Proof. We have
1. \(|f(x)| \leq |x|\) for every \(x \in X, f \neq 0\), whence \(\|f(x)\| = \frac{1}{\|x\|} \|f(x)\| < \|f(x)\|\), and \(\max |f(x)| < \lim \|f(x)\|\).
2. Let \(F\) be a linear functional defined on \(H\); then \(g_F(e) = f((a - ae)^{-1})\) is a holomorphic function defined for \(|a| < 1/a\), where \(a = \max |f(x)|\), and \(x\) is a fixed element of \(X\). It may easily be seen that \(g_F(x) = \sum_{n=0}^{\infty} F(a^n x)\). Consequently the sequence \(\sum_{n=0}^{\infty} \|F(a^n x)\|\) is bounded, being weakly convergent to 0. We have \(\|F(a^n x)\| < M_0\), and \(\|F(x)\| < \frac{M_0}{|a|}\). Consequently \(\lim \|F(x)\| < 1/|a|\) for every \(|a| < 1/a\), and \(\lim \|F(x)\| < a\).

By 1 and 2 we have \(a = \lim \|F(x)\|\), q. e. d.

References

On Banach \(*\)-semialgebras
by S. BOURNE (Berkeley)

1. Preliminaries

We shall use the term halfring in the sense given by the author in a previous paper [3]. We repeat that a halfring is a semiring which is embeddable in a ring. Since addition in our semiring \(S\) is commutative, a necessary and sufficient condition for \(S\) to be a halfring is that the additive semigroup of \(S\) be cancellative. Following [3], we construct the ring in which \(H\) is embedded. The product set \(H \times H\) again forms a halfring according to the laws of addition and multiplication: \((e_1, e_2) + (f_1, f_2) = (e_1 + f_1, e_2 + f_2), (e_1, e_2) \cdot (f_1, f_2) = (e_1 f_1 + e_2 f_2, e_1 f_2 + e_2 f_1)\). The diagonal \(D = (e, e) \in H \times H\) is an ideal in \(H \times H\). We say that \((e_1, e_2) = (f_1, f_2)(A)\) if and only if there exist elements \((x, y)\) in \(D\) such that \((e_1, e_2) + (x, y) = (f_1, f_2) + (y, y)\). The quotient ring \(N = H \times H/D\) is called the ring generated by \(H\). Let \(\nu\) denote the natural homomorphism of \(H \times H\) onto \(N\), then the halfring \(H\) is embedded in the ring \(N\), for the mapping \(h \mapsto \nu(h + a, a)\), for any \(a\), is an isomorphism of \(H\) onto \(N\). We designate by \(\nu(H)\) this isomorphic map of \(H\) in \(N\) and by \(\nu(x, y)\) the equivalence class of \((x, y)\). A division semiring is a semiring, in which the elements \(\neq 0\), form a multiplicative group. A semifield is a commutative division semiring. A semifield is a semifield which is embeddable in a field.

In a recent paper [4], we introduced the concept of a \(\textit{normed semialgebra}\). For the sake of completeness we repeat:

Definition 1. A semiring \(S\) is said to be a \(\textit{semialgebra}\) over a commutative semiring \(\Sigma\) with unit, if a law of composition \((s, e) = es\) of the product \(\Sigma \times S\) is defined such that:

(i) \((S, +)\) is a unital left \(\Sigma\)-semimodule relative to the composition \((s, e) = es\),
(ii) for all \(s \in \Sigma\) and \(s, t \in S\), \(s(t,t) = (st)t = s(ut)\).

Definition 2. A \(\textit{semivector space}\) is a semialgebra over a semifield.

Definition 3. A \(\textit{metric}\) for a semifinite space \(S\) is said to be invariant if and only if \(d(s + x, t + x) = d(x, y)\) for all \(s, t, x \in \Sigma\).
Definition 4. A norm for a semilinear space $S$, over the halffield of nonnegative reals $\mathbb{R}^+$, is a nonnegative real-valued function $\|\cdot\|$ satisfying for $s, t \in S$ and $r \in \mathbb{R}^+$

(i) $\|s\| \geq 0$,
(ii) $\|s\| = 0$ if and only if $s = 0$,
(iii) $\|rs\| = |r| \|s\|$, 
(iv) $\|s + t\| \leq \|s\| + \|t\|$.

