$$t_a(x:y) := \left(\frac{x+a(1-x\overline{y})}{1-x\overline{y}+\overline{b}(x+a(1-x\overline{y}))} : b\right).$$

It can be checked as in Theorem 5.3 that t_a is independent of the b chosen and of the representation of the equivalence class $(x:y)$. Finally, t_a is a biholomorphic map on the manifold M_X and for $\gamma \in \mathbb{R}$ sufficiently small, $t_{\gamma a}$ coincides with the flow of ξ_a at time γ through the point $(x:y)$. By uniqueness of this flow we see that ξ_a is a complete holomorphic vector field and $\exp(\xi_a) = t_a$. ■

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H^∞ functional calculus in real interpolation spaces

by

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Abstract. Let A be a linear closed densely defined operator in a complex Banach space X. If A is of type ω (i.e. the spectrum of A is contained in a sector of angle 2ω, symmetric around the real positive axis, and $\|\lambda(\lambda I - A)^{-1}\|$ is bounded outside every larger sector) and has a bounded inverse, then A has a bounded H^∞ functional calculus in the real interpolation spaces between X and the domain of the operator itself.

1. Introduction. The concept of H^∞ functional calculus has been developed by various authors; we recall the papers [1]–[3], [5]–[8]. In the present paper we follow the definitions of [3] and [7].

An H^∞ functional calculus can be defined for closed, linear, one-to-one operators with dense domain and dense range, having resolvent set that contains $\mathbb{R}^-$ and with resolvent that decreases in a maximal way on $\mathbb{R}^-$ (i.e. $\|\lambda(\lambda I - A)^{-1}\|$ is bounded). If for every $f \in H^\infty$ the operator $f(A)$ is bounded, with norm not exceeding a constant times the supremum of f, then we say that A has a bounded H^∞ functional calculus. In general it is not easy to prove that an operator has a bounded H^∞ functional calculus and there are several examples of operators that do not enjoy this property (see [8]).

In this paper we prove that every linear, closed, densely defined operator A in a complex Banach space X, with resolvent satisfying the above-mentioned hypotheses and such that $0 \in \varrho(A)$, has a bounded H^∞ functional calculus in the real interpolation spaces between X and the domain of the operator itself.

2. Preliminary definitions and results. We now recall some definitions and results from [7] and [3], to which we refer for the details.

For $\theta \in [0,\pi[$ we put

$$S_\theta = \{z \in \mathbb{C} \setminus \{0\} : |\arg z| \le \theta\} \cup \{0\}$$

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and for $\theta \in]0,\pi]$ we put
$$S^0_\theta = \{z \in \mathbb{C} \setminus \{0\} : |\arg z| < \theta\}.$$

Let X be a complex Banach space, A a linear closed operator in X. We say that A is of *type* ω (with $\omega \in [0,\pi[$) if $\sigma(A) \subseteq S_\omega$ and for each $\theta \in]\omega,\pi[$ there exists $M_\theta \in \mathbb{R}^+$ such that $\|(zI-A)^{-1}\| \le M_\theta/|z|$ for $z \in \mathbb{C}\setminus S_\theta$.

If $\mu \in]0,\pi]$ we denote by $H^\infty(S^0_\mu)$ the space of bounded holomorphic functions defined on S^0_μ with values in $\mathbb{C}$; it is a Banach algebra if endowed with the norm $\|f\|_\infty = \sup_{z\in S^0_\mu}|f(z)|$. Moreover, we put
$$\Psi(S^0_\mu) = \{f \in H^\infty(S^0_\mu) : \exists s \in \mathbb{R}^+\ f\psi^{-s} \in H^\infty(S^0_\mu)\}$$
where ψ is the rational function $\psi(z) = z/(1+z)^2$.

Suppose now that A is an operator of type ω, one-to-one, with dense domain and dense range, that we denote by $\mathcal{D}(A)$ and $\mathcal{R}(A)$ respectively. If $f \in \Psi(S^0_\mu)$ (with $\mu > \omega$) then it is possible to define the bounded linear operator $f(A)$ through the integral
$$f(A) = \frac{1}{2\pi i}\int_{\Gamma_\theta} f(\lambda)(\lambda I - A)^{-1}\,d\lambda$$
where $\theta \in]\omega,\mu[$ and Γ_θ is the path in the complex plane composed by the two half-lines $\{\varrho e^{\pm i\theta} : \varrho \in \mathbb{R}^+ \cup \{0\}\}$, oriented with decreasing imaginary part. It is easy to show that the definition is independent of θ.

