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Two-parameter maximal functions associated with degenerate homogeneous surfaces in $\mathbb{R}^3$

by

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Abstract. We consider the two-parameter maximal operator

$$Mf(x) = \sup_{a,b>0} \int_{|s|<1} |f(x-(as, b\Gamma(s)))|\, ds$$

on a homogeneous surface $x_3 = \Gamma(x_1, x_2)$ in $\mathbb{R}^3$. We assume that the curvature of the level set $\Gamma(x_1, x_2) = 1$ has a degeneracy of finite order k at a given point. We prove that the operator M is bounded on L^p if and only if $p > \max\{3/2, 2k/(k+1)\}$.

1. Introduction. This paper is a sequel to [MR], where maximal operators in $\mathbb{R}^n$ of the form

$$(1.1)\qquad Mf(x) = \sup_{a,b>0} \int_{|s|<1} |f(x-(as, b\Gamma(s)))|\, ds$$

have been introduced. In (1.1), $s \in \mathbb{R}^{n-1}$ and $\Gamma(s)$ is a function homogeneous of degree $d > 0$ and smooth away from the origin. So the integrals in (1.1) are averages taken over the surface $x_n = \Gamma(x_1, \ldots, x_{n-1})$. In [MR] the range of L^p-boundedness for M has been determined under appropriate assumptions on the curvature of the surface.

We review the general properties of M presented in [MR], restricting our attention to dimensions $n \geq 3$. To avoid trivialities, we assume that Γ is not identically 0.

If $d \neq 1$, imposing the restriction $a = b$ in (1.1), we obtain a kind of "spherical" maximal function associated with Γ. This shows that M cannot be bounded on L^p unless $p > n/(n-1)$. By a different argument, the same restriction on p holds when $d = 1$.

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Conversely, M is bounded on L^p for $p > n/(n-1)$ when one of the following conditions is satisfied:

(1) $d \neq 1$, and the Gaussian curvature of the surface $x_n = \Gamma(x_1, \dots, x_{n-1})$ does not vanish away from 0;

(2) $d = 1$, Γ does not vanish away from 0 and the level set $\Gamma(x) = \pm 1$ has non-zero Gaussian curvature in $\mathbb{R}^{n-1}$.

In this paper we extend this analysis to homogeneous surfaces in $\mathbb{R}^3$ with a degeneracy in the curvature of finite order k along a generating line. We are forced to impose the restriction that this degenerate line does not lie on the plane $x_3 = 0$. Because of the special form of the operator (1.1), the presence of degenerate points on the plane $x_3 = 0$ would require a different analysis.

Under the assumptions that will be made precise in Section 2, we prove that the operator M is bounded on L^p if and only if $p > \max\{3/2, 2k/(k+1)\}$.

We remark that, even though the surface has different geometrical properties in the cases $d \neq 1$ and $d = 1$, the final result is independent of the value of d. However, as in [MR], the proofs for the two cases present some differences.

It is interesting to observe that the critical exponent is always smaller than 2. This means in particular that M is always bounded on L^2 in the cases under consideration, regardless of the order k at which the curvature vanishes. As a matter of fact, following a remark by J. Wright, we also prove that for certain "flat" functions Γ the corresponding operator M is bounded for $p \geq 2$. This contrasts with the "completely flat" situation where Γ is linear, in which case a comparison with the Kakeya maximal function shows that M can only be bounded for $p = \infty$.

2. The maximal theorem. We shall use consistently the following notation: if $x = (x_1, x_2, x_3)$ is a point in $\mathbb{R}^3$, then x' denotes the point (x_1, x_2) in $\mathbb{R}^2$.

We localise our analysis near the degenerate line, which we make correspond to $x_1 = 0$. We assume therefore that Γ is a C^2-function defined on an angle $|x_1| \leq c x_2$ in $\mathbb{R}^2$, where c is chosen so that Γ does not vanish for $|x_1| \leq c x_2$. We can also assume that $\Gamma(0,1) = 1$.

Since

$$\frac{\partial \Gamma}{\partial x_2}(0,1) = (0,1) \cdot \nabla \Gamma(0,1) = d\Gamma(0,1) = d \neq 0,$$

if c is small enough the level set $E = \{x' : \Gamma(x') = 1\}$ coincides with the graph of a C^2-function $x_2 = 1 + \gamma(x_1)$, with $\gamma(0) = 0$.

