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# $p$-integral operators commuting with group representations and examples of quasi-$p$-integral operators which are not $p$-integral

by

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In this note we substantiate a conjecture of Persson and Pietsch (cf. [5], remark after Satz 46) that for each $p \geqslant 1$ there exists a quasi $p$-integral operator (resp. quasi $p$-nuclear operator) which is not $p$-integral (resp. $p$-nuclear). The case $p = 1$ is well known; as an example it is enough to consider in a Hilbert space any operator of a Hilbert-Schmidt type which is not nuclear. The examples for $p > 1$ will be constructed in the present note exploiting the fact that a $p$-integral operator which commutes with representations of a compact group $G$ has a "$G$-invariant factorization".

**1. Preliminaries.** The capital letters $X$, $Y$ and $Z$ will stand for Banach spaces. An *operator* means bounded linear operator. $C(K)$-denotes the space of continuous complex-valued functions on a compact Hausdorff space $K$. A *measure* on $K$ means a non-negative Borel measure on $K$ with bounded total variation. If $m$ is a measure on $K$, then $L_p(m, K)$ denotes the space of complex-valued functions $f$ on $K$ such that $m(|f|^p) < \infty$. We use the notation $m(f) = \int_K f dm$.

We recall the basic definitions from [5].

Let $1 \leqslant p < \infty$. An operator $v: C(K) \to Y$ is called *$p$-majorable* if there is a measure $m$ on $K$ such that

$$\|vf\|^p \leqslant m(|f|^p) \quad \text{for } f \in C(K). \tag{1.1}$$

An operator $u: X \to Y$ is called *$p$-integral* if there exists an isometrically isomorphic embedding $J: X \to C(K)$ and a $p$-majorable operator $v: C(K) \to Y$ such that $u = vJ$.

An operator $u: X \to Y$ is called *quasi $p$-integral* if there is an isometrically isomorphic embedding $I: Y \to Z$ such that $Iu$ is a $p$-integral operator.

We shall need the following two facts. First of them is an obvious consequence of [5], Satz 45, and [6], Theorem 2 (cf. also [3], Proposition 3.1).

PROPOSITION 1.1. *For every operator $v: C(K) \to Y$ the following conditions are equivalent*:

(1.2) $v$ is *p-majorable*,

(1.3) $v$ *is p-integral*,

(1.4) $v$ *is quasi p-integral*,

(1.5) $v$ *is p-absolutely summing, i.e. there is $a > 0$ such that*

$$\sum_{i=1}^{n} \|vf_i\|^p \leqslant a \sup_{\|m\| \leqslant 1} \sum_{i=1}^{n} |m(f_i)|^p$$

*for every finite sequence $(f_i)$ in $C(K)$.*

PROPOSITION 1.2. *For every operator $u: X \to Y$ the following conditions are equivalent*:

(1.6) $u$ *is a p-integral operator*,

(1.7) *for every isometrically isomorphic embedding $J_1: X \to C(K_1)$ ($K_1$ an arbitrary compact Hausdorff space) there exists a p-majorable operator $v_1: C(K_1) \to Y$ such that $u = v_1 J_1$.*

Proof. (1.7) $\Rightarrow$ (1.6). Obvious.

(1.6) $\Rightarrow$ (1.7). Let $u = vJ$, where $J: X \to C(K)$ is an isometrically isomorphic embedding. First observe that if $v: C(K) \to Y$ is a $p$-majorable operator, then (1.1) implies that $v$ is continuous in the $L_p(K, m)$ norm. Hence (regarding $C(K)$ as a subspace of $L_\infty(K, m)$) there is (determined in a unique way) an operator $\tilde{v}: L_\infty(K, m) \to Y$ such that

$$\|\tilde{v}f\|^p \leqslant m(|f|^p) \quad \text{for } f \in L_\infty(K, m) \tag{1.8}$$

