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Weighted Hardy inequalities and Hardy transforms of weights

by

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Abstract. Many problems in analysis are described as weighted norm inequalities that have given rise to different classes of weights, such as A_p-weights of Muckenhoupt and B_p-weights of Ariño and Muckenhoupt. Our purpose is to show that different classes of weights are related by means of composition with classical transforms. A typical example is the family M_p of weights w for which the Hardy transform is $L_p(w)$-bounded. A B_p-weight is precisely one for which its Hardy transform is in M_p, and also a weight whose indefinite integral is in A_{p+1}.

1. Introduction. If w is a weight on $\mathbb{R}^+ = [0,\infty)$, we define $W(t) = \int_0^t w(x)\,dx$, and $T : X \to Y$ indicates that T is a bounded operator between X and Y, two function spaces on $\mathbb{R}^+$. X^{d} will denote the subset of all nonincreasing and nonnegative functions (briefly, decreasing functions) of X.

We recall that A_p, for $p > 1$, is defined by the condition

$$(A_p)\qquad \sup_I \left(\frac{1}{|I|}\int_I w(x)\,dx\right)\left(\frac{1}{|I|}\int_I w(x)^{1-p'}\,dx\right)^{p-1} < \infty,$$

where the supremum is taken over all intervals I and, if $p = 1$, by $Mw \le Cw$. Here M is the Hardy–Littlewood maximal function and it is well known (see [Mu1]) that $w \in A_p$ if and only if $M : L_p(w) \to L_p(w)$ $(1 < p < \infty)$.

In [Mu2], the weights w such that $S_1 f(t) = (1/t)\int_0^t f(x)\,dx$ (the Hardy operator) is bounded on $L_p(w)$ $(1 \le p < \infty)$ are described as the weights of class M_p, defined for $1 < p < \infty$ by the estimate

$$(M_p)\qquad \sup_{t>0}\left(\int_t^\infty \frac{w(x)}{x^p}\,dx\right)^{1/p}\left(\int_0^t w(x)^{-p'/p}\,dx\right)^{1/p'} < \infty.$$

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The class M_1 is defined by $S_2w \le Cw$, where $S_2f(t) = \int_t^\infty f(x)x^{-1}\,dx$.

It is also known that $S_2 : L_p(w) \to L_p(w)$ if and only if

$$(M^p)\qquad \sup_{t>0}\Big(\int_0^t w(x)\,dx\Big)^{1/p}\Big(\int_t^\infty \frac{w(x)^{-p'/p}}{x^{p'}}\,dx\Big)^{1/p'} < \infty$$

when $1 < p < \infty$. The class M^1 is defined by $S_1w \le Cw$.

Also (see [ArM]), $S_1 : L_p(w)^{\rm d} \to L_p(w)$ for $1 \le p < \infty$ if and only if w satisfies

$$(B_p)\qquad \int_t^\infty \frac{w(x)}{x^p}\,dx \le \frac{C}{t^p}\int_0^t w(x)\,dx,$$

which defines the class B_p (for any $p \in (0,\infty)$). As shown in [So], it is easily seen that (1) is equivalent to

$$(2)\qquad \int_t^\infty \frac{W(x)}{x^{p+1}}\,dx \simeq \frac{W(t)}{t^p},$$

i.e., $\int_t^\infty W(x)x^{-(p+1)}\,dx \le CW(t)t^{-p}$, because W is increasing. As usual, $F \simeq G$ indicates the existence of a universal constant $c > 0$ so that $c^{-1}F \le G \le cF$.

For weak type estimates, it was proved in [AnM] that $S_1 : L_1(w) \to L_{1,\infty}(w)$ if and only if $w \in M_{1,\infty}$, the class of weights w such that, for some $\alpha > 0$,

$$(3)\ (M_{1,\infty})\qquad \int_t^\infty \Big(\frac{t}{x}\Big)^\alpha \frac{w(x)}{x}\,dx \le C(\alpha)\inf_{0\le x\le t} w(x).$$

Only this case $p = 1$ is interesting, since $S_1 : L_p(w) \to L_{p,\infty}(w)$ implies $S_1 : L_p(w) \to L_p(w)$ when $p > 1$ (see [AnM, Theorem 3]).

The restriction of S_1 to decreasing functions was studied in [Ne1]. Again, for $1 < p < \infty$, $S_1 : L_p(w)^{\rm d} \to L_{p,\infty}(w)$ implies $S_1 : L_p(w)^{\rm d} \to L_p(w)$.