Definition 5. A set $S$ of elements $s, t, \ldots$ is a normed semiring if and only if

1. $S$ is a semialgebra over the halffield of nonnegative reals $\mathbb{R}^+$,
2. $S$ is a semilinear space with an invariant metric $d(s, t)$,
3. $\|s\| = d(s, 0)$ is a norm for the space $S$ and $\|st\| \leq \|s\| \|t\|$ for $s, t \in S$,
4. If $S$ contains a unit $1$, then $\|e\| = 1$.

Definition 6. A Banach semiring is a complete normed semiring.

If in definition 6, the semiring is a halfring $H$, then we refer to it as a Banach halfring.

Lemma 1. If $H$ is a Banach halfring, then the halfring $H \times H$ is a Banach halfring over $\mathbb{R}$ with invariant metric $D((s_1, s_2), (t_1, t_2)) = d(s_1, t_1) + d(s_2, t_2)$ and $\|(s_1, s_2)\| = \|s_1\| + \|s_2\|$.

Proof. Immediate verification.

The ideal $I$ in $H \times H$ is a closed set in the product topology [5].

Lemma 2. The ring $\mathbb{R}$ generated by the Banach halfring $H$ is a Banach ring over the real field $\mathbb{R}$ with norm

$$\|\cdot\| = \inf_{(x, s) \in I} \|s\|.$$

Proof. In [4], we showed that $\mathbb{R}$ is a normed ring over the real field $\mathbb{R}$. There remains to be proven that the completeness of $H$ implies the completeness of $\mathbb{R}$.

Kelley [8] proved the following theorem: Let $f$ be a continuous uniformly open map of a complete pseudo-metrizable space into a Hausdorff uniform space. Then the range of the map $f$ is complete. Now a map of a uniform space $(X, \mathcal{U})$ into a uniform space $(Y, \mathcal{V})$ is uniformly open if for each $U \in \mathcal{U}$ there is $V \in \mathcal{V}$ such that $f[U \setminus \{x\}] \subseteq V[f(\{x\})]$ for each $x \in X$ [8].

In [4], we showed that the continuous homomorphism $\varphi$ of the topological halfring $H \times H$ onto the topological ring $\mathbb{R}$ is an open mapping. We proceed to show that $\varphi$ is uniformly open. We recall that $H \times H$ has the invariant metric $D((s_1, s_2), (t_1, t_2)) = d(s_1, t_1) + d(s_2, t_2)$ while $\mathbb{R}$ the invariant metric $D((s_1, s_2), (t_1, t_2)) = |s_1 - t_1| + |s_2 - t_2|$. Hence, a uniformity for $H \times H$ is defined by neighborhoods $U_{(s_1, s_2)} = \{x \in \mathbb{R} | D((x, y), (s_1, s_2)) < \varepsilon\}$ and a uniformity for $\mathbb{R}$ by neighborhoods $V_{(t_1, t_2)} = \{x \in \mathbb{R} | D((x, y), (t_1, t_2)) < \varepsilon\}$. Let $U_{(0, 0)} = \{(s, t) | (s, t) < \varepsilon\}$, then $s_1, s_2 + U_{(0, 0)} = (s_1 + s_2) + (s_1 + s_2) < \varepsilon$).

Now $D((s_1 + s_2 + \varepsilon, s_2), (s_1, s_2)) = d((s_1 + s_2, s_1) + d(s_2, s_2)) = (s_1 + s_2, s_1) < \varepsilon$. Therefore, $U_{(s_1, s_2)} \supset U_{(0, 0)}$. Since $\varphi$ is open, then $\varphi(U_{(0, 0)}) \supset V_{(s_1, s_2)}$. Hence, $\varphi(U_{(s_1, s_2)}) \supset V_{(s_1, s_2)}$ for $\mathbb{R}$ is a topological ring [4].

Hence, for any $e > 0$ there exists a $\delta > 0$ such that for each $U_{(s_1, s_2)}$ there is a $V_{\varepsilon} = V_{(s_1, s_2)}$ for which $\varphi(U_{(s_1, s_2)}) \supset V_{\varepsilon}$ for each $(e_1, e_2)$ in $H \times H$. $\mathbb{R}$ is uniformly open and this implies that $\mathbb{R}$ is a complete normed ring.