and $\tilde{v}I = v$ where $I: C(K) \to L_\infty(K, m)$ is the natural isometrically isomorphic embedding. Since the space $L_\infty(K, m)$ has the Banach-Hahn extension property ([1], p. 105), for any isometrically isomorphic embedding $J_1: X \to C(K_1)$ there exists an operator $\tilde{I}: C(K_1) \to L_\infty(K, m)$ such that $\tilde{I}J_1 = IJ$. We put $v_1 = \tilde{v}\tilde{I}$. Then

$$v_1 J_1 = \tilde{v}\tilde{I}J_1 = \tilde{v}IJ = vJ = u.$$

Finally using (1.8) one can easily check that $v_1$ satisfies condition (1.5) of Proposition 1.1. This completes the proof.

**2. $p$-integral operators commuting with group representations.** Let $G$ be a topological group and let $X$ be a Banach space. By an *$X$-representation* of $G$ we mean a homomorphism $g \to A_g$ such that



(2.1) For each $g$ in $G$ the operator $A_g: X \to X$ is an isometric isomorphism.

(2.2) For each $x$ in $X$ the function $g \to A_g x$ is continuous.

Let $u: X \to Y$ be an operator. Let $g \to A_g$ and $g \to B_g$ be $X$ and $Y$-representations of a topological group $G$. Let

$$uA_g = B_g u \quad \text{for } g \in G. \tag{2.3}$$

Then we say that $u$ *commutes with the representations $g \to A_g$ and $g \to B_g$.*

Now we are ready for the main result of this note which is similar to a result of Rudin [8].

PROPOSITION 2.1. *Let $G$ be a compact topological group. Let $g \to A_g$, $g \to B_g$ and $g \to C_g$ be $X$, $Y$ and $C(K)$ — representations of $G$. Let $u: X \to Y$ be a p-integral operator which commutes with the representations $g \to A_g$ and $g \to B_g$ and let $J: X \to C(K)$ be an isometrically isomorphic embedding which commutes with the representations $g \to A_g$ and $g \to C_g$.*

*Then there exist a measure $n$ on $K$ and an operator $w: C(K) \to Y$ such that*

(2.4) $wJ = u$,

(2.5) $w$ *commutes with the representations $g \to C_g$ and $g \to B_g$,*

(2.6) $\|wf\|^p \leqslant n(|f|^p)$ *for $f \in C(K)$,*

(2.7) $n(|C_g f|) = n(|f|)$ *for $g \in G$ and $f \in C(K)$.*

Proof. By [2], p. 442, the isometry $C_g$ is of the form $C_g f = a_g \cdot f \circ F_g^{-1}$ for $f \in C(K)$, where $a_g \in C(K)$ with $|a_g| = 1$ and $F_g: K \to K$ is a homeomorphism ($g \in G$). Since $C_{gh} = C_g \circ C_h$ for $g, h$ in $G$, we have

$$a_g \cdot (a_h \circ F_g^{-1}) \cdot f \circ F_h^{-1} \circ F_g^{-1} = a_{gh} \cdot f \circ F_{gh}^{-1}.$$

Passing to the modulus we get $f \circ F_h^{-1} \circ F_g^{-1} = f \circ F_{gh}^{-1}$ for $f \geqslant 0$. Thus $F_{gh} = F_g \circ F_h$ for $g, h$ in $G$. Next, by (2.2), we infer that $F_{(\cdot)}(\cdot)$ is a continuous function on $G \times K$.

