3. Using the above Lemma we can prove by the same method as in the paper of A. Alexiewicz the following

**Theorem 1.** If the set $B$ is linear and analytic, then there exists a decomposition $T = e_1 \cup h$ and a residual set $B$ in $X$ such that

(a) for every $x$ and every $e > 0$ there exists a set $e'$ such that $\mu(e \setminus e') < e$ and $U(x) \setminus e'$;

(b) for every $x \in B$ and every set $h' \subset h$ of positive measure $U(x) \setminus h'$.

**Theorem 2.** When the set $B$ is analytic and satisfies the following condition:

$$\text{from } y_n \in B \ (n = 1, 2, \ldots) \text{ and } e = \bigcup_{n=1}^{\infty} y_n \in B,$$

then there exists a decomposition $T = e_1 \cup h$ and a residual set $B \subset X$ such that

(a) $U(x) \setminus e_1$ for every $x$,

(b) $U(x) \setminus e_2$ for every $x \in B$ and every set $h' \subset h$ of positive measure.

**References**


**On the estimation of the norm of the $n$-linear symmetric operation**

**by**

J. KOPEC and J. MUSIELAK (Poznań)

Let $X$ and $Y$ be two Banach spaces. An operation $U(x_1, \ldots, x_n)$ from $X \times \ldots \times X$ to $Y$ is called $n$-linear if it is linear in each variable $x_i$ separately. It is called symmetric if $U(x_1, \ldots, x_n) = U(x_{i_1}, \ldots, x_{i_n})$ for every permutation $i_1, \ldots, i_n$ of the numbers $1, \ldots, n$. The operation $U(x_1, \ldots, x_n)$ being $n$-linear and symmetric, we call the operation $U(x) = U(x, x, \ldots, x)$ the power of degree $n$; $U(x_1, \ldots, x_n)$ is then called the primitive (or polar) operation of $U(x)$. Between the norms of these operations an inequality

$$\sup_{|x_j|<1, n \in C} \|U(x_1, \ldots, x_n)\| \leq B_n \sup_{|x|<1} \|U(x)\|$$

holds, with $B_n$ depending only on $n$. A. E. Taylor\(^1\) has shown that $B_n \leq \frac{n^n}{n!}$. We shall show that this estimation is the best possible.

Let $X = L^1, Y = R^1$ (the space of reals), $A_k = (k-1)/n$, $k/n > (k-1, 2, \ldots, n)$. Let us consider the operation $U(x_1, \ldots, x_n) = \sum_{(i_1, \ldots, i_n)} a_{i_1} \cdots a_{i_n} \int_0^1 \cdots \int_0^1 f(t_1) \cdots f(t_n) \, dt_1 \cdots dt_n$, where the summation is extended over all permutations $i_1, \ldots, i_n$ of the numbers $1, \ldots, n$. This operation is obviously $n$-linear and symmetric.

We shall prove that

$$\sup_{|x_j|<1, n \in C} \|U(x_1, \ldots, x_n)\| = n^n \sup_{|x|<1} \|U(x)\|.$$

Let

$$\|x_j\| = \frac{1}{n} \|x(t)\| dt \leq 1$$

On quotient-fields generated by pseudonormed rings

by

K. URBANIK (Wroclaw)

I. A linear ring \( R \) over the field of complex numbers as scalars with unit element is called a pseudonormed ring if \( R \) is a \( F \)-space (Mazur and Orlicz [4], p. 185) with respect to a sequence of pseudonorms \( \| \cdot \|_1, \| \cdot \|_2, \ldots \) submultiplicative, i.e. satisfying the condition \( \| \cdot \|_p \leq c \cdot \| \cdot \|_q \) for every \( x, y \in R \).

In this paper we shall deal exclusively with commutative pseudonormed rings without divisors of zero.

Let \( Q(R) \) be the quotient-field obtained from the pseudonormed ring \( R \) (van der Waerden [7], p. 46-49). J. G.-Mikusiński has introduced by means of the convergence in the ring \( R \) a convergence in the field \( Q(R) \) (which will be called the \( M \)-convergence) as follows: the sequence \( a_1, a_2, \ldots , a_n, \ldots \in Q(R) \) converges to \( a \in Q(R) \) (which will be denoted by \( M \)-lim \( a_n = a \)) if there exists \( x \in R \) (\( x \neq 0 \)) such that \( x a_n e R \) (\( a_n = a \)) and the sequence \( x a_n, x a_2, \ldots \) converges to \( x a \) in the ring \( R \), i.e. \( \lim_{n \to \infty} \| x a_n - x a \| = 0 \) (\( k = 1, 2, \ldots \)) (Mikusiński [5], p. 62). It is easily seen that \( Q(R) \) with the \( M \)-convergence is an \( L \)-space of Fréchet (Kuratowski [3], p. 84) and that the addition and the ring- and scalar-multiplication in \( Q(R) \) are continuous with respect to the \( M \)-convergence.

The \( M \)-convergence in \( Q(R) \) is called topological if there exists in \( Q(R) \) a topology satisfying the first axiom of countability (Kuratowski [3], p. 33), such that the convergence according to this topology is equivalent to the \( M \)-convergence, and \( Q(R) \) is a topological field with respect to this topology (we suppose here that the inverse of an element is a continuous function). In connection with the study of the \( M \)-convergence in the field of operators of Mikusiński (Urbanik [6]) arose the problem of determining those pseudonormed rings for which the \( M \)-convergence in \( Q(R) \) is topological. In this paper we shall prove that

If the \( M \)-convergence in the field \( Q(R) \) is topological, then the pseudonormed ring \( R \) is isomorphic and homeomorphic with the field of complex numbers (topologized as usual).