As a consequence of theorems 1 and 2, we have

Theorem 1. A Banach halfring $H$ over the nonnegative reals $\mathbb{R}^+$ is embeddable in the Banach ring $\mathbb{R}$ over the reals $\mathbb{R}$.

2. Introduction

Gelfand and Naimark [6] proved the following structure theorem:

Suppose $\mathbb{R}$ is a commutative Banach ring with identity and that an involution is defined in $\mathbb{R}$ satisfying the usual algebraic conditions $(x^*)^* = x$,

$$x^* = x, \quad (xy)^* = y^*x^*,$$

and, furthermore, the condition $x^* = |x|^*|x|$. Then the ring $\mathbb{R}$ is completely isomorphic to the ring $C(\mathbb{R})$ of all continuous functions $a(M)$ in the space $\mathbb{R}$ of maximal ideals of the ring $\mathbb{R}$.

Arens [1] extended this theorem to read: Let $A$ be a commutative Banach $*$-algebra satisfying $\|f\|_0 \leq \|f\|$ but having no unit. Then there exists a locally compact Hausdorff space $X$ such that $A$ is the class of all continuous complex-valued functions on $X$ vanishing at infinity. Also $f(a) = f(a)$.

Arens and Kaplansky [2] then obtained a theorem of this type for a real Banach $*$-algebra where there is defined an operation $\ast$ which satisfies $(f \ast g) = \ast f \ast g$, $(fg)^* = f^*g^* = f^*g$. Let $A$ be a commutative real Banach $*$-algebra, with [without] unit, satisfying $\|f\|_0 \leq \|f\|_0 \ast g_0 \leq \varepsilon \|f\|_0 \ast g_0 \ast f$, for $f$ and $g$ in $A$, $k$ constant. Then there exists a locally compact Hausdorff space $X$ having an involutory homomorphism $\ast$ such that $A$ is isomorphic to the ring of all continuous complex-valued functions on $X$ (vanishing at infinity) which satisfy $f(a)(x) = f(x)$ for $a \in X$. Furthermore, if $\|f\|_0 \leq \|f\|_0$ in $A$, then $\|f\|_0 = \sup |f(x)|$. 

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In section 3 we introduce the concept of a normed semi-algebra with an involution and show that it is embeddable in a real Banach *-algebra. In section 4 we extend the Arens-Kaplansky theorem [2] to read that if H is a Banach halfring with an involution, such that 
\[ \|a, b\|_2 \leq \|a, a\|_2 \leq \|a, b\|_2 + \|a, b\|_2 \] 
for all \( (a, a) \) and \( (a, b) \) in \( H \times H \), where \( k \) is a constant. Then there exists a compact space \( \mathcal{M}_H \) with an involutory homeomorphism \( \sigma \) such that \( H \) is *-isomorphic to the halfring \( C^*(\mathcal{M}_H) \) of all continuous complex-valued functions on \( \mathcal{M}_H \), for which \( f((M^*)^*) = f(M)^* \).

5. Banach halfrings with involution

**Definition 7.** An involution in a normed semi-algebra \( S \) is a mapping \( s \rightarrow s^* \) of \( S \) onto itself satisfying for \( s, t \in S \) and \( \phi \in \mathbb{R}^+ \):

1. \( (s^*)^* = s \),
2. \( (s + t)^* = s^* + t^* \),
3. \( (\phi s)^* = \phi s^* \),
4. \( (s^*)^* = s^* \).

**Lemma 3.** If \( H \) is a normed halfring with an involution \( s \rightarrow s^* \), then \( H \times H \) is a normed halfring with involution \( (s_1, s_2) \rightarrow (s_2^*, s_1^*) \).

**Proof.** Immediate verification.

If \( s_1, s_2 \in H \) and \( \phi = (u, v) \), then \( (s_1, s_2) + (s_1, s_2) = (s_1, s_2) + (s_2, s_2) = (s_2^*, s_2^*) + (s_1^*, s_1^*) = (s_2^*, s_2^*) + (s_1^*, s_1^*). \) This implies that \( (s_1, s_2) \rightarrow (s_2^*, s_1^*) \) defines an involution in the ring \( H \) and

**Lemma 4.** The ring \( \mathcal{R} \) generated by the normed halfring \( H \) with involution \( s \rightarrow s^* \) is a normed ring over the real field \( \mathbb{R} \) with involution \( (s_1, s_2) \rightarrow (s_2^*, s_1^*) \).