Since $u: X \to Y$ is a $p$-integral operator, Proposition 1.2 implies that there exist an operator $v: C(K) \to Y$ and a measure $m$ on $K$ satisfying (1.1) and such that $vJ = u$. Define for each $g$ in $G$ (via the Riesz Representation Theorem ([2], p. 265) the measure $m_g$ on $K$ by

$$m_g(f) = m(f \circ F_g^{-1}) \quad \text{for} \quad f \in C(K).$$

Let us set for $f \in C(K)$

$$n(f) = \int_G m_g(f)\, dg, \tag{2.8}$$

$$wf = \int_G B_g^{-1} v C_g f\, dg, \tag{2.9}$$

where $\int_G \ldots dg$ denotes the integral with respect to the normalized Haar measure of $G$. Clearly, formulas (2.8) and (2.9) well define the measure

$n$ on $K$ (observe that $\|n\| = \|m\|$ because $\|n\| = n(1)$ and $\|m\| = \|m_g\| = m(1) = m_g(1)$ for $g \in G$) and the operator $w: C(K) \to Y$. Using the identity $u = vJ$ and the fact that the operators $u$ and $J$ commutes with the appropriate representations we get

$$B_g^{-1} v C_g J = B_g^{-1} v J A_g = B_g^{-1} u A_g = u \quad \text{for } g \in G.$$

Thus integrating with respect to the normalized Haar measure we obtain (2.4). Next observe that for each $g$ and $h$ in $G$ we have the identity

$$B_h B_g^{-1} v C_g = (B_{gh^{-1}})^{-1} v C_{gh^{-1}} C_h.$$

Thus integrating over the variable $g$ and using the fact that the Haar measure is translation invariant we get $B_h w = w C_h$ for each $h$ in $G$. This proves (2.5).

Using the fact that $B_g^{-1}$ is an isometry and (1.1) we get

$$\|B_g^{-1} v C_g f\|^p = \|v C_g f\|^p \leqslant m(|C_g f|^p) = m(|f \circ F_g^{-1}|^p) = m_g(|f|^p) \quad (g \in G, f \in C(K)).$$

Hence, integrating, we obtain

$$\|wf\|^p = \|\int_G B_g^{-1} v C_g f dg\|^p \leqslant \int_G \|B_g^{-1} v C_g f\|^p dg \leqslant \int_G m_g(|f|^p) dg = n(|f|^p).$$

This proves (2.6).

Finally, we have

$$m_g(|C_h f|) = m_g(|f \circ F_h^{-1}|) = m(|f \circ F_{hg}^{-1}|) = m_{gh}(|f|) \quad (g, h \in G; f \in C(K)).$$

Thus integrating with respect to the Haar measure we get

$$n(|C_h f|) = \int_G m_{gh}(|f|) dg = n(|f|) \quad (h \in G; f \in C(K)).$$

This proves (2.7) and completes the proof of the Proposition.

**3. Examples of quasi $p$-integral operators which are not $p$-integral.** Let $T$ denote the unit circle on the complex plane. Clearly, $T$ is a compact abelian group. Denote by $L_p(T)$ the $L_p$-space with respect to the normalized Haar measure of $T$.

Let $M$ be a non-empty subset of the set $Z$ of the integers. Let $X_M$ (resp. $Y_{M,p}$) denote the closed subspace of $C(T)$ (resp. of $L_p(T)$) spanned by such characters $z^r$ that $r \in M$. Let, furthermore, $i_p: C(T) \to L_p(T)$ be the natural embedding (= the formal identity map, $i_p f = f$). Clearly, if $f \in X_M$, then $i_p f \in Y_{M,p}$ and the image $i_p(X_M)$ is dense in $Y_{M,p}$. Let us define $u_{M,p}: X_M \to Y_{M,p}$ by the relation

(3.1) $$J_{M,p} u_{M,p} = i_p J_M,$$



where $J_{M,p}: Y_{M,p} \to L_p(T)$ and $J_M: X_M \to C(T)$ denote the natural isometrically isomorphic embeddings.

LEMMA 3.1. *For every non-empty subset $M$ of the set $Z$ the operator $u_{M,p}: X_M \to Y_{M,p}$ is quasi $p$-integral.*

Proof. Obviously the operator $i_p: C(T) \to L_p(T)$ is $p$-majorable. Hence the operator $i_p J_M$ is $p$-integral. The desired conclusion follows now from (3.1) and the definition of quasi $p$-integral operators.