If $f \in H^\infty(S^0_\mu)$ then $f\psi \in \Psi(S^0_\mu)$, so that one can define $f(A)$ by
$$\mathcal{D}(f(A)) = \{x : (f\psi)(A)x \in \mathcal{D}((I+A)^2A^{-1})\},$$
$$f(A) = (I+A)^2A^{-1}(f\psi)(A)$$
(note that $\mathcal{D}((I+A)^2A^{-1}) = \mathcal{D}(A)\cap\mathcal{R}(A)$). Then $f(A)$ is a closed densely defined operator and this definition is consistent with the earlier one in case $f \in \Psi(S^0_\mu)$.

We say that A has a *bounded $H^\infty(S^0_\mu)$ functional calculus* if for every $f \in H^\infty(S^0_\mu)$ the operator $f(A)$ is bounded and there exists C (independent of f) such that $\|f(A)\| \le C\|f\|_\infty$.

The following theorem can be proved (see [3], Corollary 2.2).

THEOREM 2.1. *If there exists $C \in \mathbb{R}^+$ such that for every $f \in \Psi(S^0_\mu)$ we have $\|f(A)\| \le C\|f\|_\infty$ then for every $f \in H^\infty(S^0_\mu)$ we have $\|f(A)\| \le C\|f\|_\infty$, so that A has a bounded $H^\infty(S^0_\mu)$ functional calculus.*

From now on we suppose that $0 \in \varrho(A)$.

$\mathcal{D}(A)$ is a Banach space if endowed with the norm $\|x\|_{\mathcal{D}(A)} = \|x\|_X + \|Ax\|_X$. Since we suppose that A has a bounded inverse this norm is equivalent to $\|Ax\|_X$.

For $\alpha \in]0,1[$ and $p \in [1,\infty]$ we denote by $\mathcal{D}(A;\alpha,p)$ the real interpolation space $(X,\mathcal{D}(A))_{\alpha,p}$. We refer to [9], 1.3.2, for the definition of real interpolation spaces. We recall that, since $\mathcal{D}(A) \subseteq X$, we have $\mathcal{D}(A) \subseteq \mathcal{D}(A;\alpha,p)$.

Let $L^p_*(\mathbb{R}^+)$ be the space of L^p functions on $\mathbb{R}^+$, with values in $\mathbb{R}$, with respect to the measure dt/t (with obvious modifications if $p=\infty$). If A is of type ω with dense domain and $0 \in \varrho(A)$ then it can be proved ([9], Theorem 1.14.2) that the norm of x in the space $\mathcal{D}(A;\alpha,p)$ is equivalent to $\|t \mapsto t^\alpha A(tI+A)^{-1}x\|_{L^p_*(\mathbb{R}^+)}$.

We denote by $A_{\alpha,p}$ the part of the operator A in $\mathcal{D}(A;\alpha,p)$, i.e. the operator such that
$$\mathcal{D}(A_{\alpha,p}) = \{x \in \mathcal{D}(A) : Ax \in \mathcal{D}(A;\alpha,p)\},\qquad A_{\alpha,p}x = Ax.$$

THEOREM 2.2. *If A is an operator of type ω and $0 \in \varrho(A)$, then for $\alpha \in]0,1[$ and $p \in [1,\infty]$, $A_{\alpha,p}$ is an operator of type ω in $\mathcal{D}(A;\alpha,p)$ and $0 \in \varrho(A_{\alpha,p})$. Moreover, if A is densely defined and $p < \infty$ then $A_{\alpha,p}$ is densely defined.*

Proof. It is easy to prove that if $\lambda \in \varrho(A)$ then the restriction of $(\lambda I - A)^{-1}$ to $\mathcal{D}(A;\alpha,p)$ is the inverse operator of $\lambda I - A_{\alpha,p}$, so that $\lambda \in \varrho(A_{\alpha,p})$, therefore $\varrho(A_{\alpha,p})$ is not empty, hence $A_{\alpha,p}$ is closed, and $\sigma(A_{\alpha,p}) \subseteq S_\omega \setminus \{0\}$; in particular, $0 \in \varrho(A_{\alpha,p})$.

