the existence of a set $B_1 \in \mathrm{Ba}(X^{**}, w^*)$ such that $X^{**} \setminus B_1$ is negligible and for every $x^{**} \in B_1$, there exists a point $y \in \varphi(\Omega)$ such that $Sx^{**} = Sy$, which implies $Tx^{**} = Ty$ and $x_0^*(x^{**}) = x_0^*(y)$. If in addition $x^{**} \in B_0$, then $y = \psi(x^{**})$ and $x_0^*(x^{**}) = x_0^*(y)$, so we have proved $x_0^* \circ \psi(x^{**}) = x_0^*(x^{**})$ for every $x^{**} \in B_0 \cap B_1$. As $X^{**} \setminus (B_1 \cap B_0)$ is μ_F-negligible we have finished. ■

References

[AD] R. Anantharaman and J. Diestel, *Sequences in the range of a vector measure*, Comment. Math. Prace Mat. 30 (1991), 221–235.

[C] C. H. Choquet, *Lectures on Analysis, Vols. I, II, III*, Benjamin, New York, 1969.

[DJT] J. Diestel, H. Jarchow and A. Tonge, *Absolutely Summing Operators*, Cambridge Stud. Adv. Math. 43, Cambridge Univ. Press, 1995.

[DU] J. Diestel and J. J. Uhl, Jr., *Vector Measures*, Math. Surveys 15, Amer. Math. Soc. Providence, R.I., 1977.

[E] G. Edgar, *Measurability in a Banach space*, Indiana Univ. Math. J. 26 (1977), 663–677.

[FT] D. Fremlin and M. Talagrand, *A decomposition theorem for additive set functions and applications to Pettis integral and ergodic means*, Math. Z. 168 (1979), 117–142.

[K] I. Kluvánek, *Characterization of the closed convex hull of the range of a vector measure*, J. Funct. Anal. 21 (1976), 316–329.

[L] D. R. Lewis, *On integrability and summability in vector spaces*, Illinois J. Math. 16 (1972), 294–307.

[M] K. Musiał, *The weak Radon–Nikodym property in Banach spaces*, Studia Math. 64 (1979), 151–174.

[P] A. Pietsch, *Operator Ideals*, North-Holland, Amsterdam, 1980.

[R1] L. Rodríguez-Piazza, *The range of a vector measure determines its total variation*, Proc. Amer. Math. Soc. 111 (1991), 205–214.

[R2] —, *Derivability, variation and range of a vector measure*, Studia Math. 112 (1995), 165–187.

[T] M. Talagrand, *Pettis integral and measure theory*, Mem. Amer. Math. Soc. 307 (1984).

[Th] E. Thomas, *Integral representations in convex cones*, Groningen University Report ZW-7703 (1977).

Departamento de Análisis Matemático
Facultad de Matemáticas
Universidad de Sevilla
Aptdo. 1160
Sevilla 41080, Spain
E-mail: piazza@cica.es
mcromero@cica.es

Received November 6, 1996 (3771)

STUDIA MATHEMATICA 125 (3) (1997)

Best constants and asymptotics of Marcinkiewicz–Zygmund inequalities

by

ANDREAS DEFANT (Oldenburg) and MARIUS JUNGE (Kiel)

Abstract. We determine the set of all triples $1 \le p, q, r \le \infty$ for which the so-called *Marcinkiewicz–Zygmund inequality* is satisfied: There exists a constant $c \ge 0$ such that for each bounded linear operator $T : L_q(\mu) \to L_p(\nu)$, each $n \in \mathbb{N}$ and functions $f_1, \ldots, f_n \in L_q(\mu)$,

$$\Big(\int \Big(\sum_{k=1}^n |Tf_k|^r\Big)^{p/r} d\nu\Big)^{1/p} \le c\|T\| \Big(\int \Big(\sum_{k=1}^n |f_k|^r\Big)^{q/r} d\mu\Big)^{1/q}.$$

This type of inequality includes as special cases well-known inequalities of Paley, Marcinkiewicz, Zygmund, Grothendieck, and Kwapień. If such a Marcinkiewicz–Zygmund inequality holds for a given triple (p, q, r), then we calculate the best constant $c \ge 0$ (with the only exception: the important case $1 \le p < r = 2 < q \le \infty$); if such an inequality does not hold, then we give asymptotically optimal estimates for the graduation of these constants in n. Two problems of Gasch and Maligranda from [9] are solved; as a by-product we obtain best constants of several important inequalities from the theory of summing operators.

