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STUDIA MATHEMATICA, SERIA SPECJALNA, Z. I (1963)

Interpolation in Hilbert spaces of analytic functions

by

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Let H be a separable Hilbert space and let x_n be a sequence of unit vectors that span H. We wish to know when the sequence x_n will have the following two properties.

(i) $\sum|(x, x_n)|^2 \leqslant M\|x\|^2$ for some constant M and all x in H.

(ii) For each sequence $\{c_n\}$ in l_2 there is an x in H with $(x, x_n) = c_n$ and $\|x\|^2 \leqslant m\sum|c_n|^2$, where m is a constant.

Let $a_{ij} = (x_i, x_j)$ and let A be the infinite matrix (a_{ij}). Then it is not difficult to prove the following lemma:

LEMMA. *Property* (i) *is equivalent to each of the following two statements.*

1. *Given any orthonomal basis e_n, there is a bounded operator $T: H \to H$ with $T(e_n) = x_n$ (all n).*

2. *The matrix A is a bounded transformation from l_2 to itself.*

Property (ii) *is equivalent to each of the following two statements.*

1. *Given any orthonormal basis e_n, there is a bounded operator $T: H \to H$ with $T(x_n) = e_n$ (all n).*

2. *The matrix A is bounded below on l_2.*

Suppose now that H is a Hilbert space of analytic functions in some domain D of the complex plane. For example:

I. H is the set of $f = \Sigma a_n z^n$ with $\Sigma|a_n|^2 < \infty$ (this is the Hardy space H_2).

II. H is the set of f with $\sum|a_n|^2/(n+1) < \infty$ (this is the Bergman space of f analytic in the unit circle for which $\iint|f|^2 dx\,dy < \infty$).

III. H is the set of f with $\sum|a_n|^2 n! < \infty$.

We assume that H has a *reproducing kernel*, that is, a function $K_\zeta(z)$ (z, ζ in D) such that $(f, K) = f(\zeta)$ for all f in H and ζ in D. In the three examples we have:

I. $K = 1/(1-\bar{\zeta}z)$.

II. $K = 1/(1-\bar{\zeta}z)^2$.

III. $K = \exp(\bar{\zeta}z)$.

Let z_n be a sequence of points in D, and form the unit vectors $x_n = K_n/\|K_n\|$ where $K_n = K_{z_n}(z)$. We now seek necessary and sufficient conditions on the z_n in order that x_n shall have properties (i) and (ii) above. In other words, we want: $\sum|f(z_n)|^2/K_n(z_n) < M\|f\|$ for all f in H, and, given (c_n) in l_2 there is f in H such that $f(z_n) = c_n K_n(z_n)$ and $\|f\| \leqslant m\sum|c_n|^2$.

For example, in H_2 we want the necessary and sufficient condition on z_n in order that the mapping T from H_2 to the space of all sequences of complex numbers given by

$$Tf = \{f(z_n)(1-|z_n|^2)^{1/2}\}$$

shall actually be a mapping of H_2 onto l_2.

THEOREM. *T maps H_2 onto l_2 if and only if there exist functions f_k in H_2 $(k = 1, 2, \ldots)$ of uniformly bounded norm such that*

$$Tf_k = \{\delta_{nk}\}_n,$$

where $\{\delta_{nk}\}_n$ is the sequence that is zero everywhere except at the k^{th} position where it is 1.

The condition may be restated as follows:

$$\prod_{n\neq k}\left|\frac{z_k-z_n}{1-\bar{z}_n z_k}\right| \geqslant \delta > 0 \qquad (k = 1, 2, \ldots).$$

This is also the necessary and sufficient condition that T_p maps H_p onto p $(1 \leqslant p \leqslant \infty)$ where T_p is defined by

$$T_p f = \{f(z_n)(1-|z_n|^2)^{1/p}\}.$$

In the case $p = \infty$ (interpolation of arbitrary bounded values by bounded analytic functions) the result was proved by L. Carleson, and his result could be used to prove the general case. Our method is different and yields a new proof of his results.

In examples II i III it is easy to give sufficient conditions for property (i), but so far we have not obtained necessary and sufficient conditions for both (i) and (ii).

Added in proof. The lemma is due to N. Bari (Uč. Zap. Mos. Gos. Univ. 148, Matem. 4 (1951), p. 69-107).

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Determinants in Banach spaces

by

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§ 1. Terminology and notation. X is a Banach space, Ξ is the dual of X. x, y, z denote elements of X, and ξ, η, ζ — elements of Ξ. ξx is the value of ξ at x. E is the Banach algebra of all bounded endomorphisms in X (called also *operators*), with the unit I. If $y = Ax$ is a bounded endomorphism in X, then $\eta = \xi A$ denotes the adjoint endomorphism in Ξ. Endomorphisms (operators) will be often interpreted as bilinear functionals $\xi Ax = \xi(Ax) = (\xi A)x$. For fixed x_0, ξ_0 the symbol $x_0\cdot\xi_0$ denotes the one-dimensional operator $Ax = x_0\cdot\xi_0 x$.

$T \in E$ is said to be *quasinuclear* if there exists a bounded linear functional F on E such that $\xi Tx = F(x\cdot\xi)$. Then T is denoted by T_F, and F is called *quasinucleus* of T. E. g. if x_0, ξ_0 are fixed, then $F(A) = \xi_0 A x_0$ $(A \in E)$ is a functional on E, denoted by $\xi_0\otimes x_0$, which is a quasinucleus of $x_i\cdot\xi_i$. All functionals in the closure of the set of all finite sums $\sum_{i=1}^{m}\xi_i\otimes x_i$ are called *nucleus*. If F is nucleus, then $T = T_F$ is called *nuclear* and F is the nucleus of T. For any quasinucleus F, $F(I)$ is called *trace* of F and denoted by $\mathrm{Tr} F$. The space Q of all quasinucleus is a Banach algebra with multiplication: $F_1\circ F_2(A) = \frac{1}{2}\big(F_1(T_{F_2}A)+F_2(AT_{F_1})\big)$. The *canonical mapping* $F \to T_F$ is a ring homomorphism. We write sometimes $F_{\xi x}(\xi Tx)$ instead of $F(T)$.

§ 2. The determinant system for an $A \in E$ is an infinite sequence

$$D_0,\ D_1\binom{\xi_1}{x_1},\ D_2\binom{\xi_1,\ \xi_2}{x_1,\ x_2},\ \ldots,\ D_n\binom{\xi_1,\ \ldots,\ \xi_n}{x_1,\ \ldots,\ x_n},\ \ldots$$

such that: (1) D_0 is a scalar; (2) for $n > 0$, $D_n\binom{\xi_1,\ \ldots,\ \xi_n}{x_1,\ \ldots,\ x_n}$ is a $2n$-linear functional on $\Xi^n\times X^n$, skew symmetric in variables $\xi_1, \ldots, \xi_n$ and skew symmetric in $x_1, \ldots, x_n$; (3) $D_n\binom{\xi_1,\ \ldots,\ \xi_n}{x_1,\ \ldots,\ x_n}$, interpreted as a function of ξ_1 and x_1 only is a bilinear functional on $\Xi\times X$ of the form $\xi_1 C x_1$ where $C \in E$; (4) for an integer r, D_r does not vanish identically (the smallest r