If $p = 1$, it is proved in [CGS] that $S_1 : L_1(w)^{\rm d} \to L_{1,\infty}(w)$ if and only if w belongs to the class $B_{1,\infty}$ defined by the condition

$$(B_{1,\infty})\qquad \frac{1}{t}\int_0^t w(x)\,dx \le C\frac{1}{s}\int_0^s w(x)\,dx \quad \text{if } s \le t.$$

REMARK 1.1. Neugebauer also proved that the property $S_2 : L_p(w)^{\rm d} \to L_p(w)$ does not depend on $p \in [1,\infty)$ (cf. [Ne2]), and it holds if and only if

$$(B^1)\qquad \int_0^t (S_1w)(x)\,dx \le C\int_0^t w(x)\,dx,$$

i.e., $S_1S_1w \le CS_1w$. This condition defines the class B^1 of weights.

2. Monotone weights. The following facts will be applied to powers W^α of W.

PROPOSITION 2.1. *If w is decreasing, the following properties are equivalent:*

(a) *w is doubling, i.e.,* $\int_r^{r+h} w(x)\,dx \simeq \int_{r+h}^{r+2h} w(x)\,dx$ $(r, h \ge 0)$.
(b) $w \simeq S_1w$.
(c) $w \simeq Mw$, *i.e.,* $w \in A_1$.
(d) $w \in A_p$ *for one (or all)* $p > 1$.
(e) $w \in M^p$ *for one (or all)* $p \ge 1$.
(f) $w \in B^p$ *for one (or all)* $p \ge 1$.
(g) $\inf_{x>0} w(rx)/w(x) > 1/r$ *for some* $r > 1$.

Proof. If w is doubling, then

$$w(r) \le (1/r)\int_0^r w(x)\,dx \le (C/r)\int_r^{2r} w(x)\,dx \le Cw(r),$$

i.e., $w \simeq S_1w$. Since we are assuming that w is decreasing, $S_1w \simeq Mw$ and then $w \simeq Mw$. Obviously, (d) follows from (c), and it is known that A_p-weights are doubling. It was proved in [CM] that $w \in M^p$ if and only if $w \in M^1$, that is, $S_1w \simeq w$, together with the equivalence of (e), (f) and (g). ∎

COROLLARY 2.1. *If $w \in A_p$ on $\mathbb{R}^n$ $(1 \le p < \infty)$, then $w^* \in A_1$. Here* $w^*(t) = \inf\{\lambda > 0 : |\{w > \lambda\}| \le t\}$, *the nonincreasing rearrangement of w.*

Proof. Since w satisfies $[(1/|Q|)\int_Q w^s]^{1/s} \le (C/|Q|)\int_Q w$ (Q any cube) for some $s > 1$ and $(Mw^s)^{1/s} \in A_1$ (see [GR]), we have $Mw \in A_1$, since $M(Mw) \le M((Mw^s)^{1/s}) \le c(Mw^s)^{1/s} \le cCMw$.

But $(Mw)^* \simeq S_1w^*$, thus $S_1S_1w^* \simeq (MMw)^* \le C(Mw)^* \simeq S_1w^*$, and $w^* \in B^1$. It follows from Proposition 2.1 that $w^* \in A_1$. ∎

With the same proof, if $w \in A_p$ on a cube $Q \subset \mathbb{R}^n$, then $w^* \in A_1[0, |Q|]$ (cf. [Wi]).

Obviously, any decreasing weight w belongs to M_p, and hence to B_p, for all $p > 1$. This is not true if $p = 1$, as the simple example $w \equiv 1$ shows. If $p = 1$, a decreasing doubling weight w belongs to B_1 if and only if $w \in M_1$, since, if $w \in B_1$, then

$$\int_x^\infty w(s)\frac{ds}{s} \le \frac{C}{x}\int_0^x w(s)\,ds \le C'w(x)$$

by Proposition 2.1(b), and then $w \in M_1$.

PROPOSITION 2.2. *If w is decreasing, then $w \in M_1$ if and only if* $\sup_{x>0} w(rx)/w(x) < 1$ *for some* $r > 1$, *and then* $w^\alpha \in M_1$ *for any* $\alpha > 0$.