Moreover, for $x \in \mathcal{D}(A)$ and $\lambda \in \varrho(A)$ we have $(\lambda I - A)^{-1}x \in \mathcal{D}(A)$ and
$$\begin{aligned}\|(\lambda I - A)^{-1}x\|_{\mathcal{D}(A)} &= \|A(\lambda I - A)^{-1}x\|_X + \|(\lambda I - A)^{-1}x\|_X\\ &= \|(\lambda I - A)^{-1}Ax\|_X + \|(\lambda I - A)^{-1}x\|_X\\ &\le \|(\lambda I - A)^{-1}\|_{\mathcal{L}(X)}(\|Ax\|_X + \|x\|_X)\\ &= \|(\lambda I - A)^{-1}\|_{\mathcal{L}(X)}\|x\|_{\mathcal{D}(A)}\end{aligned}$$
so that the restriction of $(\lambda I - A)^{-1}$ to $\mathcal{D}(A)$ belongs to $\mathcal{L}(\mathcal{D}(A))$ and its norm in this space is less than or equal to its norm in $\mathcal{L}(X)$. By interpolation the same is true in $\mathcal{L}(\mathcal{D}(A;\alpha,p))$. Since A is of type ω we can conclude that for $\theta \in]\omega,\pi[$ we have $\|(zI - A_{\alpha,p})^{-1}\|_{\mathcal{L}(\mathcal{D}(A;\alpha,p))} \le M_\theta/|z|$ for $z \in \mathbb{C}\setminus S_\theta$, therefore $A_{\alpha,p}$ is of type ω.

Suppose $p < \infty$ and $\mathcal{D}(A)$ dense in X. We have to prove that $\mathcal{D}(A_{\alpha,p})$ is dense in $\mathcal{D}(A;\alpha,p)$; to this end we prove that $\mathcal{D}(A^2) \subseteq \mathcal{D}(A_{\alpha,p})$ and that $\mathcal{D}(A^2)$ is dense in $\mathcal{D}(A;\alpha,p)$.

If $x \in \mathcal{D}(A^2)$, then $Ax \in \mathcal{D}(A) \subseteq \mathcal{D}(A;\alpha,p)$, hence $x \in \mathcal{D}(A_{\alpha,p})$, therefore $\mathcal{D}(A^2) \subseteq \mathcal{D}(A_{\alpha,p})$.

Since $\mathcal{D}(A)$ is dense in X, if $x \in \mathcal{D}(A)$ then there exists a sequence $(y_n)_{n\in\mathbb{N}}$ in $\mathcal{D}(A)$ converging to Ax, therefore the sequence $(A^{-1}y_n)_{n\in\mathbb{N}}$ converges to x in $\mathcal{D}(A)$; this proves that $\overline{\mathcal{D}(A^2)}^{\mathcal{D}(A)} = \mathcal{D}(A)$. Since $\mathcal{D}(A)$ is continuously embedded in $\mathcal{D}(A;\alpha,p)$ it follows that $\mathcal{D}(A) \subseteq \overline{\mathcal{D}(A^2)}^{\mathcal{D}(A;\alpha,p)}$, but since $p<\infty$ we have $\overline{\mathcal{D}(A)}^{\mathcal{D}(A;\alpha,p)}=\mathcal{D}(A;\alpha,p)$ (see [9], Theorem 1.6.2) so $\overline{\mathcal{D}(A^2)}^{\mathcal{D}(A;\alpha,p)}=\mathcal{D}(A;\alpha,p)$.

3. H^∞ functional calculus

THEOREM 3.1. *Let A be an operator of type ω with dense domain and such that $0\in\varrho(A)$. Let $\mu\in\,]\omega,\pi[$, $\alpha\in\,]0,1[$ and $p\in[1,\infty]$. If $f\in\Psi(S^0_\mu)$ and $x\in\mathcal{D}(A;\alpha,p)$, then $f(A)x\in\mathcal{D}(A;\alpha,p)$ and there exists $C_\alpha\in\mathbb{R}^+$ (independent of f and x) such that*

$$\|f(A)x\|_{\mathcal{D}(A;\alpha,p)}\le C_\alpha\|f\|_\infty\|x\|_{\mathcal{D}(A;\alpha,p)}.$$

Proof. First of all we consider the case $p=\infty$. Choose $\theta\in\,]\omega,\mu[$. For $t\in\mathbb{R}^+$, from the resolvent identity we get

$$\begin{aligned}(tI+A)^{-1}f(A)&=\frac{1}{2\pi i}\int_{\Gamma_\theta}f(\lambda)(\lambda I-A)^{-1}(tI+A)^{-1}\,d\lambda\\&=\frac{1}{2\pi i}\int_{\Gamma_\theta}\frac{f(\lambda)}{t+\lambda}(\lambda I-A)^{-1}\,d\lambda+\frac{1}{2\pi i}\int_{\Gamma_\theta}\frac{f(\lambda)}{t+\lambda}(tI+A)^{-1}\,d\lambda\\&=\frac{1}{2\pi i}\int_{\Gamma_\theta}\frac{f(\lambda)}{t+\lambda}(\lambda I-A)^{-1}\,d\lambda;\end{aligned}$$

the last equality follows from the fact that f is holomorphic on S^0_μ.