As a consequence of Lemma 4 and Theorem 1, we have

**Theorem 2.** A Banach halfring \( H \) over the non-negative reals \( \mathbb{R}^+ \) with involution \( s \rightarrow s^* \) is embeddable in the Banach ring \( \mathcal{R} \) over the reals \( \mathbb{R} \) with involution \( (s_1, s_2) \rightarrow (s_2^*, s_1^*) \).

We assume that \( H \) is commutative and possesses a unit \( e \). Following Słodkowski and Zawadowski [12], we state

**Definition 8.** A commutative semi-ring \( H \) is positive if and only if \( H \) possesses a unit \( e \) and \( e + s \) has an inverse in \( S \), for every \( s \) in \( S \).

We recall that the quotient ring of a commutative real normed ring by a maximal ideal is isomorphic to the real or complex field. [7] Let \( \mathcal{M}_H \) be a set of the maximal ideals of the ring \( \mathcal{R} \). We denote the natural homomorphism of \( \mathcal{R} \) onto \( \mathcal{R}/(\mathcal{M}_H \oplus \mathcal{M}_H) \) by \( \Phi_H \), if we hold \( e \) fixed and let \( \mathbb{N} \) vary over \( \mathcal{M}_H \), we obtain a function \( f_0(e) = \Phi_H(e) \), defined on \( \mathcal{M}_H \). We propose that \( H \) is a positive Banach halfring which is isomorphically and homeomorphically embedded in the Banach ring \( \mathcal{R} \). If \( H \) is an element of \( \mathcal{R} \), then the positive nature of \( H \) implies that \( 1 + f_0(e) \neq 0 \), for \( e \in \mathbb{N} \). If \( f_1(e) = \Phi_H(e) \) and \( f_0(e) \) is a positive real number, then \( f_1(e)(e) = f_0(e) + 1 \), a contradiction. If \( f_1(e) = \Phi_H(e) = -\lambda \), \( \lambda \) a real number, then \( f_1(e)(e) = f_0(e) + 1 \) a positive real number, then \( f_1(e)(e) = f_0(e) + 1 \) a contradiction. Suppose \( f_1(e) = \Phi_H(e) = -\lambda \), \( \lambda \in \mathbb{R} \), then a power of \( f_1(e) \) would be equal to \( -\lambda \), again a contradiction. Hence, if \( f_0(e) \) must be a non-negative number.

For each \( e \) in \( \mathcal{M}_H \), the restriction of \( \Phi_H(e) \) to \( \mathcal{R} \) defines a proper homomorphism of \( H \) into the halffield \( \mathbb{R}^+ \) of non-negative real numbers, which in turn determines a maximal ideal \( \mathbb{M}^+ \in \mathcal{M}_H \), the set of maximal ideals of the halfring, such that \( f_0(e)(e) = f_0(e) \), for \( e \in \mathbb{N} \). If \( e \in \mathbb{M}^+ \), then \( f_1(e)(e) = f_1(e) \), which implies that \( f_1(e) \) is a non-negative number, \( f_1(e)(e) = f_1(e) \) a contradiction. Hence, \( f_0(e) \in \mathbb{M}^+ \).

Let \( \mathbb{M}^+ \in \mathcal{M}_H \) and \( H \) be the ideal generated by \( \mathbb{M}^+ \). \( H \) consists of all differences \( m_1 - m_2 \) with \( m_1, m_2 \in \mathbb{M}^+ \). \( H \) is a maximal ideal of \( \mathcal{R} \) for the mapping which associates to each element \( s_1 - s_2 \mathcal{R} \) the number \( f_1(e)(e) = f_0(e) - f_0(e) \). A proper homomorphism of \( \mathcal{R} \) into \( \mathbb{R}^+ \), with \( H \) as kernel, hence \( H \cdot M = \mathbb{M}^+ \). Since \( H \) is the minimal such ideal of \( \mathcal{R} \), \( \mathbb{M}^+ \) is contained in no other ideal of \( \mathcal{R} \).

We have set up a 1-1 correspondence between the sets \( \mathcal{M}_H \) and \( \mathcal{R} \) such that \( f_0(e)(e) = f_0(e) \) for any \( e \in \mathbb{N} \). Since \( f_0(e)(e) = e, e \in \mathbb{R}^+ \), \( e \in \mathbb{N} \), the quotient semiring \( H^+ / \mathbb{M}^+ \) is the halffield of non-negative real numbers \( \mathbb{R}^+ \).

