defines a topological isomorphism of $\hat{W}'/(P_1, \ldots, P_r)\hat{W}'$ *onto the direct sum of the spaces* $\hat{W}'(V_k)$ *where* $\hat{W}'(V_k)$ *is the space of all entire functions* G *on* V_k *which satisfy*

$$G(z) = O(a(z)), \quad z \in V_k,\ a \in A,$$

and the topology of $\hat{W}'(V_k)$ *is defined by the sets*

$$N_a = \{G \in \hat{W}'(V_k) : |G(z)| \leqslant a(z) \text{ for all } z \in V_k\}.$$

From the fundamental principle it follows that each $f \in W(D_1, \ldots, D_r)$ can be represented as a sum of integrals of exponential polynomials in $W(D_1, \ldots, D_r)$. In addition, we can give a complete treatment of questions of hypoellipticity, hyperbolicity, uniqueness (a la Taeklind), existence for a Cauchy-like problem, for the space $W(D_1, \ldots, D_r)$.

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Tensorial measures and homology on a compact differentiable manifold*

by

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Let $V^{(r)}$ be a differentiable manifold of the dimension r. The differential structure on $V^{(r)}$ be of class C^1_L (this means that transformations with Lipschitz-continuous first derivatives change the local systems of admissible coordinates). Let ε be an admissible map of $V^{(r)}$ (r-cell of $V^{(r)}$, with a local system of admissible coordinates on it). An object μ called *tensorial measure* is introduced. It is defined by 1°) a set of real valued measure functions $\mu^{i_1\ldots i_n}_{j_1\ldots j_k}(B)$ $(i_1, \ldots, i_n = 1, \ldots, r; j_1 \ldots j_k = 1, \ldots, r)$ (components of μ in ε) defined on the reduced σ-ring of all the Borel set contained with their closures in the support of ε; 2°) the following transformation rule for two maps ε and $\bar{\varepsilon}$ with overlapping supports:

$$\mu^{i_1\ldots i_n}_{j_1\ldots j_k}(B) = \int_B \frac{\bar{A}^m}{|\bar{A}|^{p+m}} a^{s_1}_{j_1} \ldots a^{s_k}_{j_k} \bar{a}^{i_1}_{h_1} \ldots \bar{a}^{i_n}_{h_n} d\bar{\mu}^{h_1\ldots h_n}_{s_1\ldots s_k}$$

($m = 0, 1$, p = real number, $a^i_j = \partial\bar{x}^i/\partial x^j$, $\bar{a}^i_j = \partial x^i/\partial\bar{x}^j$, $A = \det\{a^i_j\}$, $\bar{A} = \det\{\bar{a}^i_j\}$, x^i local coordinates in ε, $\bar{x}^i$ local coordinates in $\bar{\varepsilon}$). n is the first rank of μ, k the second rank; μ is called of the *first* (*second*) *kind* if $m = 0$ ($m = 1$), p is the weight of μ. Every tensorial measure is uniquely decomposed as a sum of an absolutely continuous tensorial measure μ_0 (every component of μ_0 in ε is absolutely continuous with respect to the measure $x(\tau B)$ = Lebesgue measure of the image of B in the unit sphere of the Euclidean space by the homeomorphism τ that introduces on ε the coordinate system) and a singular tensorial measure $\mathring{\mu}$ (every component of $\mathring{\mu}$ in ε is singular with respect to $x(\tau B)$). The linear space $\mathfrak{M}_0$ of the abs. cont. tens. measure (for given n, k, m) is isomorphic to the space of tensors f with locally integrable components (respect to x) of first rank n, second rank k, of the kind m, and weight $p+1$. This isomorphism is denoted by $f \leftrightarrow \int f \equiv \mu_0$.

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Let $V^{(r)}$ be compact and orientable. The particular tensorial measures obtained by assuming $n = 0$, $p = -1$, are denoted k-measures. In this case $\mathfrak{M}_0$ is isomorphic to the space of the differentiable k-forms on $V^{(r)}$ with locally integrable coefficients. On the other hand, the linear space $\overset{*}{\mathfrak{M}}$ of the singular k-measures contains a subspace that is isomorphic to the linear space of the $(r-k)$-chains on $V^{(r)}$ with real coefficients. Let t be such an isomorphism. An operation d of weak differentiation is introduced for the k-measures. The following identity hold $dt = t\beta$ $(\beta = (-1)^k\partial)$.