We next perform a linear transformation in the plane $x_3 = 0$ to reduce ourselves to the situation where γ is C^2 in a neighborhood of $[-1,1]$ and $\gamma'(0) = 0$. Such a transformation induces a conjugation on the operator M which does not alter its norm.

We can now express the degeneracy condition by requiring that

$$c_1|s|^{k-2} \leq |\gamma''(s)| \leq c_2|s|^{k-2} \tag{2.1}$$

for some $k > 2$.

Then the points on the surface $x_3 = \Gamma(x')$ that lie above the x_2 axis have one principal curvature vanishing of finite order k. The other principal curvature is non-zero when $d \neq 1$ and identically zero when $d = 1$, i.e. when the surface is a cone. Observe that we are requiring that these degenerate points do not lie on the plane $x_3 = 0$.

Parametrising the surface by

$$(r, s) \mapsto (rs, r(1+\gamma(s)), r^d), \tag{2.2}$$

the maximal function we wish to consider is

$$Mf(x) = \sup_{a,b>0} \int_{-1}^{1}\int_{0}^{1} |f(x_1 - ars, x_2 - ar(1+\gamma(s)), x_3 - br^d)| r\,dr\,ds. \tag{2.3}$$

The rest of the paper is devoted to the proof of the following result.

THEOREM 2.1. *M is bounded on L^p if and only if $p > \max\{3/2, 2k/(k+1)\}$.*

We first prove the necessity of this restriction on the values of p. The condition $p > 3/2$ appears already in [MR] in the case of a well-curved surface, and its necessity in the present context can be proved in the same way. Hence we concentrate our attention on the other condition.

For ε small, let f_ε be the characteristic function of the set $\{x : |x_1| < 1, |x_2| < \varepsilon, |x_3| < \varepsilon\}$. The value of Mf_ε at a point x such that $|x_1| < 1/2$, $1 < x_2 < \varepsilon^{-1/(k-1)}$, $x_2 < x_3 < 2x_2$ is not smaller than the integral

$$\int_{-1}^{1}\int_{0}^{1} f_\varepsilon(x_1 - ars, x_2 - ar(1+\gamma(s)), x_3 - br^d) r\,dr\,ds$$

for $a = x_2$ and $b = x_3$. This integral is extended to the set where

$$\begin{cases} |x_1 - x_2 rs| < 1, \\ x_2|1 - r(1+\gamma(s))| < \varepsilon, \\ x_3|1 - r^d| < \varepsilon. \end{cases}$$

All conditions are simultaneously satisfied if we take r close to 1 and ranging over an interval of length proportional to ε/x_2 and $|s|$ smaller than a constant times $(\varepsilon/x_2)^{1/k}$. Therefore at such a point we have

$$Mf_\varepsilon(x) > c(\varepsilon/x_2)^{1+1/k}.$$

It follows that

$$\|Mf_\varepsilon\|_p > c\varepsilon^{1+1/k}\Big(\int\limits_{|x_1|<1/2,\,1<x_2<\varepsilon^{-1/(k-1)},\,x_2<x_3<2x_2} x_2^{-p(1+1/k)}\,dx_1\,dx_2\,dx_3\Big)^{1/p}$$

$$= c\varepsilon^{1+1/k}\Big(\int\limits_{1<x_2<\varepsilon^{-1/(k-1)}} x_2^{1-p(1+1/k)}\,dx_2\Big)^{1/p}$$

$$> c\begin{cases}\varepsilon^{1+\frac{1}{k}-\frac{1}{k-1}(\frac{2}{p}-1-\frac{1}{k})} & \text{if } p<2k/(k+1),\\ (\varepsilon^2\log\varepsilon^{-1})^{1/p} & \text{if } p=2k/(k+1).\end{cases}$$

Since $\|f_\varepsilon\|_p = c\varepsilon^{2/p}$, this shows that M cannot be bounded on L^p for $p\le 2k/(k+1)$.