We have just proved the basic result which is an extension of the Mazur theorem [9]:

**Theorem 3.** If \( H \) is a positive Banach halfring, then the quotient semiring of \( H \) by a maximal ideal is the halffield of non-negative real numbers.

We topologize \( \mathcal{M}_H \) after the manner of Gelfand [7]. It is the strongest topology in which the functions \( f_0(e)(e) \) are continuous and \( \mathcal{M}_H \) is a compact Hausdorff space. Since the halfring \( H \) generates the ring \( \mathcal{R} \), \( \mathcal{M}_H \) is a compact Hausdorff space. The halfring \( H \) is the halffield \( \mathbb{R}^+ \), the topology of \( \mathcal{M}_H \) is the same as that of \( \mathcal{M}_H \). Let \( R^+(\mathcal{M}_H) \) denote the halfring of continuous functions \( f_0(e)(e) \) on space \( \mathcal{M}_H \) of maximal ideals of the halfring \( H \). Then we have an extension of Gelfand's theorem [7]:

**Theorem 4.** If \( H \) is a positive Banach halfring, then there exists...
a homomorphism of $R$ into the halfring $R^+(\mathcal{R}_0)$ of all continuous functions on a compact Hausdorff space.

4. Structure of Banach *-halfrings

In accord with Naimark [10] we give

Definition 9. A mapping $s \rightarrow s'$ of the *-semialgebra $S$ into the *-semialgebra $S'$ is a *-homomorphism if and only if (1) $s \rightarrow s'$ is a homomorphism, (2) $s \rightarrow s'$ implies $s^* \rightarrow s'^*$. 

Definition 10. A complete isomorphism of the *-semialgebra $S$ onto *-semialgebra $S'$ is an isometric mapping of the space $S$ onto the space $S'$ which is a *-isomorphism of the *-semialgebra $S$ onto the *-semialgebra $S'$.

As in [11], by embedding the Banach *-halfring $H$ into a real Banach *-algebra whose structure is given by Arens and Kaplansky [3], we obtain

Theorem 5. Let $H$ be a Banach halfring with an involution such that $\|s_1, s_2\| \leq k \|s_1, s_2\| s_1 + s_2 + (s_1, s_2)$ for all $(s_1, s_2)$ and $(t_1, t_2)$ in $H \times H$, where $k$ is a constant. Then there exists a compact space $\mathcal{R}_H$ with an involutory homeomorphism $\sigma$ such that $H$ is *-isomorphic to the halfring $C^*(\mathcal{R}_H)$ of all continuous complex-valued functions on $\mathcal{R}_H$, for which $f(\sigma(M)) = f(M)$.

Proof. We embed $H$ in $\mathcal{R} = H \times H$, then $\mathcal{R}$ is a real Banach *-algebra with the involution $\nu(s_1, s_2) = s_1^* s_1 + s_2^* t_2 + t_2^* s_2 - s_2^* t_2 - t_2^* s_2$. Now

$k [\nu(s_1, s_2) \nu(s_1, s_2) + \nu(t_1, t_2) \nu(t_1, t_2)]$

$= k \inf_{(u_1, u_2, v_1, v_2)} \sqrt{\|u_1 u_2 + u_2^* v_1 + v_1^* u_2 + u_2^* v_1 + v_1^* u_2 + u_2^* v_1 + v_1^* v_2 + v_2^* v_1\|}$

$= k \inf_{(u_1, u_2, v_1, v_2)} \|u_1 u_2 + u_2^* v_1 + v_1^* u_2 + u_2^* v_1 + v_1^* u_2 + u_2^* v_1 + v_1^* v_2 + v_2^* v_1\|$

$= k \inf_{(u_1, u_2, v_1, v_2)} \|\nu(u_1, u_2) \nu(v_1, v_2)\|$

Since $\|s_1, s_2\| \leq k \|s_1, s_2\| s_1 + s_2 + s_2 s_2 + s_2 s_2$, we have

$k [\nu(s_1, s_2) \nu(s_1, s_2) + \nu(t_1, t_2) \nu(t_1, t_2)] \geq \inf_{(u_1, u_2, v_1, v_2)} \|\nu(u_1, u_2)\|^2$

Hence, this condition which is satisfied in $\mathcal{R}$ is precisely the one which Arens and Kaplansky [2] assumed and makes $\mathcal{R}$ *-isomorphic with the ring of all those continuous complex-valued functions on $\mathcal{R}_H$ which satisfy $f_\mathcal{R}(\sigma(M)) = f_\mathcal{R}(M)$. Since for $s \in H$, $f_\mathcal{R}(M^*) = f_\mathcal{R}(M)$, we examine the map of $H$ in the ring of all those continuous complex-valued functions on $\mathcal{R}_H$.

