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On isomorphisms of standard operator algebras

by

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Abstract. We show that between standard operator algebras every bijective map with a certain multiplicativity property related to Jordan triple isomorphisms of associative rings is automatically additive.

1. Introduction. It is a surprising result of Martindale [7, Corollary] that every multiplicative bijective map from a prime ring containing a nontrivial idempotent onto an arbitrary ring is necessarily additive. Therefore, one can say that the multiplicative structure of rings of that kind completely determines their ring structure. This result has been utilized by Šemrl in [11] to describe the form of the semigroup isomorphisms of standard operator algebras on Banach spaces. The aim of this paper is to generalize this result quite significantly. Other results on the additivity of multiplicative maps (in fact, *-semigroup homomorphisms) between operator algebras can be found in [3, 8].

Besides additive and multiplicative maps (that is, ring homomorphisms) between rings, sometimes one has to consider Jordan homomorphisms. The Jordan structure of associative rings has been studied by many people in ring theory. Moreover, Jordan operator algebras have serious applications in the mathematical foundations of quantum mechanics. If $\mathcal{R}, \mathcal{R}'$ are rings and $\phi : \mathcal{R} \to \mathcal{R}'$ is a transformation, then it is called a *Jordan homomorphism* if

$$\phi(A+B) = \phi(A) + \phi(B)$$

and

$$\phi(AB+BA) = \phi(A)\phi(B) + \phi(B)\phi(A)$$

for every $A, B \in \mathcal{R}$. Clearly, every ring homomorphism is a Jordan homomorphism and the same is true for ring antihomomorphisms (a transforma-

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tion $\phi : \mathcal{R} \to \mathcal{R}'$ is called a *ring antihomomorphism* if ϕ is additive and $\phi(AB) = \phi(B)\phi(A)$ for all $A, B \in \mathcal{R}$. One can go a little further by weakening the multiplicativity property as follows. It is easy to see that if the ring $\mathcal{R}'$ is 2-torsion free (which means that $2A = 0$ implies $A = 0$), then every Jordan homomorphism $\phi : \mathcal{R} \to \mathcal{R}'$ is a *Jordan triple homomorphism*, that is, ϕ is an additive function satisfying

$$\phi(ABA) = \phi(A)\phi(B)\phi(A) \quad (A, B \in \mathcal{R}) \tag{1}$$

(see the Introduction in [1]). The aim of this paper is to show that in the same situation as in [11], that is, in the case of standard operator algebras acting on infinite-dimensional Banach spaces, every bijective map satisfying (1) is automatically linear or conjugate-linear and continuous.

2. The result. We begin with the notation and definitions that we shall use throughout.

All linear spaces are considered over the complex field. Let X be a Banach space. Denote by $B(X)$ and $F(X)$ the algebra of all bounded linear operators on X and the ideal of all finite rank operators in $B(X)$, respectively. A subalgebra of $B(X)$ which contains $F(X)$ is called a *standard operator algebra* on X. For any $n \in \mathbb{N}$, we denote by $M_n(\mathbb{C})$ the algebra of all $n \times n$ complex matrices and ^{t} stands for the transpose.

The dual space of X is denoted by X^* and A^* stands for the Banach space adjoint of the bounded linear operator A on X. If $x \in X$ and $f \in X^*$, then $x \otimes f$ denotes the operator defined by

$$(x \otimes f)(z) = f(z)x \quad (z \in X).$$

Similarly, if H is a Hilbert space and $x, y \in H$, then $x \otimes y$ denotes the operator defined by

$$(x \otimes y)(z) = \langle z, y\rangle x \quad (z \in H).$$

Two idempotents $P, Q \in B(X)$ are said to be *mutually orthogonal* (in the algebraic sense) if $PQ = QP = 0$. One can introduce a partial ordering on the set of all idempotents in $B(X)$ by defining $P \leq Q$ if and only if $PQ = QP = P$. An element R of $B(X)$ is called a *tripotent* if $R^3 = R$.

Now, the result of the paper reads as follows.

