в) Краевые задачи типа Дирихле для уравнений второго порядка с постоянными коэффициентами. В частности, задача типа Дирихле для уравнения колебания струны.

2. *Построение разложений по обобщенным собственным векторам самосопряженного оператора в гильбертовом пространстве* H_0. Доказывается, что при одном дополнительном ограничении на H_+ у такого оператора существует полная система обобщенных собственных векторов из H_-. Более детально исследуются следующие вопросы:

а) Изучается характер разложений для дифференциальных операторов в конечной и бесконечной областях. Исследуется поведение на ∞ собственных функций.

б) Изучаются вопросы, связанные с обобщением n-мерной теоремы Бохнера о положительно определенных функциях на случай разложений по собственным функциям дифференциальных операторов.

3. *Исследование спектральных свойств некоторых классов несамосопряженных операторов.* Предварительно доказывается, что аналитическая функция в верхней полуплоскости с максимум степенным ростом при приближении к вещественной оси имеет предельные значения на этой оси, являющиеся обобщенными функциями из некоторого H_-. Этот результат применяется к построению спектральных разложений вообще говоря неограниченного оператора в гильбертовом пространстве, имеющего чисто вещественный спектр и резольвента которого вблизи вещественной оси удовлетворяет оценке $\|R_z\| \leqslant C/|\operatorname{im} z|^k$ $(k \geqslant 0)$.

STUDIA MATHEMATICA, SERIA SPECJALNA, Z. I. (1963)

Approximative dimension of linear topological spaces and some of its applications

(Summary of a report)

by

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Certain classes of linear topological spaces introduced by French mathematicians seem to be more similar to finite dimensional spaces than the Banach spaces are. As a measure of this similarity one may consider the so-called *approximative dimension*.

I. Let X be a linear space. A and B — two subsets of X. Put

$$M(A, B, \varepsilon) = \sup\{n\colon \text{there exist } x_1, \ldots, x_n \in A \text{ such that } x_i - x_k \in B \text{ for } i \neq k\},$$

$$M(A, B) = \{\varphi\colon \lim_{\varepsilon\to 0}|\varphi(\varepsilon)| = \infty;\ \lim_{\varepsilon\to 0}\varphi(\varepsilon)/M(A, B, \varepsilon) = 0\},$$

where $\varphi(\varepsilon)$ are real functions defined for $\varepsilon > 0$. The quantity $M(A, B, \varepsilon)$ is called *ε-capacity* of the set A with respect to B.

Consider X with a fixed locally convex topology $\sigma\colon X = \langle X, \sigma\rangle$. Let $\mathcal{O}$ be the class of all convex centrally symmetric neighbourhoods of zero, $\mathcal{T}$ — the class of all convex bounded sets with the centre of symmetry in zero.

Definition. The family of functions

$$M(X) = \bigcup_{U\in\mathcal{O}}\bigcap_{V\in\mathcal{O}} M(V, U)$$

is called *approximative dimension* (a. d.) of the space X (see Kolmogorov [7]).

It is easy to verify that the a. d. of n-dimensional spaces is equal to $\{\varphi\colon \lim_{\varepsilon\to 0}|\varphi(\varepsilon)| = \infty, \lim_{\varepsilon\to 0}\varphi(\varepsilon)\cdot\varepsilon^n = 0\}$; the a. d. of the infinite dimensional Banach spaces is trivial, i. e. it consists of all functions φ for which $\lim_{\varepsilon\to 0}|\varphi(\varepsilon)| = \infty$.

The approximative dimension is an isomorphical invariant: moreover if Y is isomorphic to a subspace of X then $M(Y) \subset M(X)$.

Similarly two other invariants may be considered as d. a.:

$$M'(X) = \bigcup_{B\in\mathcal{T}} \bigcup_{U\in\mathcal{O}} M(B, U), \qquad M''(X) = \bigcup_{A\in\mathcal{T}} \bigcap_{B\in\mathcal{T}} M(A, B).$$

There exists another approach: instead of ε-capacity one takes as a starting-point the quantity $\delta(A, B, n)$ — the so-called *n-th diameter* of A with respect to B $\delta(A, B, n) = \inf_L \sup_{x\in A} \varrho_B(x, L)$, where L runs over all n-dimensional linear sets in X, $\varrho_B(x, L)$ is the distance between the point x and the set L "induced by B"), and we define the classes of sequences (A, B) and (X) in similar way as $M(A, B)$ and $M(X)$. This approach is especially convenient in the case of Köthe spaces.

II. Case where X is a space of type F (B_0-space) or DF.

THEOREM 1. *The a. d. of X is not trivial if and only if X is a Schwartz space* (see [17]).

THEOREM 2. $M(X) = M'(X) = M''(X)$.

THEOREM 3. *X is nuclear if and only if* $M(X) \subset \{\varphi\colon \lim_{\varepsilon\to 0}\varphi(\varepsilon)/e^{1/\varepsilon} = 0\}$.

THEOREM 4. *If X is a countable Hilbert space (i. e. the topology of X can be given by a sequence of scalar products), then $M(X) = M(X^*)$.*

III. Application.

1. THEOREM 4 (Dynin and Mitiagin [3], Dynin [21]) *Each basis in an arbitrary nuclear F-space is an unconditional basis. It follows that every nuclear F-space with a basis is a Köthe space.*

2. Application to the classification of these spaces: proofs that concrete spaces are not isomorphic.

3. Example of an infinitely dimensional F-space not isomorphic with its maximal subspace.

4. Application to the investigation of universal spaces (Pełczyński [16], Bessaga and Pełczyński [1]).

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