0. Introduction. Fix a triple (p, q, r) of scalars with $1 \le p, q, r \le \infty$. We call—for the purpose of this paper—an inequality of the following type a *Marcinkiewicz–Zygmund inequality*: There is a constant $c \ge 0$ (depending on p, q and r only) such that for each (linear and continuous) operator $T : L_q(\mu) \to L_p(\nu)$ (μ and ν arbitrary measures) and arbitrarily many functions $f_1, \ldots, f_n \in L_q(\mu)$,

$$\text{(MZ)} \qquad \Big(\int \Big(\sum_{k=1}^n |Tf_k|^r\Big)^{p/r} d\nu\Big)^{1/p} \le c\|T\| \Big(\int \Big(\sum_{k=1}^n |f_k|^r\Big)^{q/r} d\mu\Big)^{1/q}.$$

By a density and closed graph argument it is equivalent to say that each operator $T : L_q(\mu) \to L_p(\nu)$ allows an ℓ_r-valued extension, i.e. there is an operator

$$T^{\ell_r} : L_q(\mu, \ell_r) \to L_p(\nu, \ell_r)$$

1991 *Mathematics Subject Classification*: Primary 46B07, 47B10; Secondary 42B25.

such that

$$T^{\ell_r}(f \otimes x) = Tf \otimes x \quad \text{for all } f \in L_q(\mu),\ x \in \ell_r.$$

In 1932 Paley [24] showed that such an inequality holds true for $p = q$ and $r = 2$, and—together with Littlewood—he gave deep applications of this fact in what is nowadays called "Littlewood–Paley theory" (see e.g. [7]).

For $p, q, r \in [1, \infty]$ define

$$k_{q,p}(r) := \inf c \in [1, \infty]$$

to be the infimum of all $c \geq 0$ such that (MZ) holds, and for $n \in \mathbb{N}$ let

$$k_{q,p}^{(n)}(r) \in [1, \infty[$$

be its finite-dimensional graduation, i.e. with the infimum taken over all $c \geq 0$ which satisfy (MZ) for each T but for only n functions f_k. Then $k_{q,p}(r) < \infty$ means that the triple (p, q, r) satisfies the Marcinkiewicz–Zygmund inequality (for some constant $c \geq 0$).

Motivated by Paley's result, in 1939 Marcinkiewicz and Zygmund [21] proved the following:

(1) $k_{p,p}(2) = 1$ for $1 \leq p \leq \infty$,

(2) $k_{q,p}(2) \leq \dfrac{c_{2,q}}{c_{2,1}} < \infty$ for $1 \leq p, q < \infty$,

(3) $k_{q,p}(r) \leq \dfrac{c_{r,q}}{c_{r,1}} < \infty$ for $1 \leq \max(p,q) < r < 2$

([21, Thm. 1 and Thm. 3 (9), (10)]); here $c_{2,q}$ is the qth moment of the Gauss measure on $\mathbb{R}$, and $c_{r,q}$ the qth moment of the so-called r-stable Lévy measure on $\mathbb{R}$ (see Section 2). Proving his celebrated "théorème fondamental de la théorie métrique des produits tensoriels" Grothendieck [11] in 1956 added the important case

(4) $k_{\infty,1}(2) < \infty$;

see e.g. [5], [6], [15], and [16] for estimates of the so-called Grothendieck constant $K_G := k_{\infty,1}(2)$ and its n-dimensional analogue $K_G^{(n)} := k_{\infty,1}^{(n)}(2)$.

The aim of this paper is to determine the set of all triples (p, q, r) such that $k_{q,p}(r) < \infty$. More precisely, we calculate $k_{q,p}(r)$ whenever this constant is finite (except for the important case $1 \leq p < r = 2 < q \leq \infty$), and give the precise asymptotic growth of $k_{q,p}^{(n)}(r)$ in terms of n whenever $k_{q,p}(r) = \lim_n k_{q,p}^{(n)}(r) = \infty$; this way we answer several problems of Gasch and Maligranda who in [9] gave an up-to-date survey of the present topic. For estimates of $k_{q,p}^{(2)}(2)$, the so-called complexification constants of operators in L_p-spaces, see [4], [5], [9], [16], and [28].