THEOREM. *Let X, Y be complex Banach spaces, $\dim X \geq 3$, and let $\mathcal{A} \subset B(X)$ and $\mathcal{B} \subset B(Y)$ be standard operator algebras. Suppose that $\phi : \mathcal{A} \to \mathcal{B}$ is a bijective transformation satisfying*

$$\phi(ABA) = \phi(A)\phi(B)\phi(A) \quad (A, B \in \mathcal{A}). \tag{2}$$

If X is infinite-dimensional, then we have the following possibilities:

(i) *there exists an invertible bounded linear operator $T : X \to Y$ and $c \in \{-1, 1\}$ such that*

$$\phi(A) = cTAT^{-1} \quad (A \in \mathcal{A});$$

(ii) *there exists an invertible bounded conjugate-linear operator $T : X \to Y$ and $c \in \{-1, 1\}$ such that*

$$\phi(A) = cTAT^{-1} \quad (A \in \mathcal{A});$$

(iii) *there exists an invertible bounded linear operator $T : X^* \to Y$ and $c \in \{-1, 1\}$ such that*

$$\phi(A) = cTA^*T^{-1} \quad (A \in \mathcal{A});$$

(iv) *there exists an invertible bounded conjugate-linear operator $T : X^* \to Y$ and $c \in \{-1, 1\}$ such that*

$$\phi(A) = cTA^*T^{-1} \quad (A \in \mathcal{A}).$$

If X is finite-dimensional, then $\dim X = \dim Y$. So, ϕ can be supposed to act on a matrix algebra $M_n(\mathbb{C})$. In this case we have the following possibilities:

(v) *there exists a ring automorphism h of $\mathbb{C}$, an invertible matrix $T \in M_n(\mathbb{C})$ and $c \in \{-1, 1\}$ such that*

$$\phi(A) = cTh(A)T^{-1} \quad (A \in M_n(\mathbb{C}));$$

(vi) *there exists a ring automorphism h of $\mathbb{C}$, an invertible matrix $T \in M_n(\mathbb{C})$ and $c \in \{-1, 1\}$ such that*

$$\phi(A) = cTh(A)^tT^{-1} \quad (A \in M_n(\mathbb{C})).$$

Here, $h(A)$ denotes the matrix obtained from A by applying h to every entry.

REMARK. According to the referee's wish we point out that there are a lot of discontinuous ring automorphisms of the complex field. See, for example, [6].

Proof of the Theorem. First note that ϕ preserves tripotents in $\mathcal{A}$ and $\mathcal{B}$ in both directions, that is, $P \in \mathcal{A}$ is a tripotent if and only if so is $\phi(P)$.

We show that for every $n = 0, 1, \ldots$ and tripotent $P \in \mathcal{A}$ we have $\operatorname{rank} P = n$ if and only if $\operatorname{rank} \phi(P) = n$. First observe that $\phi(0) = 0$. Indeed, since $i\phi(0) \in \mathcal{B}$, there exists an $A \in \mathcal{A}$ such that $i\phi(0) = \phi(A)$. It follows that $-\phi(0) = (i\phi(0))\phi(0)(i\phi(0)) = \phi(A0A) = \phi(0)$ and this implies that $\phi(0) = 0$. So, we have the rank preserving property of ϕ for $n = 0$. It follows from the first part of the proof of [9, Theorem 4] that every tripotent on a Banach space is the difference of two mutually orthogonal idempotents (to be honest, the cited theorem is about Hilbert spaces, but the part of the proof that we need here also applies to Banach spaces). Suppose now that the equivalence "$\phi(P) \in \mathcal{B}$ is a rank-k tripotent if and only if $P \in \mathcal{A}$ is a rank-k tripotent" holds true for $k = 0, \dots, n$. Let $P \in \mathcal{A}$ be a rank-$(n+1)$ tripotent. Then the rank of $\phi(P)$ is at least $n+1$. Let $Q \in \mathcal{B}$ be a rank-$(n+1)$ tripotent such that $\phi(P)Q\phi(P) = Q$ and $Q\phi(P)Q = Q$. The existence of Q follows from the representation of tripotents as differences of mutually orthogonal idempotents mentioned above. Let $Q' = \phi^{-1}(Q)$. We have

$$PQ'P = Q' \quad \text{and} \quad Q'PQ' = Q'. \tag{3}$$

Clearly, the rank of Q' is at least $n+1$. On the other hand, the first equality in (3) shows that the range of Q' is included in the range of P, so the rank of Q' is exactly $n+1$. The tripotent P is the difference of two mutually orthogonal idempotents. These idempotents induce a splitting of X into the direct sum of three closed subspaces. With respect to this splitting every operator has a matrix representation. In particular, we can write

$$P = \begin{bmatrix} I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Let

$$Q' = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}$$

be the representation of Q'. It follows from the first equality in (3) that the only possibly nonzero entries in the matrix of Q' are Q_{11} and Q_{22}. The second equality in (3) now implies that Q_{11} and $-Q_{22}$ are idempotents. By the equality of the ranks of P and Q' we conclude that $P = Q'$. Therefore, $\phi(P) = Q$ and so $\operatorname{rank}\phi(P) = n+1$. By symmetry, we find that if $\phi(P)$ has rank $n+1$, then the same must be true for P.