Let $R(T)$ denote the set of rational functions on $T$, i.e.

$$R(T) = \{f \in C(T): f(z) = \sum_{r \in Z} a_r z^r;\ a_r = 0 \text{ for almost all } r\}.$$

Define $\tilde{P}_M: R(T) \to R(T)$ by

$$\tilde{P}_M f = \sum_{r \in M} a_r z^r \quad \text{for} \quad f = \sum_{r \in Z} a_r z^r \in R(T).$$

PROPOSITION 3.2. *The following conditions are equivalent:*

(3.2) *$u_{M,p}$ is a $p$-integral operator,*

(3.3) *$\tilde{P}_M$ is bounded in $L_p(T)$ norm,*

(3.4) *$Y_{M,p}$ is a complemented subspace of $L_p(T)$.*

Proof. (3.2) $\Rightarrow$ (3.3) First we define the action of the group $T$ on $X_M$, $Y_{M,p}$ and $C(T)$ by

(3.5) $$f \to f_a \quad \text{for } a \in T,$$

where $f_a(z) = f(az)$ $(a \in T, z \in T)$.

Since $(z^r)_a = a^r z^r$ for each $a$ in $T$ and for each $r$ in $Z$, we easily check that (3.5) defines $X_M$, $Y_{M,p}$ and $C(T)$-representations. Clearly, $u_{M,p}$ and $J_M$ commutes with this representations. Hence, by Proposition 2.1 there is an operator $w: C(T) \to Y_{M,p}$ satisfying conditions (2.4)-(2.7) with $G = K = T$, $u = u_{M,p}$, $J = J_M$, $X = X_M$ and $Y = Y_{M,p}$. Let us put for conveniency $w(z^r) = f_r$ $(r \in Z)$. By (2.4) and (3.1), we have

(3.6) $$f_r = z^r \quad \text{for } r \in M.$$

On the other hand, (2.5) implies that

$$(f_r)_a = w\big((z^r)_a\big) = w\big((az)^r\big) = a^r f_r \quad (a \in T; r \in Z).$$

Hence

$$(f_r)_a(1) = f_r(a) = a^r f_r(1).$$

Thus, taking into account that $f_r \in Y_{M,p}$ and $z^r \notin Y_{M,p}$ for $r \notin M$, we infer that

(3.7) $$f_r = 0 \quad \text{for } r \notin M.$$

Clearly, (3.6), (3.7) and the linearity of $w$ imply that $wf = \tilde{P}_M f$ for $f$ in $R(T)$. Hence, by (2.6), $\|\tilde{P}_M f\|^p \leqslant n(|f|^p)$ for $f \in C(T)$ and for some measure $n$ on $T$. By (2.7) and the definition of the action of $T$ on $C(T)$ we infer that the measure $n$ is translation-invariant and therefore $n$ is a Haar measure on $T$ (in general $n$ is not the normalized Haar measure of $T$!). Thus $\tilde{P}_M$ is bounded in $L_p$ $(T)$ norm.

(3.3) $\Rightarrow$ (3.4). Since $R(T)$ is dense in $L_p(T)$ and $\tilde{P}_M(R(T)) \subset Y_{M,p}$, (3.3) implies that there is the unique extension of $\tilde{P}_M$ to an operator, say $P$, from $L_p(T)$ into $Y_{M,p}$. Clearly, $P$ is a projection ($=$ *a* bounded linear idempotent) from $L_p(T)$ onto $Y_{M,p}$.

(3.4) $\Rightarrow$ (3.2). Let $Q: L_p(T) \xrightarrow[\text{onto}]{} Y_{M,p}$ be a projection. We put $w = Qi_p$. Then clearly $u_{M,p} = wJ_M$ and $\|wf\|^p \leqslant \|Q\|^p n_1(|f|^p)$, where $n_1$ denotes the normalized Haar measure of $T$. This shows that $u_{M,p}$ is a $p$-integral operator and completes the proof of the Proposition.