Our method is to relate the constants $k_{q,p}(r)$ and $k_{q,p}^{(n)}(r)$ with some useful invariants from local Banach space theory—e.g. the stable type (q, p) of ℓ_r and ℓ_r^n, and certain quotient norms of the identity on ℓ_r and ℓ_r^n with respect to s-integral and s-summing norms. As a by-product we obtain the best constants for some useful inequalities from the theory of p-summing operators due to Kwapień [17] and Saphar [27]. The main results are given in Sections 4–6, whereas Sections 1–3 have a preparatory character.

Most of our notation is standard—we use [5], [6], and [26] as general references for Banach spaces, operator ideals and tensor products, and [5], [19], and [26] for all information needed on p-stable random variables. All Banach spaces are real (although most of our results can be easily extended to the complex case). An "operator" means a linear and continuous operator between Banach spaces.

1. Characterizations of Marcinkiewicz–Zygmund inequalities via s-integral and s-summing norms. In this section we collect some abstract formulations of the above inequalities (most of which in a more general context can be found in [5]).

Denote by Δ_p the natural norm on $L_p(\mu) \otimes E$ ($1 \leq p \leq \infty$, μ an arbitrary measure and E a Banach space) induced by the embedding of this tensor product in the space $L_p(\mu, E)$ of all Bochner p-integrable functions f with values in E.

For $1 \leq p, q \leq \infty$ define

$$k_{q,p}(E) := \sup \|T \otimes \mathrm{id}_E : L_q(\mu) \otimes_{\Delta_q} E \to L_p(\nu) \otimes_{\Delta_p} E\| \in [0, \infty],$$

the supremum taken over all operators $T : L_p(\mu) \to L_p(\nu)$ with norm ≤ 1. It is known (see [5, 29.12]) that this supremum does not change if it is only taken with respect to two fixed measures μ_0 and ν_0 such that $L_q(\mu_0)$ and $L_p(\nu_0)$ are infinite-dimensional—in particular, with respect to ℓ_q and ℓ_p. Obviously, for all n,

$$k_{q,p}(\ell_r^n) = k_{q,p}^{(n)}(r),$$

and an easy density argument yields

$$k_{q,p}(\ell_r) = k_{q,p}(r) = \lim_n k_{q,p}(\ell_r^n) = \lim_n k_{q,p}^{(n)}(r).$$

Note that whenever $q = 1$ or $p = \infty$, then for every non-trivial E,

$$k_{q,p}(E) = 1,$$

since $\Delta_1 = \pi$ (the projective norm) and $\Delta_\infty = \varepsilon$ (the injective norm). The constants $k_{q,p}(E)$ are increasing in q and decreasing in p:

$$k_{q_1,p_1}(E) \leq k_{q_2,p_2}(E) \quad \text{whenever} \quad q_1 \leq q_2,\ p_2 \leq p_1$$

(see [9, Thm. 1] and [5, Ex. 28.14], and for an elementary direct proof [28]). Moreover, we mention the following obvious duality result which will be used frequently:

$$k_{q,p}(E) = k_{p',q'}(E').$$

Recall from [5], [6] or [26] the notion of s-integral and s-summing operators $T : E \to F$ between two Banach spaces; here we write $i_s(T)$ and $\pi_s(T)$ for the s-integral and s-summing norm of T, respectively. The following quotient formula for $k_{q,p}(E)$ in terms of s-integral and s-summing norms will be useful; its proof is a direct consequence of [5, 29.12 Lemma], the trace formula $\Delta_q \otimes_{L_q} \varepsilon \otimes_{L_p} \Delta^t_{p'} = d_q \otimes g_{p'}$ from the proof of [5, 29.12 Corollary], and the abstract quotient formula from [5, 25.7].

PROPOSITION. *For every Banach space E and $1 \le p, q \le \infty$,*

$$k_{q,p}(E) = \sup \pi_{q'}(T'),$$

the supremum taken over all Banach spaces X and all operators $T : E \to X$ with $i_{p'}(T) \le 1$.

For a modification of this characterization with an interesting geometric application see also [14]; for example, for Banach lattices E the Proposition combined with [14, Prop. 1.3] yields that

$$k_{q,p}(E) \le K^{q'}(E)\, K_{p'}(E),$$

where $K^{q'}(E)$ (resp. $K_{p'}(E)$) denotes the q'-convexity (resp. p'-concavity) constant of E.

