E_2$ of the Banach spaces E_1 and E_2, we shall (try to) use consistently the following notation: $P_i \in \mathfrak{B}(X, E_i)$ is the projection $P_i(x_1, x_2) = x_i$ for $i = 1, 2$; $J_i \in \mathfrak{B}(E_i, X)$ are the injections $J_1(x_1) = (x_1, 0)$ and $J_2(x_2) = (0, x_2)$. Every $T \in \mathfrak{B}(X)$ has a matrix representation that will be denoted by $T = (T_{ij})$, where $T_{ij} \in \mathfrak{B}(E_j, E_i)$ for $i = 1, 2$ and $j = 1, 2$, and

$$T(x_1, x_2) = (T_{11}x_1 + T_{12}x_2,\ T_{21}x_1 + T_{22}x_2).$$

Clearly,

$$T_{ij} = P_i T J_j \quad \text{and} \quad T = \sum_{i,j} J_i T_{ij} P_j.$$

For any set $\mathfrak{a} \subset \mathfrak{B}(X)$ we shall also denote by $\mathfrak{a}_{ij} \subset \mathfrak{B}(E_j, E_i)$ the set $\mathfrak{a}_{ij} = \{T_{ij}, T \in \mathfrak{a}\}$; in particular, $\mathfrak{B}(X)_{ij} = \mathfrak{B}(E_j, E_i)$ and if $\mathfrak{a} \subset \mathfrak{B}(X)$ is a two-sided ideal, $\mathfrak{a}_{ij} = \{S \in \mathfrak{B}(E_j, E_i);\ J_i S P_j \in \mathfrak{a}\}$. Finally, $\mathfrak{m}_i \subset \mathfrak{B}(X)$ will denote the set $\mathfrak{m}_i = \mathfrak{m}_{E_i}(X)$ (in the notation of [1]) of all operators $T \in \mathfrak{B}(X)$ of the form $T = SQ$ for suitable $Q \in \mathfrak{B}(X, E_i)$ and $S \in \mathfrak{B}(E_i, X)$, and $\mathfrak{a}_i$ will denote the (norm) closure of $\mathfrak{m}_i$. Clearly, $P_i \in \mathfrak{m}_i$ and for every $T \in \mathfrak{B}(X)$, $J_i T_{ij} P_j \in \mathfrak{m}_i \cap \mathfrak{m}_j$.

2. Strictly singular and factorable operators. The following general hypothesis is assumed all throughout this section:

E_1 and E_2 are Banach spaces isomorphic to their cartesian squares $E_1 \times E_1$ and $E_2 \times E_2$, respectively, and $X = E_1 \times E_2$.

It follows from [7] (or [1], Theorem 5.13) that $\mathfrak{m}_i$ (whence $\mathfrak{a}_i$) is a two-sided ideal of $\mathfrak{B}(X)$ for $i = 1, 2$,.

2.1. Lemma. (a) *For every $T \in \mathfrak{B}(X)$, $T - J_1 T_{11} P_1 \in \mathfrak{m}_2$ and $T - J_2 T_{22} P_2 \in \mathfrak{m}_1$;*

(b) *if $T \in \mathfrak{k}(X)$, then $T_{ij} \in \mathfrak{k}(E_j, E_i)$ for $i = 1, 2$ and $j = 1, 2$, and therefore also $T - J_i T_{ij} P_j \in \mathfrak{k}(X)$;*

(c) *if $T \in \mathfrak{C}(X)$, then $T_{ij} \in \mathfrak{C}(E_j, E_i)$ for $i = 1, 2$ and $j = 1, 2$, and therefore also $T - J_i T_{ij} P_j \in \mathfrak{C}(X)$.*

Proof. Since $T = J_1 T_{11} P_1 + S$, where $S = \sum_{i+j \geqslant 3} J_i T_{ij} P_i \in \mathfrak{m}_2$ (and, similarly, for $i = 2$), (a) follows. Since $T_{ij} = P_i T J_j$, (b) and (c) follow.