In view of Proposition 3.2 and Lemma 3.1 the existence for every $p \geqslant 1$ of quasi $p$-integral operators which are not $p$-integral is a simple consequence of the following known fact (cf. [7], p. 36):

LEMMA 3.3. *Let $1 \leqslant p < +\infty$. If $\tilde{P}_M$ is bounded in $L_p(T)$ norm for every subset $M$ of $Z$, then $p = 2$.*

Proof. We consider first the case where $1 \leqslant p \leqslant 2$. By a standard "glinding hump" procedure we conclude that the assumption of the Lemma implies that there is $c > 0$ such that for every subset $M$ of $Z$

$$\|\tilde{P}_M f\|_p \leqslant c\|f\|_p \quad \text{for } f \in R(T).$$

Here $\|f\|_p = (n_1(|f|^p))^{1/p}$ denotes the $L_p(T)$ norm of $f$. Hence for $f = \sum_{r \in Z} a_r z^r \in R(T)$ and for every sequence $(s_r)$ such that $s_r = \pm 1$ for $r \in Z$ we have

$$\Big\|\sum_{r \in Z} a_r s_r z^r\Big\|_p \leqslant (1+2c)\|f\|_p.$$

Therefore (remembering that $1 \leqslant p \leqslant 2$), by a theorem of Orlicz [4],

$$\|f\|_2 = \Big(\sum_{r \in Z} |a_r|^2\Big)^{1/2} \leqslant (1+2c)b_p\|f\|_p \leqslant (1+2c)b_p\|f\|_2 \quad \text{for } f \in R(T), \tag{3.8}$$

where $b_p$ is a constant depending only on $p$. Thus on $R(T)$ the $L_p(T)$-norm and $L_2(T)$-norm are equivalent. Since $R(T)$ is dense in every $L_q(T)$ for $1 \leqslant q < +\infty$, (3.8) implies that $L_p(T)$ and $L_2(T)$ coincide as the classes of functions. Hence $p = 2$.

The proof in the case where $2 < p < +\infty$ reduces by a standard duality argument to the previous case. This completes the proof.



**4. The existence of quasi $p$-nuclear operators which are not $p$-nuclear.** We recall (cf. [5], [9]) that an operator $u: X \to Y$ is said to be *$p$-nuclear* if there exist a sequence $(x_q^*)$ of linear functionals on $X$ and a sequence $(y_q)$ of elements of $Y$ such that

$$ux = \sum_{q=1}^{\infty} x_q^*(x) y_q \quad \text{for } x \text{ in } X,$$

$$\Big(\sum_{q=1}^{\infty} \|x_q^*\|^p\Big)^{1/p} < +\infty \quad \text{and} \quad \sup_{\|y^*\| \leqslant 1} \Big(\sum_{q=1}^{\infty} |y^*(y_q)|^{p^*}\Big)^{1/p^*} < +\infty,$$

where $p^* = p(p-1)^{-1}$.

An operator $u: X \to Y$ is called *quasi $p$-nuclear* if there exist a Banach space $Z$ and an isometrically isomorphic embedding $I: Y \to Z$ such that the operator $Iu$ is $p$-nuclear.