In the case $p \le q$ there are two important corollaries: the first one is a reformulation of a result of Kwapień [18] (here an immediate consequence of the Proposition and [18, Corollary 8] (see also [5, 25.9 Corollary])):

COROLLARY 1. *For every Banach space E and $1 \le p \le \infty$,*

$$k_{p,p}(E) = \inf \|R\| \cdot \|S\|,$$

where the infimum is taken with respect to all subspaces G of quotients of $L_p(\mu)$'s ($=$ all quotients of subspaces of $\ldots$), and all operators $R : E \to G$, $S : G \to E$ such that $\mathrm{id}_E = SR$. In particular, E is isomorphic to a subspace of a quotient of some $L_p(\mu)$ iff $k_{p,p}(E) < \infty$.

For $p = 2$ this is a well-known result of [20]; an analogue for $p < q$ can be found in [5, 28.4] and reads as follows:

COROLLARY 2. *For every Banach space E and $1 \le p < q \le \infty$,*

$$k_{q,p}(E) = \inf \|R\| \cdot \|S\|,$$

the infimum taken over all factorizations

$$\begin{array}{ccc} E & \xrightarrow{\ \mathrm{id}_E\ } & E \\ \downarrow R & & \uparrow S \\ M/N & \xrightarrow{\ \pi\ } & L/K \end{array}$$

where $M, N \subset L_q(\mu)$ and $L, K \subset L_p(\mu)$ (μ a probability measure) are closed subspaces satisfying

$$\begin{array}{ccccc} N & \subset & M & \subset & L_q(\mu) \\ \cap & & \cap & & \cap \\ K & \subset & L & \subset & L_p(\mu). \end{array}$$

Note that in the language of Banach operator ideals these two corollaries read as follows (see again [5, 28.4]): For $1 \le p \le q \le \infty$,

$$k_{q,p}(E) = \mathbf{L}^{\mathrm{inj\,sur}}_{p,q'}(\mathrm{id}_E),$$

the norm taken in the injective and surjective hull of the ideal $\mathcal{L}_{p,q'}$ of all (p, q')-factorable operators.

We finish with some remarks which will also be needed later.

Remark. Let E be a Banach space. Then for every $1 \le p, q \le \infty$,

$$k_{q,p}(E) \le K_G\, k_{2,2}(E),$$

and for $1 \le p \le 2 \le q \le \infty$ even

$$k_{2,2}(E) \le k_{q,p}(E) \le K_G\, k_{2,2}(E).$$

The first inequality is a special case of [5, 26.3 Prop. 1], and the second then follows by monotonicity.

Using Corollary 1 it can be easily seen that $k_{2,2}(E)$ for an n-dimensional space E is nothing but the Banach–Mazur distance $d(E, \ell^n_2)$; moreover, recall that $d(\ell^n_r, \ell^n_2) = n^{|1/2-1/r|}$ (see e.g. [5, p. 360]).

2. Stable measures. For the discussion of Marcinkiewicz–Zygmund inequalities in the cases $1 \le p \le q \le 2$ and $2 \le p \le q \le \infty$ the so-called stable measures are essential; this deep observation was first made in [21].

For $1 \le r \le 2$ let μ^1_r be the unique probability measure on $\mathbb{R}$ having as its Fourier transform the positive definite function $e^{-|\cdot|^r}$ (see e.g. [5, Sec. 24]). It is well known that for $1 \le p < r \le 2$ the pth moment

$$c_{r,p} := \Big(\int_{\mathbb{R}} |x|^p \, d\mu^1_r(x) \Big)^{1/p}$$

exists; see [26] for the following formula:

$$c_{r,p} = 2 \left[\frac{\Gamma\left(\frac{r-p}{r}\right)}{\Gamma\left(\frac{2-p}{2}\right)} \, \frac{\Gamma\left(\frac{1+p}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \right]^{1/p}.$$

Clearly, μ_2^1 is the Gauss measure (up to normalization), and in this case the pth moment exists for all $1 \le p < \infty$:

$$c_{2,p} = 2\left[\frac{\Gamma(\frac{1+p}{2})}{\Gamma(\frac{1}{2})}\right]^{1/p}.$$

