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Corrigenda to my paper "A stochastic dam process with non-homogeneous Poisson inputs"

(Studia Mathematica 21 (1962), p. 307-315)

by

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Professor D. G. Kendall has pointed out to me that there is a gap in the proof of formula (3.2) for the probability of emptiness on page 311. The equivalent formula (3.1) for the discrete case has previously been proved for inputs whose arrival times form a homogeneous Poisson process in [2] (J. Gani and N. U. Prabhu, *The time-dependent solution for a storage model with Poisson input*, J. Math. and Mechanics 8 (1959), p. 653-663). In the present case with a non-homogeneous Poisson process, the method of proof is identical.

The result for the continuous input case may be derived from this either by a limiting method or by an argument on the following lines. Starting with a content $u \geqslant 0$ at time τ, emptiness may occur at time $\tau+t$ $(t \geqslant u)$ as a result of total inputs of size 0, or lying between $(0, \delta x)$ $\ldots, (x-\delta x, x), \ldots, (t-u-\delta x, t-u)$, these forming mutually exclusive events. The probability of emptiness due to the input 0 is clearly $\exp[-\{\varrho(\tau+t)-\varrho(\tau)\}]$. For the probability of emptiness at time $\tau+t$ related to a total input $(x-\delta x, x)$, we note that to each possible path of the actual realisation of the process, there corresponds a unique path with identical arrival times and sizes of inputs starting from the content $t-x$ and terminating in the interval $\tau+t-\delta x$, $\tau+t$. Thus the probability of this event is $g(t-x; \tau, \tau+t)\,\delta x$; summing over all possible inputs, we obtain

$$F(u, 0; \tau, \tau+t) = e^{-\{\varrho(\tau+t)-\varrho(\tau)\}} + \int_0^{t-u} g(t-x; \tau, \tau+t)\,dx,$$

which, on writing $v = t-x$, gives (3.2).

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