Following [5] let the symbols $I_p(X, Y)$, $I_p^Q(X, Y)$, $N_p(X, Y)$, and $N_p^Q(X, Y)$ denote the Banach spaces (under the appropriate norms defined in [5]) of $p$-integral, quasi $p$-integral, $p$-nuclear and quasi $p$-nuclear operators from $X$ into $Y$, respectively. Let $1 < p < +\infty$. According to the Persson-Pietsch duality theory (cf. [5], Satz 52 and Satz 53) if

(4.1) $Y$ is a reflexive Banach space,

(4.2) $X$ and $Y$ have the metric approximation property (cf. e.g. [5] for the definition),

then there are natural isometric isomorphisms (onto!)

$$d: (N_p(Y, X))^* \to I_{p^*}^Q(X, Y),$$

$$d_Q: (N_p^Q(Y, X))^* \to I_{p^*}(X, Y),$$

where $p^* = p(p-1)^{-1}$ and $Z^*$ denotes the dual space of a Banach space $Z$. Furthermore if $j_N: N_p(Y, X) \to N_p^Q(Y, X)$ and $j_I: I_{p^*}(X, Y) \to I_{p^*}^Q(X, Y)$ denotes the natural embeddings, then

$$dj_N^* = j_I d_Q, \tag{4.3}$$

where $j_N^*$ denotes the adjoint operator of $j_N$. Since an operator $u$ is an isomorphism ($=$ a linear homeomorphism) if and only if the adjoint operator $u^*$ has the same property, (4.3) implies the following fact:

COROLLARY 4.1. *If the spaces $X$ and $Y$ satisfy conditions* (4.1) *and* (4.2), *then $N_p(Y, X) = N_p^Q(Y, X)$ (i.e. $j_N$ is an isomorphism) if and only if $I_{p^*}(X, Y) = I_{p^*}^Q(X, Y)$ (i.e. $j_I$ is an isomorphism).*

We shall apply this Corollary in the case where $X = X_M$ and $Y = Y_{M,p^*}$. Clearly, $Y_{M,p^*}$ is reflexive as a closed linear subspace of $L_{p^*}(T)$. To verify (4.2) observe that the Cesàro means form in the spaces

$X_M$ and $Y_{M,p^*}$ a sequence of finite-dimensional operators (each of norm one) which tends pointwise to the identity operators of the spaces.

Combining Corollary 4.1 with Proposition 3.2 we obtain

PROPOSITION 4.2. *If* $\tilde{P}_M$ *is unbounded in* $L_{p^*}(T)$*-norm, then* $N_p(Y_{M,p^*}, X_M) \neq N_p^Q(Y_{M,p^*}, X)$ $(1 < p < +\infty)$.

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# Multipliers and tensor products of $L^p$-spaces of locally compact groups*

by

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In an earlier paper concerned with induced representations [12] we introduced a definition of tensor product for Banach modules. Section 1 of the present paper contains general remarks on the relationship between multipliers of Banach modules and this definition of tensor product. In the following sections, motivated by theorems of Figà-Talamanca, Gaudry, Hörmander, and Eymard [5, 6, 10, 4] concerning multipliers of the $L^p$-spaces of locally compact groups, we give concrete representations as function spaces for the tensor products of these $L^p$-spaces, and we indicate how the theorems of the above-named authors can be reformulated in terms of these representations.

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**1. Multipliers and tensor products.** Let $A$ be a Banach algebra. By a *left (right) Banach $A$-module* we mean [12] a Banach space, $V$, which is a left (right) $A$-module in the algebraic sense, and for which

$$\|av\| \leqslant \|a\|\,\|v\| \quad \text{for all } a \in A \text{ and } v \in V.$$

If $V$ and $W$ are left (right) Banach $A$-modules, then $\mathrm{Hom}_A(V, W)$ will denote the Banach space of all continuous $A$-module homomorphisms from $V$ to $W$ with the operator norm. The elements of $\mathrm{Hom}_A(V, W)$ are traditionally called *multipliers* from $V$ to $W$. If $V$ is a left (right) Banach $A$-module, then $V^*$, the dual of $V$, is a right (left) Banach $A$-module under the adjoint action of $A$.

For completeness we include the definition of the tensor product of Banach modules which was introduced in [12]. Let $V$ and $W$ be respectively a left and right Banach $A$-module. Let $V \otimes_\gamma W$ denote the

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