The n-fold product μ_r^n of μ_r^1 (sometimes called the r-stable Lévy measure on $\mathbb{R}^n$) has the following fundamental stability property: For each $\alpha \in \mathbb{R}^n$,

$$c_{r,p}\|\alpha\|_{\ell_r^n} = \Big(\int_{\mathbb{R}^n} \Big|\sum_{k=1}^n h_k\alpha_k\Big|^p \, d\mu_r^n\Big)^{1/p},$$

where $h_k : \mathbb{R}^n \to \mathbb{R}$ is the kth projection; for $r = 2$ this equality holds for all $1 \le p < \infty$. If μ_r denotes the countable product measure of μ_r^1 on $\mathbb{R}^{\mathbb{N}}$, then the above stability property is equivalent to the fact that the mapping

$$I_{r,p} : \ell_r \hookrightarrow L_p(\mu_r, \mathbb{R}^{\mathbb{N}}), \quad I_{r,p}(e_k) := \frac{1}{c_{r,p}} h_k,$$

is a well-defined isometry (h_k again the kth projection). Clearly, $I_{2,p} : \ell_2 \hookrightarrow L_p(\mu_2)$ is a well-defined isometry for all $1 \le p < \infty$ (and not only $1 \le p \le 2$).

Recall that for $1 \le p < q \le 2$ a Banach space E is said to be *of stable type* (q,p) if there is a constant $c \ge 0$ such that for each set of finitely many elements $x_1, \ldots, x_n \in E$,

$$\frac{1}{c_{q,p}}\Big(\int_{\mathbb{R}^n} \Big\|\sum_{k=1}^n h_k x_k\Big\|_E^p \, d\mu_q^n\Big)^{1/p} \le c\Big(\sum_{k=1}^n \|x_k\|_E^q\Big)^{1/q},$$

in other words,

$$\mathbf{ST}_{q,p}(E) := \|I_{q,p} \otimes \mathrm{id}_E : \ell_q \otimes_{\Delta_q} E \to L_p(\mu_q) \otimes_{\Delta_p} E\| < \infty.$$

For $1 \le q \le 2$ and $1 \le p < \infty$ a Banach space E is said to have *Gauss type* (q,p) whenever

$$\mathbf{T}_{q,p}(E) := \|I_{2,p} j_{q,2} : \ell_q \otimes_{\Delta_q} E \to L_p(\mu_2) \otimes_{\Delta_p} E\| < \infty,$$

where $j_{q,2} : \ell_q \hookrightarrow \ell_2$ is the canonical embedding. For our purpose the following trivial estimates will be crucial:

Remark. For every Banach space E and $1 \le p < q \le 2$,

$$\mathbf{ST}_{q,p}(E) \le k_{q,p}(E),$$

and for $1 \le p, q < \infty$,

$$\mathbf{T}_{\min(2,q),p}(E) \le k_{\min(2,q),p}(E) \le k_{q,p}(E).$$

The class of all Banach spaces which are of stable type (q,p) (resp. Gauss type (q,p)) is actually independent of p: By a result of Hoffmann-Jørgensen [13] for $1 \le p_1 \le p_2 < q \le 2$ (resp. $q = 2$ and $1 \le p_1 \le p_2 < \infty$) there is a constant $c_{q,p_1,p_2} > 0$ such that for every Banach space E,

$$\Big(\int_{\mathbb{R}^n} \Big\|\sum_{k=1}^n h_k x_k\Big\|_E^{p_2} \, d\mu_q^n\Big)^{1/p_2} \le c_{q,p_1,p_2}\Big(\int_{\mathbb{R}^n} \Big\|\sum_{k=1}^n h_k x_k\Big\|_E^{p_1} \, d\mu_q^n\Big)^{1/p_1}$$

for all n and $x_1, \ldots, x_n \in E$. It is well known that every Banach space E of Gauss type q ($1 \le q \le 2$) has stable type s for all $s < q$, and each E with stable type q has Gauss type q (see e.g. [5, 24.8]). Moreover, we recall that by a result of Maurey and Pisier [23] the Banach spaces E with stable type q are precisely those which do not contain the ℓ_q^n uniformly.