If $x\in\mathcal{D}(A;\alpha,\infty)$ then we have

$$\begin{aligned}&\|t^\alpha A(tI+A)^{-1}f(A)x\|\\&=\Big\|t^\alpha\frac{1}{2\pi i}\int_{\Gamma_\theta}\frac{f(\lambda)}{t+\lambda}A(\lambda I-A)^{-1}x\,d\lambda\Big\|\\&\le\frac{1}{2\pi}\int_{\mathbb{R}^+}\frac{t^\alpha\|f\|_\infty}{|t+\varrho e^{i\theta}|}\|A(\varrho e^{i\theta}I-A)^{-1}x\|\,d\varrho\\&\quad+\frac{1}{2\pi}\int_{\mathbb{R}^+}\frac{t^\alpha\|f\|_\infty}{|t+\varrho e^{-i\theta}|}\|A(\varrho e^{-i\theta}I-A)^{-1}x\|\,d\varrho\\&\le\frac{1}{2\pi}\int_{\mathbb{R}^+}\frac{t^\alpha}{\varrho^\alpha}\frac{1}{|t+\varrho e^{i\theta}|}\,d\varrho\,\|f\|_\infty\sup_{\varrho\in\mathbb{R}^+}\|\varrho^\alpha A(\varrho e^{i\theta}I-A)^{-1}x\|\\&\quad+\frac{1}{2\pi}\int_{\mathbb{R}^+}\frac{t^\alpha}{\varrho^\alpha}\frac{1}{|t+\varrho e^{-i\theta}|}\,d\varrho\,\|f\|_\infty\sup_{\varrho\in\mathbb{R}^+}\|\varrho^\alpha A(\varrho e^{-i\theta}I-A)^{-1}x\|\\&=\frac{1}{2\pi}\int_{\mathbb{R}^+}\frac{1}{\sigma^\alpha}\frac{1}{|1+\sigma e^{i\theta}|}\,d\sigma\,\|f\|_\infty\sup_{\varrho\in\mathbb{R}^+}\|\varrho^\alpha e^{-i\theta}A(\varrho-e^{-i\theta}A)^{-1}x\|\\&\quad+\frac{1}{2\pi}\int_{\mathbb{R}^+}\frac{1}{\sigma^\alpha}\frac{1}{|1+\sigma e^{-i\theta}|}\,d\sigma\,\|f\|_\infty\sup_{\varrho\in\mathbb{R}^+}\|\varrho^\alpha e^{i\theta}A(\varrho-e^{i\theta}A)^{-1}x\|.\end{aligned}$$

The operators A, $-e^{i\theta}A$ and $-e^{-i\theta}A$ have the same domain and for every $x\in\mathcal{D}(A)$ we have $\|Ax\|=\|-e^{i\theta}Ax\|=\|-e^{-i\theta}Ax\|$ so that the spaces $\mathcal{D}(A)$, $\mathcal{D}(-e^{i\theta}A)$ and $\mathcal{D}(-e^{-i\theta}A)$ coincide and have equal norms, therefore $\mathcal{D}(A;\alpha,\infty)=\mathcal{D}(-e^{i\theta}A;\alpha,\infty)=\mathcal{D}(-e^{-i\theta}A;\alpha,\infty)$ (with equal norms). From this fact it follows that there exists a constant C such that for $x\in\mathcal{D}(A;\alpha,\infty)$ we have

$$\sup_{\varrho\in\mathbb{R}^+}\|\varrho^\alpha e^{i\theta}A(\varrho-e^{i\theta}A)^{-1}x\|\le C\|x\|_{\mathcal{D}(A;\alpha,\infty)},$$

$$\sup_{\varrho\in\mathbb{R}^+}\|\varrho^\alpha e^{-i\theta}A(\varrho-e^{-i\theta}A)^{-1}x\|\le C\|x\|_{\mathcal{D}(A;\alpha,\infty)},$$

and we can conclude that

$$\sup_{t\in\mathbb{R}^+}\|t^\alpha A(tI+A)^{-1}f(A)x\|\le C_\alpha\|f\|_\infty\|x\|_{\mathcal{D}(A;\alpha,\infty)}.$$