2.2. Proposition. *Assume $\mathfrak{k}(E_1) = \mathbf{cl}(\mathfrak{F}(E_1))$ ($\mathbf{cl}(\ )$ means "closure"). Then $\mathfrak{k}(X) \subset \mathfrak{a}_2$. In particular, if $\mathfrak{k}(E_i) = \mathbf{cl}(\mathfrak{F}(E_i))$ for $i = 1, 2$, then $\mathfrak{k}(X) \subset \mathfrak{a}_1 \cap \mathfrak{a}_2$.*

Proof. Let $T \in \mathfrak{k}(X)$. From 2.1(a), $T = J_1 T_{11} P_1 + S$, where $S \in \mathfrak{m}_2$. But since $T_{11} \in \mathfrak{k}(E_1)$ (from 2.1(b)) and $\mathfrak{k}(E_1) = \mathbf{cl}(\mathfrak{F}(E_1))$, we have $T_{11} = \lim F_n$, where rank $F_n < +\infty$. Clearly, $J_1 F_n P_1 \in \mathfrak{m}_2$ and, therefore, $J_1 T_{11} P_1 \in \mathbf{cl}(\mathfrak{m}_2) = \mathfrak{a}_2$. Thus $T = J_1 T_{11} P_1 + S \in \mathfrak{a}_2$.

Remark. All $\ell^p, 1 \leqslant p < \infty$ (and no L^p, $1 \leqslant p < \infty$, $p \neq 2$), have the property $\mathfrak{k} = \mathbf{cl}(\mathfrak{F})$. See [3] for a sufficient condition for this property.

2.3. Definition ([9]). The Banach spaces E_1 and E_2 are *totally incomparable* if E_1 and E_2 have no infinite-dimensional isomorphic subspaces.

It is clear that if E_1 and E_2 are totally incomparable, every $T\colon E_1 \to E_2$ is strictly singular.

2.4. Proposition. *Assume E_1 and E_2 are totally incomparable. Then $\mathfrak{a}_1 \cap \mathfrak{a}_2 \subset \mathfrak{k}(X)$.*

Proof. Let $T \in \mathfrak{a}_1 \cap \mathfrak{a}_2$ and $S_n^i\colon E_i \to X$, $Q_n^i\colon X \to E_i$ for $i = 1, 2$ and $n = 1, 2, \ldots$ such that $T = \lim S_n^1 Q_n^1 = \lim S_n^2 Q_n^2$ for $n \to \infty$. If $i \neq j$, $Q_n^i J_j\colon E_j \to E_i$ is strictly singular and, therefore, $S_n^1 Q_n^1 J_2 P_2$ and $S_n^2 Q_n^2 J_1 P_1$ are also strictly singular for all n. Hence

$$T = \lim (S_n^1 Q_n^1 J_2 P_2 + S_n^2 Q_n^2 J_1 P_1)$$

is strictly singular, and the Proposition follows.

From 2.2 and 2.4 we obtain the following:

2.5. Theorem. *If $\mathfrak{k}(E_i) = \mathbf{cl}(\mathfrak{F}(E_i))$ for $i = 1, 2$, and E_1 and E_2 are totally incomparable, then $\mathfrak{k}(X) = \mathfrak{a}_1 \cap \mathfrak{a}_2$.*

Now we turn to the consideration of the other possible ideals in $\mathfrak{B}(X)$.

2.6. Proposition. *Under the same hypotheses of Theorem 2.5, and assuming moreover that $\mathfrak{k}(E_i) = \mathbf{cl}(\mathfrak{F}(E_i))$ is a maximal ideal, let $\mathfrak{a} \subset \mathfrak{B}(X)$ be a closed two-sided ideal. Then one of the following must be satisfied:*

(a) $\mathfrak{a} \subset \mathfrak{k}(X)$,

(b) $\mathfrak{a} \supset \mathfrak{a}_i$ *for some* $i = 1, 2$.

Proof. Every $T\in\mathfrak{a}$ can be decomposed as $T=J_1T_{11}P_1+J_2T_{22}P_2+S$, where $S\in\mathfrak{m}_1\cap\mathfrak{m}_2$, so that the only problem is to determine the possible T_{11} and T_{22}. Clearly, $\mathfrak{a}_{ii}=\{T_{ii}\}=\{S\in\mathfrak{B}(E_i);\, J_iSP_i\in\mathfrak{a}\}$ is a closed two-sided ideal of $\mathfrak{B}(E_i)$ for $i=1,2$. If $\mathfrak{a}_{ii}\subset\mathfrak{C}(E_i)$ for both $i=1$ and $i=2$, then clearly $\mathfrak{a}\subset\mathfrak{C}(X)\subset\mathfrak{f}(X)$. Otherwise, for some $i=1,2$ we have $\mathfrak{a}_{ii}=\mathfrak{B}(E_i)$ and, therefore, $P_i\in\mathfrak{a}$, whence $\mathfrak{a}\supset\mathfrak{a}_i$.