Generalizing the definition of $c_{r,p}$ we define for $n \in \mathbb{N}$ and $1 \le p < r \le 2$,

$$c_{r,p}^{(n)} := \Big(\int_{\mathbb{R}^n} \|x\|_{\ell_r^n}^p \, d\mu_r^n(x)\Big)^{1/p},$$

and recall (see e.g. [1]) that

$$c_{r,p}^{(n)} \asymp (n(1+\log n))^{1/r};$$

as usual we use for positive sequences (a_n) and (b_n) the notation $a_n \asymp b_n$ whenever the ratio a_n/b_n is bounded from above and below by positive constants not depending on n. The following technical lemma will become essential:

LEMMA. *For $1 \le p < r \le 2$ and $1 \le q < r \le 2$,*

$$\lim_n c_{r,p}^{(n)}/c_{r,q}^{(n)} = 1.$$

Proof. Clearly, we may assume that $1 < p$ and $q = 1$. Since $1 \le c_{r,p}^{(n)}/c_{r,1}^{(n)}$ for all n, we have to check that

$$\lim_n c_{r,p}^{(n)}/c_{r,1}^{(n)} \le 1.$$

For the vector-valued random variable $X_n := \sum_{k=1}^n h_k \otimes e_k : (\mathbb{R}^n, \mu_r^n) \to \ell_r^n$ we have

$$c_{r,p}^{(n)} = (\mathbb{E}\|X_n\|^p)^{1/p},$$

and (as just mentioned)

$$\mathbb{E}\|X_n\| \asymp (n(1+\log n))^{1/r}.$$

Define $t_n := (1-\delta_n)^{-1}\mathbb{E}\|X_n\|$, where $[0,1] \ni \delta_n \searrow 0$ will be determined later. Then for large n,

$$\mathbb{E}\|X_n\|^p = \int_0^\infty pt^{p-1}\,\mu_r^n(\|X_n\| \ge t)\,dt \le t_n^p + \int_{t_n}^\infty pt^{p-1}\mu_r^n(\|X_n\| \ge t)\,dt,$$

so that it suffices to show that the sequence (δ_n) can be chosen in such a way that

$$\lim_n \frac{1}{(\mathbb{E}\|X_n\|)^p}\int_{t_n}^{\infty} pt^{p-1}\mu_r^n(\|X_n\|\ge t)\,dt = 0.$$

Since for each n, w and t with $\|X_n(w)\| \ge t > t_n$,

$$\|X_n(w)\| - \mathbb{E}\|X_n\| \ge t - \mathbb{E}\|X_n\| > \delta_n t,$$

we conclude from a result of [10] (see also [19, p. 136, 132]) that for all $t > t_n$,

$$\mu_r^n(\|X_n\|\ge t) \le \mu_r^n(|\,\|X_n\| - \mathbb{E}\|X_n\|\,| > \delta_n t) \le c_r\frac{n}{(\delta_n t)^r},$$

where the constant $c_r > 0$ depends on r only. Therefore

$$\begin{aligned}\int_{t_n}^{\infty} pt^{p-1}\mu_r^n(\|X_n\|\ge t)\,dt &\le c_r p n\delta_n^{-r}\int_{t_n}^{\infty} t^{p-r-1}\,dt\\ &= c_r p n\delta_n^{-r}\frac{1}{r-p}t_n^{p-r}\\ &= c_r n\delta_n^{-r}\frac{p}{r-p}[(1-\delta_n)^{-1}\mathbb{E}\|X_n\|]^{p-r}\\ &\asymp \delta_n^{-r}n^{p/r}(1+\log n)^{(p-r)/r},\end{aligned}$$

which implies

$$\frac{1}{(\mathbb{E}\|X_n\|)^p}\int_{t_n}^{\infty} pt^{p-1}\mu_r^n(\|X_n\|\ge t)\,dt \prec \delta_n^{-r}(1+\log n)^{-1}.$$

Hence, if δ_n is (for example) chosen such that $\delta_n^r = (1+\log n)^{-p/r}$, then as desired

$$\frac{1}{(\mathbb{E}\|X_n\|)^p}\int_{t_n}^{\infty} pt^{p-1}\mu_r^n(\|X_n\|\ge t)\,dt \prec (1+\log n)^{p/r-1} \to 0. \ \blacksquare$$

3. Characterizations of Marcinkiewicz–Zygmund inequalities via mixing norms and stable type. For $p, q \in [1,\infty]$ the (q,p)-*mixing norm* of an operator $T : E \to F$ is given by

$$\mathbf{M}_{q,p}(T) := \sup \pi_p(ST) \in [0,\infty],$$

where the supremum is taken over all $S : F \to G$ with $\pi_q(S) \le 1$. We write $\mathbf{M}_{q,p}(E) := \mathbf{M}_{q,p}(\mathrm{id}_E)$.