The key step of the proof now follows. We prove that if $P', P'' \in \mathcal{A}$ are mutually orthogonal rank-1 idempotents, then $\phi(P'+P'') = \phi(P') + \phi(P'')$. In order to verify this, let $P \in \mathcal{A}$ be a rank-3 idempotent. Then $Q = \phi(P)$ is a rank-3 tripotent. Let $Q = R_1 - R_2$, where $R_1, R_2 \in B(Y)$ are idempotents with $R_1R_2 = R_2R_1 = 0$. If $A \in \mathcal{A}$ is any operator satisfying $PAP = A$, then $Q\phi(A)Q = \phi(A)$. We compute

$$R_1\phi(A)R_2 = R_1(Q\phi(A)Q)R_2 = -R_1\phi(A)R_2,$$

which implies that $R_1\phi(A)R_2 = 0$. Similarly, $R_2\phi(A)R_1 = 0$. It follows that $\phi(A) = R_1\phi(A)R_1 + R_2\phi(A)R_2$. Conversely, if $\phi(A) = R_1\phi(A)R_1 + R_2\phi(A)R_2$, then

$$\begin{aligned} Q\phi(A)Q &= Q(R_1\phi(A)R_1 + R_2\phi(A)R_2)Q \\ &= R_1\phi(A)R_1 + R_2\phi(A)R_2 = \phi(A). \end{aligned}$$

The algebra of all operators $A \in \mathcal{A}$ for which $PAP = A$ is isomorphic to $M_3(\mathbb{C})$. Let r_1 be the rank of R_1 and let r_2 be the rank of R_2. Clearly, $r_1 + r_2 = 3$. The algebra of all operators $B \in \mathcal{B}$ for which $B = R_1BR_1 + R_2BR_2$ is isomorphic to $M_{r_1}(\mathbb{C}) \oplus M_{r_2}(\mathbb{C})$. Therefore, ϕ induces a bijective transformation

$$\psi : M_3(\mathbb{C}) \to M_{r_1}(\mathbb{C}) \oplus M_{r_2}(\mathbb{C})$$

which satisfies (2). We assert that either $r_1 = 3$ or $r_2 = 3$. Suppose on the contrary that, for example, $r_1 = 2$ and $r_2 = 1$. One can see that there are five rank-1 tripotents $P_1, \dots, P_5$ on the Hilbert space $\mathbb{C}^3$ such that $P_iP_jP_i = 0$ $(i \neq j)$. Indeed, choose an orthonormal basis x, y, z in $\mathbb{C}^3$ and consider the operators

$$(x+y)\otimes x, \quad y\otimes(x+y), \quad \tfrac{1}{2}(x-y)\otimes(x-y), \quad z\otimes(x+z), \quad (x-z)\otimes z.$$

They fulfil the requirements. It follows that there are five rank-1 tripotents in $M_2(\mathbb{C})\oplus M_1(\mathbb{C})$ with similar properties. This readily implies that there are four rank-1 tripotents $Q_1, \dots, Q_4$ in $M_2(\mathbb{C})$ for which $Q_iQ_jQ_i = 0$ $(i \neq j)$. But this cannot happen. In fact, applying a similarity transformation or the negative of a similarity transformation we can suppose that $Q_1 = a \otimes a$ for some unit vector a in $\mathbb{C}^2$. Choose a unit vector $b \in \mathbb{C}^2$ which is orthogonal to a. Since $Q_1Q_jQ_1 = 0$ $(j = 2, 3, 4)$, it follows that in the "vector-tensor-vector" representation of any of Q_2, Q_3, Q_4 either the first or the second component is a scalar multiple of b. Clearly, at least in two of Q_2, Q_3, Q_4, the vector b appears in the same component. For example, suppose that $Q_2 = c\otimes b$ and $Q_3 = d\otimes b$. Since $Q_2Q_3Q_2 = 0$, we see that either $\langle c, b\rangle = 0$ or $\langle d, b\rangle = 0$. But this implies that either $Q_2^2 = 0$ or $Q_3^2 = 0$, which contradicts the fact that Q_2, Q_3 are nonzero tripotents.