Now $f_\mathcal{R}(s \sigma(M)) = f_\mathcal{R}(s)(M) = f_\mathcal{R}(M^*) = f_\mathcal{R}(M)$. Since $s_1 - s_2 = s_1 - s_2$ in $\mathcal{R}$, $f_\mathcal{R}(s \sigma(M)) = f_\mathcal{R}(M^*) = f_\mathcal{R}(M)$. Therefore we have $f_\mathcal{R}(M^*) = f_\mathcal{R}(M) = f_\mathcal{R}(M^*)$ for all $s \in H$. In particular, if $s_2 = 0$ we obtain that $f_\mathcal{R}(M^*) = f_\mathcal{R}(M^*)$. This states that the mapping $s \rightarrow f_\mathcal{R}(M^*)$ is a *-isomorphism of $H$ into $C(\mathcal{R}_H)$.

The involutory homeomorphism $\sigma(M)$ of the compact space $(\mathcal{R}_H)$ onto itself is defined by $f_\mathcal{R}(\sigma(M^*)) = f_\mathcal{R}(M)$. Since $f_\mathcal{R}(M^*) = f_\mathcal{R}(M^*)$ the map of $H$ in $C^*(\mathcal{R}_H)$ is contained in the set of those functions which satisfy $f_\mathcal{R}(\sigma(M^*)) = f_\mathcal{R}(M^*)$.

Let $C^*(\mathcal{R}_H, \sigma)$ denote the halfring of all those continuous functions on $(\mathcal{R}_H)$ for which $f_\mathcal{R}(\sigma(M^*)) = f_\mathcal{R}(M^*)$ for all $M^* \in \mathcal{R}_H$. For any $f \in C^*(\mathcal{R}_H, \sigma)$ then $f(\sigma(M^*)) = f(\sigma(M^*))$. Since $f$ is also in $C^*(\mathcal{R}_H)$, then $f(\sigma(M^*)) = f(\sigma(M^*)) + f(\sigma(M^*))$. Thus, $f(\sigma(M^*)) = f(\sigma(M^*)) + f(\sigma(M^*))$. Since $f_\mathcal{R}(\sigma(M^*)) = f_\mathcal{R}(M^*)$ and $f_\mathcal{R}(\sigma(M^*)) = f_\mathcal{R}(M^*)$ then $f_\mathcal{R}(\sigma(M^*)) = f_\mathcal{R}(M^*) - f_\mathcal{R}(M^*)$. Therefore $f(\sigma(M^*)) = 0$ and $f(\sigma(M^*)) = f(\sigma(M^*))$, for all $M^* \in \mathcal{R}_H$, and $f$ is contained in the map of $H$ in $C^*(\mathcal{R}_H)$. Hence, the map of $H$ in $C^*(\mathcal{R}_H)$ is precisely $C^*(\mathcal{R}_H, \sigma)$. In the case $k = 1$, the *-isomorphism is complete.

Bibliography

Asymptotische Auffassung der Operatorenrechnung

von

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$$ f_g = \frac{d}{dt} \int_0^t f(t - \tau) g(\tau) d\tau \quad (1) $$

als Grundlage der Operatorenrechnung wählen. Die Menge der für $t > 0$ einmal stetig differenzierbaren Funktionen bildet dann mit der gewöhnlichen Addition und der Funktionenmultiplikation (1) einen kommutativen Ring $\mathfrak{F}$. Dabei schreiben wir die Funktionenmultiplikation (1) zur Unterscheidung von der gewöhnlichen Wertemultiplikation $f(t) g(t)$ entweder ohne Argument oder mit Malpunkt

$$ fg = f(t) g = f(t) g(t) $$

und bezeichnen Funktionenpotenzen mit $f^n = f^n(t)$. Jeder Funktion