The following result from [3] (see also [5, 32.3 Corollary]) shows that there is an intimate relation between vector-valued extensions of operators in L_p-spaces and mixing norms.

LEMMA. *Let $T : L_q(\mu) \to F$ and $S : E \to L_p(\nu)$ be operators. Then for all $p, q, s \in [1,\infty]$,*

$$\|T\otimes S : L_q(\mu)\otimes_{\Delta_q} E \to F\otimes_{\Delta_p^t} L_p(\nu)\| \le \mathbf{M}_{s',q'}(T)\mathbf{M}_{s,p}(S'),$$

where Δ_p^t stands for the norm on $F\otimes L_p(\nu)$ defined by

$$F\otimes_{\Delta_p^t} L_p(\nu) := L_p(\nu)\otimes_{\Delta_p} F, \quad y\otimes f \rightsquigarrow f\otimes y.$$

For our investigations of Marcinkiewicz–Zygmund inequalities in the case $1 \le p < q \le 2$ (and dually for $2 \le p < q \le \infty$) the following estimates will be the main abstract tools:

PROPOSITION. *For every Banach space E and $1 \le p < q \le 2$,*

$$\mathbf{M}_{q,p}(E') \le \mathbf{ST}_{q,p}(E) \le k_{q,p}(E),$$

and if E is an isometric subspace of some $L_p(\eta)$, then even equality holds.

In particular (see the preceding section), for each triple (p,q,r) such that $1 \le p < q \le 2$ and $p \le r \le 2$,

$$k_{q,p}(\ell_r^n) = \mathbf{ST}_{q,p}(\ell_r^n) = \mathbf{M}_{q,p}(\ell_{r'}^n) \quad \text{for all } n,$$

an equality which will be used in the next section.

Proof of Proposition. For the first inequality see [26, p. 292]; this result seems to be due to Maurey (implicitly contained in [22]). The second inequality was already mentioned in the remark of the preceding section. Let us check that equality holds whenever E is a subspace of some $L_p(\eta)$: Fix an operator $T : L_q(\mu) \to L_p(\nu)$ and look at

$$L_q(\mu)\otimes_{\Delta_q} E \xrightarrow{T\otimes \mathrm{id}_E} L_p(\nu)\otimes_{\Delta_p} E \overset{\mathrm{id}\otimes j_E}{\hookrightarrow} L_p(\nu)\otimes_{\Delta_p^t} L_p(\eta),$$

$j_E : E \hookrightarrow L_p(\eta)$ the canonical embedding. Then by Fubini's theorem $\mathrm{id}\otimes j_E$ is an isometry, and hence by the Lemma $(s = q)$

$$\begin{aligned}\|T\otimes \mathrm{id}_E\| &= \|T\otimes j_E\,\mathrm{id}_E\| \le \mathbf{M}_{q',q'}(T)\mathbf{M}_{q,p}((j_E\,\mathrm{id}_E)')\\ &\le \|T\|\mathbf{M}_{q,p}(\mathrm{id}_{E'})\|j_E'\| = \|T\|\mathbf{M}_{q,p}(E').\end{aligned}$$

Taking the supremum over all T gives $k_{q,p}(E) \le \mathbf{M}_{q,p}(E')$ as desired. $\blacksquare$

4. Best constants for the cases $1 \le p \le q \le 2$ and $2 \le p \le q \le \infty$. In [4] it was shown that for $1 \le p \le q \le 2$,

$$k_{q,p}^{(n)}(2) = \frac{c_{2,q}}{c_{2,q}^{(n)}}\cdot\frac{c_{2,p}^{(n)}}{c_{2,p}},$$

and since

$$\lim_n n^{1/2}\frac{c_{2,p}}{c_{2,p}^{(n)}} = \frac{c_{2,p}}{c_{2,2}}$$

(see [26, p. 299]), we obtain

$$k_{q,p}(2) = \frac{c_{2,q}}{c_{2,p}}.$$

For the special case $q = 2$ this formula was shown in [9, Thm. 3] and [
p. 377], the case $q = 2$ and $p = 1$ is due to [21, p. 118] (upper bound) an
[11, p. 52] (lower bound):

$$k_{2,1}(2) = \sqrt{\pi/2};$$

note that up to duality this is (by the very definition of 2-summing ope
ators) the so-called Little Grothendieck Theorem (!). The following resu
complements these formulas.