We have proved that the induced transformation ψ is a bijection from $M_3(\mathbb{C})$ onto itself which satisfies (2). Moreover, observe that we have also shown that either $\psi(I) = I$ (this is the case if $r_1 = 3$) or $\psi(I) = -I$ (if $r_2 = 3$). Without loss of generality we can suppose that $\psi(I) = I$. It follows from (2) that

$$\psi(A^2) = \psi(AIA) = \psi(A)I\psi(A) = \psi(A)^2,$$

which shows that ψ preserves idempotents in both directions. If P, Q are idempotents in $M_3(\mathbb{C})$ such that $PQ = QP = P$ (that is, if $P \leq Q$), then

$$\psi(P)\psi(Q)\psi(P) = \psi(P) \quad \text{and} \quad \psi(Q)\psi(P)\psi(Q) = \psi(P). \tag{4}$$

Since $\psi(P), \psi(Q)$ are idempotents, multiplying the second equality in (4) by $\psi(Q)$ from the left and from the right, we find that $\psi(Q)\psi(P) = \psi(P)\psi(Q) = \psi(P)$. So, ψ preserves the partial ordering $\leq$ between the idempotents in $M_3(\mathbb{C})$ in both directions. We now apply a nice result of Ovchinnikov [10] describing the automorphisms of the poset of all idempotents on a Hilbert space of dimension at least 3. It trivially implies that ψ is orthoadditive on the set of all idempotents in $M_3(\mathbb{C})$, that is, if P, Q are mutually orthogonal idempotents in $M_3(\mathbb{C})$, then $\psi(P+Q) = \psi(P) + \psi(Q)$. Returning to our original transformation ϕ we see that if $P, Q \in \mathcal{A}$ are mutually orthogonal rank-1 idempotents, then $\phi(P+Q) = \phi(P) + \phi(Q)$.

If P is a rank-1 tripotent and $\lambda \in \mathbb{C}$ is a scalar, then

$$\phi(\lambda P) = \phi(P(\lambda P)P) = \phi(P)\phi(\lambda P)\phi(P) = h_P(\lambda)\phi(P)$$

for some scalar $h_P(\lambda) \in \mathbb{C}$. This follows from the fact that $\phi(P)$ has rank 1. We have

$$\begin{aligned} h_P(\lambda^2\mu)\phi(P) &= \phi(\lambda^2\mu P) = \phi((\lambda P)(\mu P)(\lambda P)) \\ &= \phi(\lambda P)\phi(\mu P)\phi(\lambda P) = h_P(\lambda)^2 h_P(\mu)\phi(P), \end{aligned}$$

which gives

$$h_P(\lambda^2\mu) = h_P(\lambda)^2 h_P(\mu) \tag{5}$$

for every $\lambda, \mu \in \mathbb{C}$. Choosing $\mu = 1$, we see that $h_P(\lambda^2) = h_P(\lambda)^2$. From (5) we see that h_P is a multiplicative function.

We next assert that h_P does not depend on P. Let $Q \in \mathcal{A}$ be a rank-1 tripotent with $PQP \neq 0$. We compute

$$\begin{aligned} \phi((\lambda P)(\mu^2 Q)(\lambda P)) &= \phi(\lambda P)\phi(\mu^2 Q)\phi(\lambda P) \\ &= h_P(\lambda)^2 h_Q(\mu^2)\phi(P)\phi(Q)\phi(P). \end{aligned}$$

On the other hand, we also have

$$\phi((\lambda P)(\mu^2 Q)(\lambda P)) = \phi((\mu P)(\lambda^2 Q)(\mu P)) = h_P(\mu)^2 h_Q(\lambda^2)\phi(P)\phi(Q)\phi(P).$$

This yields

$$h_P(\lambda)^2 h_Q(\mu^2) = h_P(\mu)^2 h_Q(\lambda^2)$$

and so $h_P = h_Q$. If $PQP = 0$, then we can choose a rank-1 tripotent $R \in \mathcal{A}$ such that $PRP \neq 0$ and $RQR \neq 0$. Hence, we can infer that $h_P = h_R = h_Q$. This means that h_P really does not depend on P. In what follows $h : \mathbb{C} \to \mathbb{C}$ denotes this common scalar function.