We now prove the sufficiency of the condition $p>\max\{3/2, 2k/(k+1)\}$. As in [MR], we can reduce matters to considering the operator

$$M'f(x) = \sup_{i\in\mathbb{Z},\,b>0}\Big|\iint f(x_1-2^irs, x_2-2^ir(1+\gamma(s)), x_3-br^d)\varphi(r)\psi(s)\,dr\,ds\Big|$$

for some smooth cut-off functions $\varphi\in C_0^\infty(1/2,3)$ and $\psi\in C_0^\infty(-1-\delta,1+\delta)$, which are identically 1 on a consistent part of their support.

We then consider the multiplier corresponding to M',

$$m(\xi) = \iint e^{-2\pi i(\xi_1rs+\xi_2r(1+\gamma(s))+\xi_3r^d)}\varphi(r)\psi(s)\,dr\,ds; \tag{2.4}$$

it is such that

$$M'f(x) = \sup_{i\in\mathbb{Z},\,b>0}|\mathcal{F}^{-1}(m(2^i\xi', b\xi_3)\widehat{f}(\xi))(x)|. \tag{2.5}$$

We decompose m (and consequently M') into three parts by means of appropriate cut-off functions in the ξ variable. The first, obtained by means of a smooth cut-off which is identically 1 near the origin, gives rise to a maximal operator which can be easily controlled by the strong maximal function. This part will therefore be disregarded in the following discussion.

Next we take a cone C in $\mathbb{R}^3$ of the form

$$C = \{\xi : c_1 < |\xi'|^2/\xi_3^2 < c_2\}$$

which contains, in a proper subcone, the normal directions to the surface $(rs, r(1+\gamma(s)), r^d)$ for $r\in[1/2,3]$ and $s\in\operatorname{supp}\psi$. By means of a smooth cut-off η which is homogeneous of degree zero, identically 1 on C and supported on a small neighborhood of C, we split the remaining part of m into two parts, one supported near C, the other, called $\widetilde{m}$, supported outside C.

LEMMA 2.2. *The operator*

$$\widetilde{M}f(x) = \sup_{a,b>0}|\mathcal{F}^{-1}(\widetilde{m}(2^i\xi', b\xi_3)\widehat{f}(\xi))(x)|$$

is bounded on L^p for $p>6/5$.

Proof. The argument used in [MR] cannot be repeated here, because $\widetilde{m}$ does not decay sufficiently rapidly to allow a control of $\widetilde{M}$ by means of the strong maximal function. Observe in fact that, since the phase function in (2.4) is assumed to be only twice differentiable, the best we can say is that $\widetilde{m}(\xi) = O(|\xi|^{-2})$ together with all its derivatives.

We take instead a smooth two-parameter partition of unity $\omega_{jl}(\xi) = \omega(2^{-j}\xi', 2^{-l}\xi_3)$ to decompose $\widetilde{m}$ as a sum of $\widetilde{m}_{jl} = \widetilde{m}\omega_{jl}$, in such a way that $\widetilde{m}_{jl}$ is supported where $|\xi'|\sim 2^j$ and $|\xi_3|\sim 2^l$.

The constants c_1, c_2 in the definition of the cone C can be chosen so that, for some $n>0$, only the indices $j,l>0$ such that $|j-l|>n$ intervene in the decomposition of $\widetilde{m}$. For such values of j and l, we have the estimate

$$|\partial^\alpha\widetilde{m}_{jl}(\xi)| \le C_\alpha 2^{-2\max\{j,l\}}, \tag{2.6}$$

for every multi-index α.

Let $\widetilde{M}_{jl}$ be the maximal operator defined by the multiplier $\widetilde{m}_{jl}$. We apply a variant of the argument used in the proof of Lemma 2.1 in [MR], based on the following estimate: if $g(a,b)$ is a C^1 function defined for $a,b\ge 0$ and such that $g(a,0)=g(0,b)=0$, then

$$\sup_{a,b>0}|g(a,b)| \le C\Bigg(\Big(\int_0^\infty\!\!\int_0^\infty |g(a,b)|^2\frac{da\,db}{ab}\Big)^{1/2} \times\Big(\int_0^\infty\!\!\int_0^\infty\Big|ab\frac{\partial^2 g}{\partial a\partial b}(a,b)\Big|^2\frac{da\,db}{ab}\Big)^{1/2} + \Big(\int_0^\infty\!\!\int_0^\infty\Big|a\frac{\partial g}{\partial a}(a,b)\Big|^2\frac{da\,db}{ab}\Big)^{1/2} \times\Big(\int_0^\infty\!\!\int_0^\infty\Big|b\frac{\partial g}{\partial b}(a,b)\Big|^2\frac{da\,db}{ab}\Big)^{1/2}\Bigg)^{1/2}.$$