The space $H_k^{(m)}$ of m-homology is the quotient space of the closed k-measure (k-measure with a vanishing differential) modulo the space of the k-measure which are homologous to zero (i. e. k-measures that are the differential of $(k-1)$-measures). The space $H_n^{(f)}$ of f-homology is the quotient space of the closed regular k-form (k-form with C_L^0 coefficients and vanishing differential) modulo the space of regular and homologous to zero k-forms. The space $H_k^{(c)}$ of c-homology is the quotient space of the $(r-k)$-cicles modulo the space of the bounding $(r-k)$-cicles. The isomorphism $\int$ induces an homomorphism of $H_k^{(f)}$ into $H_k^{(m)}$. This is called the imbedding of $H_k^{(f)}$ in $H_k^{(m)}$. Analogously — by using t — the imbedding of $H_k^{(c)}$ in $H_k^{(m)}$ is defined. The two main theorems are: I) The imbedding of $H_k^{(f)}$ in $H_k^{(m)}$ is an isomorphism of $H_k^{(f)}$ onto $H_k^{(m)}$. II) The imbedding of $H_k^{(c)}$ in $H_k^{(m)}$ is an isomorphism of $H_k^{(c)}$ onto $H_k^{(m)}$. The de Rham theorems are easily derived from I) and II). When a metric is introduced on $V^{(r)}$ and harmonic forms defined, the following theorem (Hodge) is proved: III) Each m-homology class of $H_k^{(m)}$ contains one and only one harmonic form.

The main analytical tool in proving theorems I) and II) is an inequality for regular differential forms. Let C_k be the Banach space of continuous k-form with a C norm, C_k^0 be the quotient Banach space of C_k modulo the closure of the manifold of the closed regular k-forms. The main inequality is the following that holds for every regular k-form: $\|[v]\|_{C_k^0} \leqslant K_p \|dv\|_{L_k^p}$; $[v]$ denotes an equivalence class of C_k^0, K_p is a constant depending on p, p any real number greater than r. The norms are respectively taken in C_k^0 and in L_k^p (Banach space of k-forms with locally L^p-integrable coefficients).

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R-fastperiodische Funktionen *

von

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Bekanntlich ist die reelle Achse D stetig isomorph einer dichten Untergruppe der als Bohrsches Kompaktum bezeichneten kompakten Gruppe K mit folgenden Eigenschaften: jede (im Sinne von Bohr) fastperiodische (fp.) Funktion einer reellen Variablen läßt sich zu einer stetigen Funktion auf K erweitern und umgekehrt, jede auf K stetige Funktion ist auf D fastperiodisch. Beschränkt man sich auf fp. Funktionen, deren Fourierexponenten zu einer abzählbaren additiven Gruppe Λ von reellen Zahlen gehören, so kann man eine metrische kompakte Gruppe K_Λ (Untergruppe des $\aleph_0$-dimensionalen Torusses) konstruieren, welche diesen Funktionen gegenüber dieselbe Rolle spielt, wie K gegenüber der Gesamtheit aller fp. Funktionen. Man kann dann jeder auf K_Λ nach dem invarianten Maße μ integrierbaren Funktion eine Besicovitchsche fp. Funktion (B-Funktion) auf D mit Exponenten aus Λ so zuordnen, daß die Fourierreihe erhalten bleibt, wenn man den Koordinaten x_j der Punkte aus K_Λ die Charaktere $e^{i\lambda_j t}$ $(\lambda_j \epsilon \Lambda)$ von D entsprechen läßt. Um diese Korrespondenz eineindeutig zu machen, muß man einzelnen Klassen von μ-äquivalenten Funktionen auf K_Λ (μ-Klassen) volle Klassen B-äquivalenter Funktionen auf D (B-Klassen) zuordnen, wobei zwei Funktionen f und g *B-äquivalent* heißen, wenn

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |f(t) - g(t)|\, dt = 0$$

gilt.

Die nach Riemann integrierbaren (d. h. μ-fast überall stetigen) Funktionen auf K_Λ bilden einen linearen Unterraum (Unterring) der Gruppenalgebra $L(K_\Lambda)$. Es liegt die Frage nahe, wie die entsprechenden B-Funktionen beschaffen sind. Dazu werde folgender Begriff einer R-fp. Funktion eingeführt:

Definition. Eine B-Funktion f ist *R-fastperiodisch*, wenn es für jedes $\varepsilon > 0$ zwei Bohrsche fp. Funktionen φ_1 und φ_2 gibt, so daß

* Für ausführliche Darstellung siehe Verfassers die Arbeit des *Über Niveaulinien fastperiodischer Funktionen*, Studia Math. 20 (1961), S. 313-325.