2.7. THEOREM. *Assume that $\mathfrak{f}(E_i)=\mathbf{cl}(\mathfrak{F}(E_i))$ is a maximal two-sided ideal of $\mathfrak{B}(E_i)$ and that E_1 and E_2 are totally incomparable. Then:*

(a) *for $i=1,2$, if a closed two-sided ideal $\mathfrak{a}\subset\mathfrak{B}(X)$ satisfies $\mathfrak{f}(X)\subset\mathfrak{a}\subset\mathfrak{a}_i$, then either $\mathfrak{f}(X)=\mathfrak{a}$ or $\mathfrak{a}=\mathfrak{a}_i$;*

(b) *for any closed two-sided ideal $\mathfrak{b}\subset\mathfrak{B}(X)$, either $\mathfrak{b}\subset\mathfrak{f}(X)$ or $\mathfrak{f}(X)\subset\mathfrak{b}$.*

From a lattice theoretic standpoint, (a) means that $\mathfrak{a}_i$ *covers* $\mathfrak{f}(X)$, while (b) means that the singleton $\{\mathfrak{f}(X)\}$ is *crosscut* of the lattice of closed two-sided ideals of $\mathfrak{B}(X)$.

Proof of 2.7. Let $\mathfrak{f}(X)\subset\mathfrak{a}\subset\mathfrak{a}_1$; 2.6(a) and 2.6(b) imply $\mathfrak{a}\supset\mathfrak{a}_1$ or $\mathfrak{a}\supset\mathfrak{a}_2$. The former implies $\mathfrak{a}=\mathfrak{a}_1$ and the latter leads to the contradiction $\mathfrak{a}_2\subset\mathfrak{a}\subset\mathfrak{a}_1$. Similarly, for $\mathfrak{f}(X)\subsetneq\mathfrak{a}\subset\mathfrak{a}_2$. Finally, (b) follows from 2.5 and 2.6.

3. The case $E_i=\ell^p$ or $E_i=c_0$. We shall consider now the special case in which E_1 and E_2 are the spaces ℓ^p, $1<p<\infty$, or c_0.

3.1. THEOREM. *Let $X=\ell^p\times\ell^q$ $(1<p\neq q<\infty)$ or $X=\ell^p\times c_0$ $(1<p<\infty)$. Then:*

(a) $\mathfrak{f}(X)=\mathfrak{a}_1\cap\mathfrak{a}_2$;

(b) *there are no closed two-sided ideals $\mathfrak{a}\subset\mathfrak{B}(X)$ satisfying $\mathfrak{f}(X)\subsetneq\mathfrak{a}\subsetneq\mathfrak{a}_1$ or $\mathfrak{f}(X)\subsetneq\mathfrak{a}\subsetneq\mathfrak{a}_2$;*

(c) *for every closed two-sided ideal $\mathfrak{b}\subset\mathfrak{B}(X)$, either $\mathfrak{b}\subset\mathfrak{f}(X)$ or $\mathfrak{f}(X)\subset\mathfrak{b}$;*

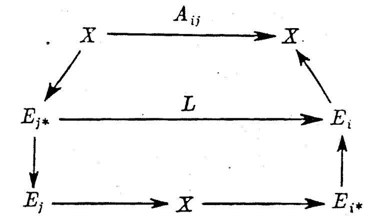
(d) *there are no closed two-sided ideals $\mathfrak{c}\subset\mathfrak{B}(X)$ satisfying $\{0\}\subsetneq\mathfrak{c}\subsetneq\mathfrak{C}(X)$;*

(e) $[\mathfrak{f}(X)]^2\subset\mathfrak{C}(X)$, *i. e., if $T,S\in\mathfrak{f}(X)$, then $TS\in\mathfrak{C}(X)$.*

Proof. (a), (b) and (c) follow from 2.5 and 2.7, and (d) from the fact that X has a basis. For $T\in\mathfrak{f}(X)$, it is clear that $T_{11}\in\mathfrak{f}(\ell^p)=\mathfrak{C}(\ell^p)$ and $T_{22}\in\mathfrak{f}(\ell^q)=\mathfrak{C}(\ell^q)$ (resp. $T_{22}\in\mathfrak{f}(c_0)=\mathfrak{C}(c_0)$) (see [2]) so that

$$T=K+J_1T_{12}P_2+J_2T_{21}P_1,\qquad \text{where } K\in\mathfrak{C}(X).$$

Similarly, for $S\in\mathfrak{f}(X)$, $S=K'+J_1S_{12}P_2+J_2S_{21}P_1$. Hence $ST=K''+ A_{11}+A_{12}+A_{21}+A_{22}$ with $A_{ij}=J_iT_{ii^*}P_{i^*}J_jS_{jj^*}P_{j^*}$, where for $h=1,2$, h^* is defined by $h^*=3-h$. It follows immediately from [8], Lemma 3, that in the diagram



the operator L is always compact, whence each A_{ij} and, therefore, ST, are also compact. This proves (e).

