be replaced in Theorem 1 by a somewhat larger class of convex sets which includes, in particular, all those which are either *closed*, or *finite dimensional*, or possess an *interior point* [1].

Our second method of strengthening Theorem 1 is to weaken the requirement of convexity. Let us call a subset A of a Banach space *a-convex* $(0 \leqslant a < \frac{1}{2})$ if, whenever $S \subset A$ and x is in the convex hull of S, then $\varrho(x, S) \leqslant a$ (diam S). (Note that if A is closed, 0-convex = convex.) The sets which are a-convex for some $a < \frac{1}{2}$ are a strange lot; they include, for instance, all finite unions of intervals radiating from a point, but do not include a semi-circle in the plane (where the subset S consisting of the two end points causes trouble). Anyhow, Theorem 1 remains true if $\mathscr{C}(Y)$ is replaced by the family of all closed, a-convex subsets of Y, for some *fixed* $a < \frac{1}{2}$. (See [3].)

Finally, Theorem 1 may be strengthened by no longer requiring Y to be a Banach space, but only a complete metric space with an axiomatically defined *convex structure*, which permits one to take "convex combinations" of some (but not necessarily all) n-tuples in a suitably continuous fashion. (This includes the locally convex F-spaces mentioned earlier.)

With convex sets then defined in the obvious way, Theorem 1 remains true in this considerably more general context. Among the consequences of this new result, let us mention that Corollary 1 remains true if E is merely assumed to be a metrizable group, and F a closed subgroup which is isomorphic to a Banach space. (See [2]).

In conclusion, let us mention that, for *finite dimensional* X, it is possible to place purely topological conditions on the sets $\Phi(x)$ which are not only sufficient but, in a sense, also necessary. Without dimensional restrictions on X, however, the search for such a set of conditions has so far remained unsuccessful.

References

[1] E. Michael, *Continuous selections I*, Annals of Math. 63 (1956), p. 361-382.

[2] — *Convex structures and continuous selections*, Canadian J. Math. 11 (1959), p. 556-575.

[3] — *Paraconvex sets and continuous selections*, Math. Scand. 7 (1959), p. 372-376.

STUDIA MATHEMATICA, SERIA SPECJALNA, Z. I. (1963)

Operators and distributions

by

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I recall first the definition of operator which I have introduced in my book *Operational Calculus*.

We start from the ring of continuous functions of a real variable t $(0 \leqslant t < \infty)$, the product being defined by the convolution

$$fg = \int_0^t f(t-\tau)g(\tau)d\tau.$$

By a Titchmarsh theorem, that ring has no divisors of zero. The elements of the quotient field, obtained from that ring are by definition operators.

This algebraic method seems to be the most adequate for the notion of operators. However an equivalent definition can be given by the sequential method, as for distributions. Then instead of distributional convergence we should introduce the following one:

We say that a sequence of continuous functions $\varphi_n(x)$ is *convergent* or *fundamental* if, given any non-vanishing identically function ω, the sequence $\omega\varphi_n$ (convolution) converges almost uniformly.

Since this convergence is more general than the distributional one, the notion of operator is more general than that of distribution. But this is only so when the interval where the functions are defined is bounded from below. It if is not, distributions can be obtained from operators by an additional limiting process.

It is important to answer the question whether the algebraic (or sequential) method of introducing operators is essentially more general than that of the Laplace transform. In order to give a positive answer we should show that there exist operators or functions which can not be represented in the form

$$x = \frac{f}{g}$$

(the division being meant here as the inversion of the convolution), where f and g are Laplace transformable. In other words, we should find an L-transformable function g, non-vanishing identically, such that the convolution

$$f(t) = \int_0^t x(t-\tau)g(\tau)\,d\tau$$

cannot be transformable, no matter what is the continuous function $x(t)$. Last year I proved that $g(t) = e^{t^2}$ is such a function. Till now it is the only one function with this property known. L. Berg (Halle/Saale) has given recently a whole class of non-analytical functions with this property.

There exist several problems where the most natural approach is the distributional one. But there exist other problems where the operational calculus is necessary. J. Wloka (Heidelberg) showed that a conjunction of a distribution and of an operator may also be useful in some problems. Roughly speaking, this is a generalization of a function which can be considered as a distribution in one of the variables and as an operator in another variable. Another interpretation is also possible. This is a distribution whose values are operators. The notion of operator-valued distributions occurs in the recent theory of Schwartz, this being concerned with distributions whose values are in some vector spaces. It is always supposed that those spaces are topological locally convex spaces. Since the space of operators is not topological, the operator-valued distributions of Wloka do not enter into the theory of vector-valued distributions.

It is important to know whether there is an appropriate abstract space which makes possible a common consideration of all distributional spaces we meet in practice.

I say that a linear space with a given sequential topology is *partially normable*, if it is possible to consider it as a directed union of normed subspaces X such that, if $X_1 \subset X_2$, the identical mapping of X_1 onto X_2 is continuous. The space is complete if it can be considered as such a union of Banach spaces.

Although such abstract spaces are very general, and embrace, as particular cases, the spaces of distributions, the spaces of operators, the space of Mazur and Orlicz etc., it is easy to build in them an Analysis very similar to the classical Analysis. The proofs are very easy, since they are reduced, as a matter of fact, to proofs in Banach spaces.

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О теоремах вложения для классов дифференцируемых функций многих переменных

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Этот доклад посвящен некоторым вопросам, относящимся к теории которую принято называть в настоящее время теорией вложения классов дифференцируемых функций.

Результаты, о которых я буду здесь говорить, будут приводиться в самых простейших случаях. О возможностях распространения их на более сложные случаи мы будем делать только отдельные замечания, далеко не исчерпывающие вопрос.

Пусть R_n обозначает n-мерное пространство точек $\bar{x} = (x_1, \ldots, x_n)$ с действительными координатами. Будем считать всегда, что число p удовлетворяет неравенствам $1 \leqslant p \leqslant \infty$. Некоторые результаты, о которых будет идти речь ниже, будут верны только при $1 < p < \infty$.

Если $g \subset R_n$ область и $f(\bar{x})$ — измеримая на g функция, модуль которой интегрируем в p-й степени на g, то положим

$$\|f\|_{L_p(g)} = \Big(\int_g |f|^p dg\Big)^{1/p} \quad (1 \leqslant p \leqslant \infty).$$

Как обычно, при $p = \infty$ будем считать

$$\|f\|_{L_\infty(g)} = \sup_{\bar{x}\in g} \operatorname{vrai} |f(\bar{x})|.$$

Пусть r есть неотрицательное целое число. Говорят, что функция f *принадлежит к классу $W_p^{(r)}(g)$ Соболева*, если она интегрируема в p-й степени на g вместе со своими обобщенными производными до порядка r включительно. При этом вводится в рассмотрение норма функции f в метрике $W_p^{(r)}(g)$ при помощи равенства

$$\|f\|_{W_p^{(r)}(g)} = \|f\|_{L_p(g)} + \sum \|f^{(r)}\|_{L_p(g)},$$

где $f^{(r)}$ условно обозначает любую обобщенную производную от f порядка r и сумма распространена на все такие производные.

При $r = 0$ считается, что $W_p^{(0)}(g) = L_p(g)$ есть пространство функций, абсолютные величины которых интегрируемы в p-й степени на g.