Proof. If $\int_x^\infty (w(s)/s)\,ds \le Cw(x)$ and $r > e^C$, then

$$(\ln r)w(rx) \le \int_x^{rx} \frac{w(s)}{s}\,ds \le \int_x^\infty \frac{w(s)}{s}\,ds \le Cw(x)$$

and $w(rx)/w(x) \le C/\ln r < 1$. Conversely, if $\delta := \sup_{x>0} w(rx)/w(x) < 1$, then $w \in M_1$, since

$$\int_x^\infty \frac{w(s)}{s}\,ds = \sum_{n=0}^\infty \int_{xr^n}^{xr^{n+1}} \frac{w(s)}{s}\,ds \le (\ln r)w(x)\sum_{n=0}^\infty \delta^n = \frac{\ln r}{1-\delta}w(x). \ \blacksquare$$

PROPOSITION 2.3. *For an increasing weight w and $1 < p < \infty$, the following properties are equivalent*:

(a) $w \in A_p$,
(b) $w \in M_p$,
(c) $w \in B_p$ *and* $\int_t^\infty w(x)x^{-p}\,dx \simeq w(t)t^{1-p}$, *and*
(d) $\inf_{x>0} w(rx)/w(x) > r^{p-1}$ *for some* $r < 1$.

Proof. By [CU; Corollary 6.3], (c) implies (a) and, since $S_1 f \le Mf$ for any $f \ge 0$, (a) implies (b). Also (b) implies (c), since if $w \in M_p$ then $w \in B_p$ and

$$\frac{1}{p}\frac{w(t)}{t^{p-1}} \le \int_t^\infty \frac{w(x)}{x^p}\,dx \le C\frac{1}{t^p}\int_0^t w(x)\,dx \le C\frac{w(t)}{t^{p-1}}.$$

If (d) holds and $0 < a < 1$ is such that $w(rx)/w(x) \ge a^{p-1} > r^{p-1}$, then

$$\int_x^\infty \frac{w(s)}{s^p}\,ds = \sum_{n=0}^\infty \int_{x/r^n}^{x/r^{n+1}} \frac{w(s)}{s^p}\,ds$$
$$\le \sum_{n=0}^\infty w\left(\frac{x}{r^{n+1}}\right)\frac{1}{r^{n(1-p)}}\frac{x^{1-p}}{1-p}\left(1-\frac{1}{r^{1-p}}\right).$$

Since $w(x) \ge a^{p-1}w(x/r)$, also $w(x/r^{n+1}) \le w(x)a^{(n+1)(1-p)}$ and it follows that

$$\int_x^\infty \frac{w(s)}{s^p}\,ds \le \frac{1}{1-p}\cdot\frac{1-r^{p-1}}{a^{p-1}-r^{p-1}}w(x)x^{1-p},$$

and w has property (c). Conversely, if $w \in A_p$, then $w^{1-p'} \in A_{p'}$. By Proposition 2.1, $S_1(w^{1-p'}) \simeq w^{1-p'}$ and there exists $s > 1$ such that $\inf_{x>0}(w(sx)/w(x))^{1-p'} > 1/s$, and $\inf_{x>0} w(rx)/w(x) > r^{p-1}$ with $r = 1/s$. $\blacksquare$

3. B_p-weights as derivatives of A_{p+1}-weights.

The main result of this section states that $w \in B_p$ if and only if $W \in A_{p+1}$.

THEOREM 3.1. *Let w be a weight on $\mathbb{R}^+$ and $0 < p < \infty$. Then $w \in B_p$ if and only if $W^\alpha \in A_{p\alpha+1}$ for one (or for all) $\alpha > 0$.*

Proof. Since $w \in B_p$ is equivalent to (2), it follows from Proposition 2.3 applied to W that (2) holds if and only if $W \in A_{p+1}$. Now $W \in A_{p+1}$ if and only if $\inf_{x>0} W(rx)^\alpha/W(x)^\alpha > r^{p\alpha}$ for some $r < 1$, i.e. $W^\alpha \in A_{p\alpha+1}$. $\blacksquare$

REMARK 3.1. The above results can be used to see that $w \in B_p$ if and only if $W^{\alpha-1}w \in B_{p\alpha}$ (cf. [Ne1; Theorem 6.5]), since $W^\alpha(t) = \alpha\int_0^t W^{\alpha-1}w$.

Neugebauer ([Ne1]) presented some properties of B_p suggested by the analogous properties of A_p, and gave short proofs of facts such as B_p implies $B_{p-\varepsilon}$ (see also [Ma]). Here we give a very easy proof from the corresponding result for A_p.

COROLLARY 3.1. (a) *If $w \in B_p$ $(0 < p < \infty)$, then $w \in B_{p-\varepsilon}$ for some $\varepsilon \in (0,p)$.*

(b) *$w \in B_\infty = \bigcup_{p>0} B_p$ if and only if $W \in \Delta_2$, i.e., $W(2t) \le CW(t)$.*

(c) *$w \in B_p$ $(p \in (0,\infty))$ if and only if $\int_0^t W(x)^{-1/p}\,dx \simeq tW(t)^{-1/p}$* (cf. [So]).