By specializing a little further, we can see that there are no ideals between $\mathfrak{a}_1$ and $\mathfrak{B}(X)$. First we observe that:

3.2. LEMMA. *Let $X=\ell^p\times\ell^2$, where $2<p<\infty$. Then:*

(a) *Every subspace of X isomorphic to ℓ^2 is complemented.*

(b) *Every subspace of X isomorphic to ℓ^p contains a complemented subspace isomorphic to ℓ^p.*

The proof follows from [4], § 3, Cor. 1 and Cor. 2, and the remark that X is isomorphic to a (complemented) subspace of

$$L^p\times(\ell^p\times\ell^2)\sim(L^p\times\ell^p)\times\ell^2\sim L^p\times\ell^2\sim L^p$$

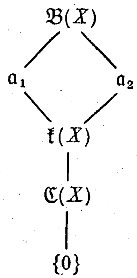
(where $\sim$ means "isomorphic to").

3.3. PROPOSITION. *Let $X=\ell^p\times\ell^2$, where $2<p<\infty$. Then $\mathfrak{a}_1$ and $\mathfrak{a}_2$ are maximal two-sided ideals of $\mathfrak{B}(X)$.*

Proof. Let $\mathfrak{a}\subset\mathfrak{B}(X)$ be a two-sided ideal satisfying $\mathfrak{a}_1\subsetneqq\mathfrak{a}$ and $T\in\mathfrak{a}$ such that $T\notin\mathfrak{a}_1$. Since $T=V+U$ with $V\in\mathfrak{m}_1$ and $U\in\mathfrak{m}_2$, we conclude that $U\in\mathfrak{a}$ and $U\notin\mathfrak{a}_1$. Then 3.1(a) implies that $U\notin\mathfrak{f}(X)$ and therefore there is an infinite-dimensional subspace $Y\subset X$ such that $U\colon Y\to U(Y)$ is an isomorphism. Moreover, since $U\in\mathfrak{m}_2$, there exist $S\colon\ell^2\to X$ and $S'\colon X\to\ell^2$ such that $U=SS'$, whence $S'\colon Y\to S'(Y)\subset\ell^2$ must also be an isomorphism; it follows that Y is isomorphic to ℓ^2 and, according to 3.2(a), necessarily complemnted. Let $P\in\mathfrak{B}(X)$ be a projection on Y. Since the inverse $(S')^{-1}\colon S'(Y)\to Y$ can be extended (to ℓ^2 (by 0 on the complement of $S'(Y)\subset\ell^2$), and therefore also) to an operator $L\in\mathfrak{B}(X)$, we have $LUP=P\in\mathfrak{a}$ and this implies [7] that $\mathfrak{m}_2\subset\mathfrak{a}$. This means that $\mathfrak{a}\supset\mathfrak{a}_1+\mathfrak{m}_2=\mathfrak{B}(X)$ and therefore $\mathfrak{a}_1$ is maximal. The maximality of $\mathfrak{a}_2$ can be handled in a similar fashion, except that the corresponding subspace $S'(Y)\subset\overline{\ell}^p$ need not be complemented and, therefore, the operator $(S')^{-1}$: $S'(Y) \to Y$ cannot be extended to ℓ^p, and hence to X. But from [6], Lemma 2, p. 214, $S'(Y)$ contains a subspace Z complemented in ℓ^p and isomorphic to ℓ^p. The proof proceeds as above with Y replaced by $(S')^{-1}(Z)$ and using 3.2 (b) instead of 3.2 (a).

From 3.1 and 3.3 we get:

3.4. THEOREM. *Let $X = \ell^p \times \ell^2$, where $2 < p < \infty$. Then the lattice of all closed two-sided ideals of $\mathfrak{B}(X)$ contains the following*:



and in fact every other closed two sided ideal is necessarily between $\mathfrak{C}(X)$ and $\mathfrak{f}(X)$.

It is clear that the same result holds for $1 < p < 2$ by standard duality arguments: just observe that if X is a reflexive space, then $\mathfrak{B}(X)$ and $\mathfrak{B}(X^*)$ (where X^* is the dual of X) are antiisomorphic algebras and therefore their families of two sided ideals are lattice isomorphic.

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