In this way we have proved that for $x\in\mathcal{D}(A;\alpha,\infty)$ we have $f(A)x\in\mathcal{D}(A;\alpha,\infty)$ and there exists $C_\alpha\in\mathbb{R}^+$ such that

$$\|f(A)x\|_{\mathcal{D}(A;\alpha,\infty)}\le C_\alpha\|f\|_\infty\|x\|_{\mathcal{D}(A;\alpha,\infty)}.$$

If $p<\infty$ choose $\alpha_0\in\,]0,\alpha[$ and $\alpha_1\in\,]\alpha,1[$; then, by the reiteration theorem for real interpolation ([9], Theorem 1.10.2), we have

$$\mathcal{D}(A;\alpha,p)=(\mathcal{D}(A;\alpha_0,\infty),\mathcal{D}(A;\alpha_1,\infty))_{(\alpha-\alpha_0)/(\alpha_1-\alpha_0),p}$$

with equivalence of the norms. Since we have proved that $f(A)$ is a bounded operator in $\mathcal{D}(A;\alpha_0,\infty)$ and in $\mathcal{D}(A;\alpha_1,\infty)$, with norm not greater than $C_{\alpha_0}\|f\|_\infty$ and $C_{\alpha_1}\|f\|_\infty$ respectively, we can conclude, by interpolation, that $f(A)$ is a bounded operator in $\mathcal{D}(A;\alpha,p)$ whose norm is less than or equal to a constant (depending only on α) times $\|f\|_\infty$.

THEOREM 3.2. *Let A be an operator of type ω with dense domain and such that $0\in\varrho(A)$. Let $\mu\in\,]\omega,\pi[$, $\alpha\in\,]0,1[$ and $p\in[1,\infty[$. The operator $A_{\alpha,p}$ has a bounded $H^\infty(S^0_\mu)$ functional calculus.*

Proof. By Theorem 2.2, $A_{\alpha,p}$ is an operator of type ω with dense domain and 0 belongs to its resolvent set. From the fact that $(\lambda I-A_{\alpha,p})^{-1}$ is the restriction of $(\lambda I-A)^{-1}$ to $\mathcal{D}(A;\alpha,p)$ it follows that if $f\in\Psi(S^0_\mu)$ then $f(A_{\alpha,p})$ is the restriction of $f(A)$ to $\mathcal{D}(A;\alpha,p)$, so that by Theorem 3.1 we have $\|f(A_{\alpha,p})\|_{L(\mathcal{D}(A;\alpha,p))}\le C_\alpha\|f\|_\infty$ and the conclusion follows from Theorem 2.1.

It must be noted that, under the hypotheses of this theorem, Theorem 7.1b of [6] yields the existence of a Banach space Y, continuously embedded in X, such that A has a bounded $H^\infty(S^0_\mu)$ functional calculus in Y;

moreover, for all $\alpha > 0$, $\mathcal{D}(A^\alpha)$ is continuously embedded in Y, hence the same is true for $\mathcal{D}(A;\alpha,p)$.

As a consequence of the existence of a bounded H^∞ functional calculus we obtain the following theorem concerning the imaginary powers of an operator.

THEOREM 3.3. *Let A be an operator of type ω with dense domain and such that $0 \in \varrho(A)$. Let $\alpha \in]0,1[$ and $p \in [1,\infty[$. For every $s \in \mathbb{R}$ the operator $A^{is}_{\alpha,p}$ is bounded in $\mathcal{D}(A;\alpha,p)$ and for every $\mu > \omega$ there exists $C \in \mathbb{R}^+$ such that $\|A^{is}_{\alpha,p}\| \le Ce^{\mu|s|}$.*

Proof. For $s \in \mathbb{R}$ and $\mu > \omega$ the function $z \mapsto z^{is}$ is in $H^\infty(S^0_\mu)$ and $\sup_{z\in S^0_\mu} |z^{is}| = e^{\mu|s|}$ so that the theorem is an immediate consequence of Theorem 3.2.

We observe that in [4], Theorem 2.1, it is proved that from the estimate of the norm of the imaginary powers of an operator obtained in Theorem 3.3 it follows that the operator is of type μ. Therefore the estimate of the norm of the imaginary powers obtained here cannot be improved.

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