Following the same lines as in [MR], this gives the L^2-estimate

$$\|\widetilde{M}_{jl}\|_{2,2} \le C2^{-2\max\{j,l\}}.$$

In order to obtain an estimate for p near 1, consider the inverse Fourier transform $\mathcal{F}^{-1}\widetilde{m}_{jl} = \Phi_{jl}$. By (2.6), for every multi-index α, we have

$$|x^\alpha\Phi_{jl}(x)| \le \int|\partial^\alpha\widetilde{m}_{jl}(\xi)|\,d\xi \le C2^{2j+l}2^{-2\max\{j,l\}}.$$

This implies that $\widetilde{M}_{jl}$ is controlled by the strong maximal function in $\mathbb{R}^3$ multiplied by a factor of the order of $2^{2j+l-2\max\{j,l\}}$. So if s is near 1, the norm $\|\widetilde{M}_{jl}\|_{s,s}$ is controlled by a constant times $2^{2j+l-2\max\{j,l\}}$.

If $6/5 < p < 2$, we can write $1/p = (1-\theta)/s + \theta/2$ for some $s > 1$ with $1/(3\theta) < 1$. By interpolation,

$$\|\widetilde{M}_{jl}\|_{p,p} \leq C2^{(2j+l-2\max\{j,l\})(1-\theta)-2\theta\max\{j,l\}},$$

and it is a simple exercise to show that the series

$$\sum_{j,l>0} 2^{(2j+l-2\max\{j,l\})(1-\theta)-2\theta\max\{j,l\}}$$

converges for $1/(3\theta) < 1$. ∎

Since $6/5 < 3/2$, this shows, as was to be expected, that the crucial part of the maximal operator comes from $m\eta$, i.e. the restriction of the multiplier to a neighborhood of the cone C.

We recall that η is homogeneous of degree 0, but since the part of the multiplier supported near the origin has already been dealt with, we can modify it by making it equal to 0 near the origin. In order to analyse this part, we decompose dyadically the cut-off function η by introducing another cut-off function, depending this time on $|\xi|$, so that η can be written as $\sum_{j\geq 0}\eta_j$, where $\eta_j(\xi) = \eta_0(2^{-j}\xi)$ and η_0 is supported where $1/2 \leq |\xi| \leq 3$.

We also introduce a dyadic splitting of the integral in ds in the definition (2.4) of m. This is obtained by means of the cut-off function $\widetilde{\psi}(s) = \psi(s) - \psi(2s)$, which is supported away from 0 and is such that $\psi(s) = \sum_{l\geq 0}\widetilde{\psi}(2^l s)$.

After this double decomposition, $m(\xi)\eta(\xi) = \sum_{j,l\geq 0} m_{jl}(\xi)$, where

$$m_{jl}(\xi) = \eta_j(\xi)\iint e^{-2\pi i(\xi_1 rs + \xi_2 r(1+\gamma(s)) + \xi_3 r^d)}\varphi(r)\widetilde{\psi}(2^l s)\,dr\,ds. \tag{2.7}$$

We call M_{jl} the maximal operator corresponding to m_{jl} and apply the following result, which is essentially Lemma 2.1 of [MR].

LEMMA 2.3. *Assume that m is a smooth multiplier supported on the set where $|\xi'| \sim |\xi_3| \sim 2^j$ and satisfying the condition*

$$\int\left(|m(\xi',\xi_3)|^2 + \left|\frac{\partial m}{\partial\xi_3}(\xi',\xi_3)\right|^2\right)\frac{d\xi_3}{|\xi_3|} \leq A^2|\xi'|^{-2}. \tag{2.8}$$

Then the maximal operator

$$Mf(x) = \sup_{i\in\mathbb{Z},\, b>0}|\mathcal{F}^{-1}(m(2^i\xi', b\xi_3)\widehat{f}(\xi))(x)|$$

is bounded on L^2 and $\|Mf\|_{2,2} \leq CA2^{-j/2}$.