Proof. (a) From Theorem 3.1, $W \in A_{p+1-\varepsilon}$ and $w \in B_{p-\varepsilon}$.

(b) If $w \in B_p$ then $W \in A_{p+1}$ and $W \in \Delta_2$. If $W \in \Delta_2$, then $W(t/2)/W(t) \ge 1/C > (1/2)^q$ $(q > 1)$, so $W \in A_{q+1}$ (cf. Proposition 2.3) and $w \in B_q$ (Theorem 3.1).

(c) Since, for $1 < q < \infty$, $w \in A_q$ if and only if $w^{1-q'} \in A_{q'}$ (see [GR]), we have $W \in A_{p+1}$ if and only if $W^{1-(p+1)'} \in A_{(p+1)'}$, which means that $W^{-1/p} \in A_{1+1/p}$ and $W^{-1/p}$ is a doubling and decreasing weight, and Proposition 2.1 applies. $\blacksquare$

4. Hardy transforms of B_p-weights.

Let us see that $w \in B_p$ if and only if $S_1 w \in M_p$.

THEOREM 4.1. *If $1 \le p < \infty$, then*

$$S_1 : L_p(w)^{\mathrm{d}} \to L_p(w) \quad \textit{if and only if} \quad S_1 : L_p(S_1 w) \to L_p(S_1 w).$$

Proof. First assume $S_1 w \in M_p$, i.e. $S_1 : L_p(S_1 w) \to L_p(S_1 w)$. If $1 < p < \infty$, then $w_1(t) := (S_1 w)(t) = W(t)/t$ satisfies

$$\left(\int_t^\infty \frac{w_1(x)}{x^p}\,dx\right)^{1/p}\left(\int_0^t w_1(x)^{-p'/p}\,dx\right)^{1/p'} \le C$$

and, W being increasing,

$$\int_0^t w_1^{-p'/p}(x)\,dx = \int_0^t (x/W(x))^{p'/p}\,dx \ge \frac{1}{p'}\frac{t^{p'}}{W(t)^{p'/p}}.$$

Thus

$$\left(\int_t^\infty \frac{W(x)}{x^{p+1}}\,dx\right)^{1/p}\left(\frac{t^{p'}}{W(t)^{p'/p}}\right)^{1/p'} \le p'^{1/p'} C$$

and $w \in B_p$ follows from (2).

In the case $p = 1$, $S_1w \in M_1$ means that $S_2S_1w \le CS_1w$ and then $S_1S_2w \le CS_1w$. Since S_2w is decreasing, $S_2w \le S_1S_2w \le CS_1w$ and hence $w \in B_1$.

Let now $w \in B_p$. If $p = 1$, then $S_2S_1w = S_2w + S_1w \le (C+1)S_1w$ and $S_1w \in M_1$. In the case $1 < p < \infty$, to prove that $w_1 = S_1w$ satisfies

$$\left(\int_t^\infty \frac{w_1(x)}{x^p}\,dx\right)^{1/p}\left(\int_0^t w_1(x)^{-p'/p}\,dx\right)^{1/p'} \le C \tag{4}$$

observe that (2) implies

$$I_1 := \left(\int_t^\infty \frac{W(x)}{x^{p+1}}\,dx\right)^{1/p} \le C\frac{W(t)^{1/p}}{t}.$$

On the other hand $\widetilde{w} := W^{\alpha-1}w \in B_{p'}$ for $\alpha = p'/p$ (cf. Remark 3.1), which means that

$$\int_0^t \frac{x^{p'-1}}{\widetilde{W}(x)}\,dx \le C'\frac{t^{p'}}{\widetilde{W}(t)}$$

(see [So; Theorem 2.5(ii)]). Then

$$I_2^{p'} := \int_0^t \frac{x^{p'-1}}{W(x)^\alpha}\,dx \simeq \int_0^t \frac{x^{p'-1}}{\int_0^x \widetilde{w}(s)\,ds}\,dx = \int_0^t \frac{x^{p'-1}}{\widetilde{W}(x)}\,dx \le \frac{C't^{p'}}{\widetilde{W}(t)} = \frac{C't^{p'}}{W^\alpha(t)}.$$

Thus, $I_1 \cdot I_2 \le CC'$ gives (4). ∎

A similar result holds for the weak type Hardy inequalities:

THEOREM 4.2. *$S_1 : L_1(w)^{\mathrm{d}} \to L_{1,\infty}(w)$ if and only if $S_1 : L_1(S_1w) \to L_{1,\infty}(S_1w)$.*

