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Topological structure of infinite-dimensional linear spaces: the classification problem*

by

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In studying a class T of topological spaces, it is natural to ask what topological possibilities are represented by the members of T. We call this the *classification problem* for T. A solution consists of exhibiting, in a more or less constructive way, a minimal class M of topological representatives for T — that is, a class M of topological spaces such that every member of T is homeomorphic with some member of M but no two members of M are homeomorphic. The problem is of fundamental importance and has been completely solved in a few significant cases (Seifert and Threlfall for the two-dimensional manifolds, de Groot for the countable closed subsets of E^n). But for most interesting choices of T, the problem is unsolved and appears to be exceedingly difficult. Even in these cases, consideration of the problem may be a good approach to the study of T, and partial results may illuminate the topological structure of individual members of T. Papakyriakopoulos has supplied an interesting survey of this development in the case of three-dimensional manifolds. In the present report, we consider the classification problem for certain classes of normed linear spaces and their convex subsets. Extant fragments of the theory originated largely in response to specific questions raised by various authors, and of course the answers have stimulated new questions. These questions, both old and new, play a prominent role in our exposition. It is especially fitting that this topic should be discussed at a conference in honor of Stefan Banach, since some of the problems mentioned in his path-breaking book of 1932 are still the basic unsolved problems in the area under discussion, and since Banach himself made some of the initial contributions to the area.

Our report consists mainly of a summary of known results and unsolved problems which are at least indirectly related to the classification problem for convex sets in normed linear spaces, together with an outline

* This is extracted from a long report which will soon appear; it cover also the methods of Bessaga and Pełczyński.

of three methods of attack (due to Kadeč, Keller, and Klee) which have proved at least partially successful for various parts of the problem. Since our report is itself a summary, it does not seem appropriate to summarize it here. Instead, we shall merely repeat some of the basic problems which appear still to be unsolved.

1) *Are all infinite-dimensional separable Banach spaces homeomorphic?*

This is the basic question raised by Banach, and earlier by Fréchet. Despite some recent very nice contributions by Kadeč, the problem remains unsolved.

2) *How many different topological possibilities are represented by the infinite-dimensional separable normed linear spaces?*

Assuming the general continuum hypothesis, we know that the answer is c or 2^c.

3) *How many different dimension types are represented by the infinite-dimensional separable normed linear spaces?*

We mean "topological dimension type" in the sense of Fréchet. We know only that the answer is between 3 and 2^c.

4) *Can an $\aleph_0$-dimensional normed linear space be homeomorphic with one of dimension $>\aleph_0$?*

If the answer is negative, then any two normed linear spaces must have the same algebraic dimension if they are homeomorphic.

5) *Are all Banach spaces which have density character c of the same dimension type?* (1)

(We conjecture that the answer is negative, even though it is known that all infinite-dimensional separable Banach spaces are of the same dimension type.)

6) *Must an $\aleph_0$-dimensional normed linear space be homeomorphic with its unit cell?* (2)

For most infinite-dimensional Banach spaces, the answer is affirmative.

7) *How many topological possibilities are represented by the closed convex subsets of Hilbert space which are not locally compact?*

Those which are locally compact have been completely classified. Those which have an interior point are known to be homeomorphic with the entire Hilbert space.

(1) Recent progress on these questions has been made by C. Bessaga and A. Pełczyński, *Some remarks on homeomorphisms of Banach spaces*, Bull. Acad. Pol. Sci., Série math., astr. et phys., 8 (1960), p. 757-761, and *Some remarks on homeomorphisms of F-spaces*, ibidem 10 (1962), p. 265-270.

(2) It is now known that every $\aleph_0$-dimensional normed linear space is homeomorphic with all its closed convex bodies. See H. Corson and V. Klee, *Topological classification of convex sets*, Proc. Symp. Pure Math. (Convexity) 7, Amer. Math. Soc., to appear.

8) *If J is a Hilbert manifold, must J be homeomorphic with $H\times M$ for some finite-dimensional manifold M?* (*H is Hilbert space.*)

9) *For what pairs of finite-dimensional manifolds M and N will the Hilbert manifolds $H\times M$ and $H\times N$ be homeomorphic?* (3)

It may be that the first of these questions has an obvious negative answer, and that the second question is hopelessly difficult. However, there seems to be some evidence that the classification problem for Hilbert manifolds may be much easier than that for finite-dimensional manifolds.

10) *For what spaces X is the product space $H\times X$ homeomorphic with H?*

It is easily seen that X must be a separable metric absolute retract. The simplest spaces X for which the problem is unsolved are the triod and the Hilbert cube. (4)

(3) This has been answered negatively by K. Borsuk, *On a problem of V. Klee concerning the Hilbert manifolds*, Coll. Math. 8 (1961), p. 239-242.

(4) For the Hilbert cube, an affirmative answer is given by Bessaga and Pełczyński in the second paper cited above.