At this point we have to consider $d \neq 1$ and $d = 1$ separately. In both cases we shall obtain the estimate

$$\|M_{jl}\|_{2,2} \leq C\min\{2^{l(k-2)/2-j/2}, 2^{-l}\}, \tag{2.9}$$

where k is the type of the curve γ.

In the case $d \neq 1$, (2.8) follows from the pointwise estimate

$$\left|\iint e^{-2\pi i(\xi_1 rs + \xi_2 r(1+\gamma(s)) + \xi_3 r^d)}\varphi(r)\widetilde{\psi}(2^l s)\,dr\,ds\right| \leq C\min\{2^{l(k-2)/2}/|\xi|, 2^{-l}/|\xi_3|^{1/2}\},$$

together with a similar estimate on the derivative in ξ_3.

The first quantity on the right-hand side comes from the standard stationary phase estimate on the double integral, since the Hessian of the phase function is controlled from below by a constant times $|\gamma''(s)| \sim 2^{-l(k-2)}$. The other quantity is obtained by integrating first in r, which gives us a bound of the order of $|\xi_3|^{-1/2}$, again by standard oscillatory integral methods, and then by integrating in s over an interval of length 2^{-l}.

At this point (2.9) follows directly from Lemma 2.3.

In the case $d = 1$, (2.8) does not come from a pointwise estimate. We have to use Plancherel's theorem in the variable ξ_3 instead, as in the proof of Theorem 2.2 in [MR]. Bearing in mind that $\varphi(t)$ cuts off to about $t \sim 1$, we have

$$\begin{aligned}
\int|m_{jl}(\xi',\xi_3)|^2\frac{d\xi_3}{|\xi_3|} &\leq \frac{C}{|\xi'|}\int\left|\iint e^{-2\pi i(\xi_1 rs + \xi_2 r(1+\gamma(s)) + \xi_3 r)}\varphi(r)\widetilde{\psi}(2^l s)\,dr\,ds\right|^2 d\xi_3\\
&= \frac{C}{|\xi'|}\int\left|\int\widehat{\varphi}(\xi_1 s + \xi_2(1+\gamma(s)) + \xi_3)\widetilde{\psi}(2^l s)\,ds\right|^2 d\xi_3\\
&= \frac{C}{|\xi'|}\int|\varphi(t)|^2\left|\int e^{-2\pi it(\xi_1 s + \xi_2(1+\gamma(s)))}\widetilde{\psi}(2^l s)\,ds\right|^2 dt\\
&\leq \frac{C}{|\xi'|}\min\left\{\frac{2^{l(k-2)}}{|\xi'|}, 2^{-2l}\right\}.
\end{aligned}$$

The first of the last two estimates follows by van der Corput's Lemma, the second by the trivial majorisation by the size of the interval of integration. A similar estimate holds for $\partial m_{jl}/\partial\xi_3$. Again, Lemma 2.3 gives (2.9) immediately.

We now wish to interpolate the L^2 estimate (2.9) with a restricted weak-type estimate at $p = 1$. We write $M_{j,l}$ as a convolution operator:

$$M_{j,l}f(x) = \sup_{i\in\mathbb{Z},\, b>0}\left|2^{-2i}b^{-1}\int K_{j,l}(2^{-i}(x'-y'), b^{-1}(x_3-y_3))f(y)\,dy\right|,$$

where $K_{j,l}(x) = \mathcal{F}^{-1}(m_{j,l})(x)$. By (2.7), $K_{j,l} = \mu_l * (\mathcal{F}^{-1}\eta_j)$, where μ_l is a positive measure, supported on the part of the surface corresponding to $r \sim 1$ and $s \sim 2^{-l}$ and of total mass of the order of 2^{-l}.