Proof. If $w \in B_{1,\infty}$, from $(S_1w)(x) \le C(S_1w)(t)$ $(t < x)$ we see that S_1w satisfies (3) with $\alpha = 1$, since

$$\int_t^\infty \frac{t}{x}(S_1w)(x)\frac{dx}{x} \le C(S_1w)(t)\int_t^\infty \frac{t}{x^2}\,dx = C(S_1w)(t).$$

Assume now $S_1w \in M_{1,\infty}$. Then, as in [AnM; proof of Theorem 2],

$$\frac{1}{y}\int_t^y (S_1w)(x)\,dx \le C\inf_{0\le s\le t}(S_1w)(s) \quad (0 < t < y),$$

and, for $y = 2t$,

$$\frac{1}{2t}\int_t^{2t}(S_1w)(x)\,dx \ge \frac{1}{2t}\int_t^{2t}\left(\frac{1}{x}\int_0^t w(s)\,ds\right)dx = \frac{\ln 2}{2}(S_1w)(t).$$

Hence $(S_1w)(t) \le C\inf_{0\le s\le t}(S_1w)(s)$ and $w \in B_{1,\infty}$. ∎

THEOREM 4.3. (a) *$S_2 : L_1(w)^{\mathrm{d}} \to L_1(w)$ if and only if $S_2 : L_1(S_1w) \to L_1(S_1w)$.*

(b) *If $S_2 : L_{p_0}(w)^{\mathrm{d}} \to L_{p_0}(w)$ with $p_0 \in [1,\infty)$, then $S_2 : L_p(S_1w) \to L_p(S_1w)$ for all $p \in [1,\infty)$.*

(c) *If $W \in \Delta_2$ and $S_2 : L_p(S_1w) \to L_p(S_1w)$ for some $p \in [1,\infty)$, then $S_2 : L_q(w)^{\mathrm{d}} \to L_q(w)$ for any $q \in [1,\infty)$.*

Proof. (a) If $w \in B^1$, then $\int_0^t (S_1w)(x)\,dx \le C\int_0^t w(x)\,dx$, i.e. $S_1w \in M^1$. If $S_1w \in M^1$, then

$$\int_0^\infty f(x)(S_1S_1w)(x)\,dx = \int_0^\infty (S_2f)(x)(S_1w)(x)\,dx \le C\int_0^\infty f(x)(S_1w)(x)\,dx$$

when $f \ge 0$, and then $S_1S_1w \le CS_1w$, i.e., $w \in B^1$.

(b) Since $S_2 : L_1(w)^{\mathrm{d}} \to L_1(w)$ (see Remark 1.1), $S_1w \in M^1$ and also $S_2 : L_p(S_1w) \to L_p(S_1w)$, since $M^p \subset M^q$ for $q > p$ (cf. [BMR]).

(c) Let $1 < p < \infty$ and $S_2 : L_p(S_1w) \to L_p(S_1w)$ (if $p = 1$, see Theorem 3.3). Then

$$\left(\int_0^t S_1w(x)\,dx\right)^{1/p}\left(\int_t^\infty \frac{S_1w(x)^{-p'/p}}{x^{p'}}\,dx\right)^{1/p'} \le C$$

and in our case

$$\int_t^{2t} \frac{S_1w(x)^{-p'/p}}{x^{p'}}\,dx = \int_t^{2t} \frac{W(x)^{-p'/p}}{x}\,dx \ge (\ln 2)W(2t)^{-p'/p}.$$

Thus

$$\left(\int_0^t (S_1w)(x)\,dx\right)^{1/p} \le C\left(\int_t^{2t} \frac{W(x)^{-p'/p}}{x}\,dx\right)^{-1/p'} \le C(\ln 2)^{-1/p'}W(2t)^{-1/p}.$$

Since $W \in \Delta_2$, $\int_0^t (S_1w)(x)\,dx \le C'W(t)$ and now we apply Remark 1.1. ∎

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[6] D. Beck, *Introduction to Dynamical Systems*, Vol. 2, Progr. Math. 54, Birkhäuser, Basel, 1978.

[7] R. Hill and A. James, *An index formula*, J. Differential Equations 15 (1982), 197–211.

[8] J. Kowalski, *Some remarks on $J(X)$*, in: Algebra and Analysis (Edmonton, 1973), E. Brook (ed.), Lecture Notes in Math. 867, Springer, Berlin, 1974, 115–124.

[Nov] A. S. Novikov, *An existence theorem for planar graphs*, preprint, Moscow University, 1980 (in Russian).

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