[6] K. Honda, *A characteristic property of L_ϱ-spaces* ($\varrho \geqslant 1$), *III*, ibidem 39 (1963), p. 348-351.

[7] — *A characterization of the Orlicz space L_Φ^**. Proc. Acad. Amsterdam 67 (1964), p. 144-151 and 580-581.

[8] — *On smoothness of normed lattices*, Proc. Japan Acad. 42 (1966), p. 589-592.

[9] M. A. Krasnosel'skiĭ and Ya. B. Rutickiĭ, *Convex functions and Orlicz spaces*, Groningen 1961 (Translated from the Russian edition).

[10] W. A. J. Luxemburg, *Banach function spaces*, Thesis, Delft Institute of Techn., Assen 1955.

[11] H. Nakano, *Stetige lineare Funktionale auf dem teilweisegeordneten Modul*, Jour. Fac. Sci. Imp. Univ. Tokyo 4 (1942), p. 201-382.

[12] — *Modulared semi-ordered linear spaces*, Tokyo, Marzen 1950.

[13] — *Modern spectral theory*, Tokyo, Marzen 1950.

[14] S. Yamamuro, *On conjugate spaces of Nakano spaces*, Trans. Amer. Math. Soc. 90 (1959), p. 291-311.

[15] A. C. Zaanen, *Linear Analysis*, Amsterdam 1953.

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STUDIA MATHEMATICA, T. XXIX. (1968)

A norm satisfying the Bernstein condition

by

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In the research that recently culminated ([1], [3], [4]) in the proof that all separable infinite-dimensional Fréchet spaces are homeomorphic, one step (not however used in that proof) was the introduction into c_0 of a new norm, equivalent to the original norm but in addition satisfying the "Bernstein condition". Bessaga [2] gives a rather complicated construction and proof, communicated to him by Kadets. The purpose of the present note is to point out that the very simple norm

$$|||x||| = \|x\| + \sum_{i=1}^{\infty} a_i |x_i| \quad \text{for} \quad x = (x_i) \in c_0,$$

where $\sum a_i$ is any fixed convergent series of positive numbers, will serve the purpose equally well. In view of the inequalities

$$\|x\| \leqslant |||x||| \leqslant \Big(1 + \sum_{i=1}^{\infty} a_i\Big)\|x\|$$

it is obvious that $|||\cdot|||$ is an admissible norm, equivalent to $\|\cdot\|$, and it remains to be shown that it satisfies the Bernstein condition. Thus, we have to prove the following

THEOREM. *If* $x_i \geqslant 0$, $y_i \geqslant 0$ $(i = 1, 2, \ldots)$, $x_i \to 0$, $y_i \to 0$, *and*

$$\sup_{i \geqslant j} x_i + \sum_{i=j}^{\infty} a_i x_i = \sup_{i \geqslant j} y_i + \sum_{i=j}^{\infty} a_i y_i = \delta_j \quad (say) \tag{1}$$

for $j = 1, 2, \ldots$, *then* $x_i = y_i$ *for all* $i = 1, 2, \ldots$

Proof. Suppose not. If k is the first index for which $x_k \neq y_k$, and say $x_k > y_k$, then the inequality $x_i \geqslant y_i$ cannot hold for all $i \neq k$, otherwise (1) would fail for $j = 1$. Hence there exist indices m and n such that

$$1 \leqslant m < n, \; x_m > y_m, \; x_n < y_n, \text{ and } x_i = y_i \text{ for } m < i < n.$$

By considering the two cases $x_j \leqslant \sup_{i \geqslant j+1} x_i$ and the contrary, it is easy to see that

(2) $$\delta_j - \delta_{j+1} = \max\{a_j x_j,\ a_j x_j + (x_j - \sup_{i \geqslant j+1} x_i)\} \quad (j = 1, 2, \ldots),$$

and of course similarly for (y_i). Since, by (2), $a_m y_m < a_m x_m \leqslant \delta_m - \delta_{m+1}$, it follows with the help of (2) for (y_i) that

$$a_m y_m + (y_m - \sup_{i \geqslant m+1} y_i) = \delta_m - \delta_{m+1} \geqslant a_m x_m + (x_m - \sup_{i \geqslant m+1} x_i),$$

and consequently

(3) $$\sup_{i \geqslant m+1} x_i \geqslant (a_m + 1)(x_m - y_m) + \sup_{i \geqslant m+1} y_i > \sup_{i \geqslant m+1} y_i.$$

Hence for $m+1 \leqslant i \leqslant n$

$$x_i \leqslant y_i \leqslant \sup_{i \geqslant m+1} y_i < \sup_{i \geqslant m+1} x_i,$$

and therefore

(4) $$\sup_{i \geqslant n+1} x_i = \sup_{i \geqslant m+1} x_i > \sup_{i \geqslant m+1} y_i \geqslant \sup_{i \geqslant n+1} y_i.$$

On the other hand, in the same way that from $x_m > y_m$ we derived (3), from $x_n < y_n$ we can derive

$$\sup_{i \geqslant n+1} x_i < \sup_{i \geqslant n+1} y_i,$$

which contradicts (4). The proof is complete.

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References

[1] R. D. Anderson, *Hilbert space is homeomorphic to the countable infinite product of lines*, Bull. Amer. Math. Soc. 72 (1966), p. 515-519.

[2] C. Bessaga, *On topological classification of complete linear metric spaces*, Fund. Math. 55 (1965), p. 251-288.

[3] — and A. Pełczyński, *Some remarks on homeomorphisms of F-spaces*, Bull. Acad. Pol. Sci., Sér. Sci. Math., Astr. et Phys., 10 (1962), p. 265-270.

[4] M. I. Kadets, *Topological equivalence of all separable Banach spaces*, Doklady Akad. Nauk SSSR 167 (1966), p. 23-25 (Russian).

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On the probability measures in Hilbert spaces

by

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Introduction. In a topological semigroup we may consider integrals of the following kind. Let $g = g(t)$, $a \leqslant t \leqslant b$ be a mapping of the interval $[a, b]$ into G and $F(\Delta)$, $\Delta \subset [a, b]$ — a function of interval whose values are transformations of G into itself. If the limit of the sums

$$s(P) = \sum_i F(\Delta_i) g(t_i)$$

(where $P = (\Delta_1, \Delta_2, \ldots, \Delta_n)$ is a partition of $[a, b]$ and $t_i \in \Delta_i$) exists as P runs over a normal sequence of partitions, then this limit will be called an *integral of g with respect to F on the interval* $[a, b]$.

The aim of this paper is to study some properties of integrals of this kind, where G is a semigroup of probability measures in a Hilbert space (with the convolution as a semigroup operation and with the topology generated by the weak convergency of measures), while F is assumed to take the values from the space of linear bounded operators in a Hilbert space (and induces a transformation of the semi-group of distributions into itself). § 1 contains the basic definitions and facts of the theory of probability measures in a Hilbert space. In § 2 we define the convolution integral and prove its fundamental properties; § 3 contains some theorems illustrating the applications of the convolution integral (the continual analogous of classical theorems on the limit distributions of sums of independent random variables); finally in § 4, employing the notion of a convolution integral, we construct the Gaussian stochastic process with special properties.

§ 1

1.1. Let H be a real separable Hilbert space with the scalar product $(\cdot, \cdot)$ and with the norm $\|\cdot\|$. Denote by $\mathfrak{M}$ the set of all probability measures in H (i.e. the set of normed regular measures defined on a σ-field $\mathfrak{B}$ of