As $\mathcal{F}^{-1}\eta_j$ is a Schwartz function and $(\mathcal{F}^{-1}\eta_j)(x) = 2^{3j}(\mathcal{F}^{-1}\eta_0)(2^j x)$, we can write

$$|\mathcal{F}^{-1}\eta_j| \leq \sum_{\nu\geq 0}\lambda_\nu\widetilde{\chi}_{\nu-j},$$

where $\{\lambda_\nu\}$ is a rapidly decreasing sequence of positive numbers and $\widetilde{\chi}_\nu$ is the normalised characteristic function of the ball of radius 2^ν. Correspondingly,

$$(2.10)\quad M_{j,l}f(x) \le \sum_{\nu\ge 0}\lambda_\nu \sup_{i\in\mathbb{Z},\, b>0}\Big|2^{-2i}b^{-1}\int(\mu_l*\widetilde{\chi}_{\nu-j})(2^{-i}(x'-y'),b^{-1}(x_3-y_3))f(y)\,dy\Big|.$$

We estimate the size and support of the convolutions $\mu_l*\widetilde{\chi}_{\nu-j}$, in order to compare each summand in (2.10) with the strong maximal function.

If $\nu-j\le -l$, the convolution is supported in $|x_1|<2^{-l}$, $|x_2|<1$, $|x_3|<1$ and its size is controlled by $2^{-\nu+j}$. The corresponding maximal operator is then essentially dominated by $\lambda_\nu 2^{-\nu+j-l}$ times the strong maximal function.

If $-l<\nu-j<0$, the convolution is supported in $|x_1|<2^{\nu-j}$, $|x_2|<1$, $|x_3|<1$ and its size is controlled by $2^{-2(\nu-j)-l}$. The corresponding maximal operator is again dominated by $\lambda_\nu 2^{-\nu+j-l}$ times the strong maximal function.

Finally, if $\nu-j\ge 0$, the convolution is supported in $|x|<2^{\nu-j}$ and its size is controlled by $2^{-3(\nu-j)-l}$. So the corresponding maximal operator is dominated by $\lambda_\nu 2^{-l}$ times the strong maximal function.

Summing over ν we can conclude that M_{jl} is of restricted weak type $(1,1)$ with "norm" controlled by

$$C\Big(2^{j-l}\sum_{\nu<j}\lambda_\nu 2^{-\nu}+2^{-l}\sum_{\nu\ge j}\lambda_\nu\Big)\le C2^{j-l}.$$

Interpolating the above estimates at $1/p=(1-\theta)+\theta/2$, we now want to sum in j,l. Because of (2.9), the estimate of the L^p norm leads to two separate sums:

$$\sum_{j,l\ge 0}\|M_{jl}\|_{p,p}\le C\sum_{j\le lk}2^{(j-l)(1-\theta)-l\theta} + C\sum_{lk<j}2^{(j-l)(1-\theta)+(l(k-2)/2-j/2)\theta}.$$

The first sum converges if $\theta>(k-1)/k$, i.e. $p>2k/(k+1)$, and the second converges if $\theta>2/3$, i.e. if $p>3/2$, and $\theta>(k-1)/k$. This gives the required result.

REMARK. The proof of Theorem 2.1 can be adapted to certain flat surfaces, i.e. surfaces defined by (2.2) with γ vanishing of infinite order at $s=0$. We consider here the special case $\gamma(s)=e^{-|s|^{-\sigma}}$ with $\sigma>0$.

If $|s|\sim 2^{-l}$, then $|\gamma''(s)|\ge C2^{l(2\sigma+2)}e^{-c2^{l\sigma}}$ for some $c>0$. It follows that, if M_{jl} is as above,

$$\|M_{jl}\|_{2,2}\le C\min\{2^{-l(\sigma+1)-j/2}e^{(c/2)2^{l\sigma}},2^{-l}\}.$$

We have

$$\sum_{j,l\ge 0}\|M_{jl}\|_{2,2}\le C\sum_{j<c2^{l\sigma}-2l\sigma}2^{-l} + C\sum_{j\ge c2^{l\sigma}-2l\sigma}2^{-l(\sigma+1)-j/2}e^{(c/2)2^{l\sigma}}.$$

If $0<\sigma<1$, both sums converge. The other arguments used in the proof of Theorem 2.1 for what concerns the L^2-boundedness apply with essentially no change. We can then conclude that M is bounded if and only if $p\ge 2$.

References

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