Let $A \in \mathcal{A}$. Then

$$\begin{aligned} \phi(P)\phi(\lambda^2 A)\phi(P) &= \phi(P(\lambda^2 A)P) = \phi((\lambda P)A(\lambda P)) \\ &= \phi(\lambda P)\phi(A)\phi(\lambda P) = h(\lambda)^2\phi(P)\phi(A)\phi(P). \end{aligned}$$

Since this holds for every rank-1 tripotent P on X and $\phi(P)$ runs through the whole set of rank-1 tripotents on Y, we obtain $\phi(\lambda^2 A) = h(\lambda)^2\phi(A)$ for every $\lambda \in \mathbb{C}$, which yields

$$\phi(\lambda A) = h(\lambda)\phi(A) \quad (\lambda \in \mathbb{C}).$$

We prove that h is additive. Let $x, y \in X$ be linearly independent vectors, and choose linear functionals $f, g \in X^*$ such that $f(x) = 1$, $f(y) = 0$ and $g(x) = 0$, $g(y) = 1$. Let $\lambda, \mu \in \mathbb{C}$ and let $A = (\lambda x + \mu y) \otimes (f + g)$, $P = x \otimes f$, $Q = y \otimes g$. By the orthoadditivity property of ϕ we can compute

$$\begin{aligned} h(\lambda+\mu)\phi(A) &= \phi((\lambda+\mu)A) = \phi(A(P+Q)A) \\ &= \phi(A)\phi(P+Q)\phi(A) = \phi(A)\phi(P)\phi(A) + \phi(A)\phi(Q)\phi(A) \\ &= \phi(APA) + \phi(AQA) = \phi(\lambda A) + \phi(\mu A) \\ &= (h(\lambda) + h(\mu))\phi(A) \end{aligned}$$

and this proves that h is additive.

We now verify that ϕ is additive. Let $A, B \in \mathcal{A}$ and pick any rank-1 tripotent P on X. There are $x \in X$ and $f \in X^*$ such that $P = x \otimes f$. We compute

$$\begin{aligned} \phi(P)\phi(A+B)\phi(P) &= \phi(P(A+B)P) = \phi(f((A+B)x)P) \\ &= h(f((A+B)x))\phi(P) = h(f(Ax))\phi(P) + h(f(Bx))\phi(P) \\ &= \phi(f(Ax)P) + \phi(f(Bx)P) = \phi(PAP) + \phi(PBP) \\ &= \phi(P)\phi(A)\phi(P) + \phi(P)\phi(B)\phi(P) = \phi(P)(\phi(A) + \phi(B))\phi(P). \end{aligned} \tag{6}$$

Since this holds true for every rank-1 tripotent P on X, we deduce that $\phi(A+B) = \phi(A) + \phi(B)$. Consequently, $\phi : \mathcal{A} \to \mathcal{B}$ is an additive bijection satisfying (2).

Since every standard operator algebra $\mathcal{R}$ on a Banach space is prime (this means that for every $A, B \in \mathcal{R}$, the equality $A\mathcal{R}B = \{0\}$ implies $A = 0$ or $B = 0$), we can apply a result of Brešar [1, Theorem 3.3] (also see [4]) to conclude that ϕ is necessarily a homomorphism, or an antihomomorphism, or the negative of a homomorphism, or the negative of an antihomomorphism. In the homomorphic cases the satement follows from [11], while in the antihomomorphic cases one can apply analogous ideas (cf. [2, Proposition 3.1]). ■

REMARK. We should explain why we have supposed in our theorem that $\dim X \geq 3$. First, it is easy to see that the conclusion does not hold true if $\dim X = 1$. Indeed, the function $z \mapsto z|z|$ is a multiplicative bijection of $\mathbb{C}$ which is not additive. The place in the proof where we used the assumption that $\dim X \geq 3$ is where we applied Ovchinnikov's result. If we knew that every bijective function $\phi : M_2(\mathbb{C}) \to M_2(\mathbb{C})$ satisfying (2) preserves the mutual orthogonality between rank-1 idempotents (that is, $\phi(P)\phi(Q) = \phi(Q)\phi(P) = 0$ whenever $P, Q \in M_2(\mathbb{C})$ are rank-1 idempotents with $PQ = QP = 0$), then the use of this deep result could be avoided. Unfortunately, we do not know whether ϕ has this preservation property. However, if we suppose that ϕ satisfies the stronger equality

$$\phi(\{ABC\}) = \{\phi(A)\phi(B)\phi(C)\} \tag{7}$$

where $\{ABC\}$ denotes the so-called *Jordan triple product* $\frac{1}{2}(ABC+CBA)$, then one can check that ϕ preserves the mutual orthogonality between rank-1 idempotents and hence we get the conclusion in the Theorem also in the case $\dim X = 2$. Observe that if ϕ is additive, then (7) is equivalent to (2).

To conclude, we note that our approach was mainly functional-analytic. In our opinion, it is a challenging question how one can generalize the "additive part" of our result to general rings to obtain results similar to Martindale's theorem.

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