THEOREM. *For all triples* (p, q, r) *which satisfy* $1 \le p \le q \le r = 2$ (
$1 \le p \le q < r < 2$,

$$k_{q,p}(r) = \frac{c_{r,q}}{c_{r,p}} = \left[\frac{\Gamma\left(\frac{r-q}{r}\right)}{\Gamma\left(\frac{2-q}{2}\right)}\,\frac{\Gamma\left(\frac{1+q}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}\right]^{1/q}\left[\frac{\Gamma\left(\frac{2-p}{2}\right)}{\Gamma\left(\frac{r-p}{r}\right)}\,\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1+p}{2}\right)}\right]^{1/p}.$$

In particular (see the preceding proposition),

$$\mathbf{ST}_{q,p}(\ell_r) = \mathbf{M}_{q,p}(\ell_{r'}) = \frac{c_{r,q}}{c_{r,p}},$$

where in the case of $\mathbf{ST}_{q,p}$ *we assume that* $p < q$.

The upper estimate for $k_{q,p}(r)$ is due to [9, Thm. 3] and based on th
techniques from [21] (see also [5, 28.4 and Ex. 28.3]), and for $\mathbf{TS}_{q,p}(\ell_r)$ an
$\mathbf{M}_{q,p}(\ell_{r'})$ to Maurey (see [22, p. 52] and [26, 22.3.6]). It remains to sho
that $c_{r,q}c_{r,p}^{-1}$ serves as a lower bound for $k_{q,p}(r)$.

Proof of Theorem. In view of the preceding proposition, we prov
that

$$c_{r,q}/c_{r,p} \le \mathbf{M}_{q,p}(\ell_{r'}) \quad \text{for } 1 \le p \le q < r < 2.$$

By a result of [25, §5] (for $r = 2$ due to Garling [8])

$$\pi_p(\mathrm{id} : \ell_{r'}^n \to \ell_r^n) = c_{r,p}^{(n)}/c_{r,p},$$

hence

$$\frac{c_{r,p}^{(n)}}{c_{r,p}} \cdot \frac{c_{r,q}}{c_{r,q}^{(n)}} = \frac{\pi_p(\mathrm{id} : \ell_{r'}^n \to \ell_r^n)}{\pi_q(\mathrm{id} : \ell_{r'}^n \to \ell_r^n)} \le \mathbf{M}_{q,p}(\mathrm{id}_{\ell_{r'}^n}).$$

But then the conclusion follows from the Lemma in Section 2, and the fac
that $\mathbf{M}_{q,p}(\ell_{r'}^n)$ tends to $\mathbf{M}_{q,p}(\ell_{r'})$ as n tends to infinity. ■

By $\mathcal{L}_p$-local techniques (see e.g. [5, 23.1]) it can be easily seen that ℓ_r ca
be replaced by any $L_r(\mu)$-space, and in the case of $\mathbf{ST}_{q,p}$ and $k_{q,p}$ even b

any (isometric) subspace of a quotient (= quotient of a subspace) of every $L_r(\mu)$-space. Moreover, the proof shows that for all n,

$$\frac{c_{r,p}^{(n)}}{c_{r,p}} \cdot \frac{c_{r,q}}{c_{r,q}^{(n)}} \le k_{q,p}^{(n)}(r).$$

It would be interesting to know whether or not equality holds (as in the case of $r = 2$).

As a by-product we find that the constants appearing in many important inequalities of the theory of p-summing operators are even best possible; for a whole collection of the inequalities we have in mind see [25]. We give four examples; the first is an obvious reformulation of what was just proved.

COROLLARY. (1) *Let* $1 \le p \le q \le 2$. *Then for* $r = 2$ *or* $2 < r < q'$ *every* q*-summing operator* T *on* ℓ_r *is* p*-summing. More precisely,*

$$\sup\{\pi_p(T) \mid \pi_q(T : \ell_r \to E) \le 1,\ E \text{ Banach space}\} = c_{r',q}/c_{r',p}.$$

(2) *Let* $2 \le q \le \infty$. *Then for* $r = 2$ *or* $2 < r < q$ *every operator* $T : \ell_\infty \to \ell_r$ *is* q*-summing. More precisely,*

$$\sup\{\pi_q(T) \mid \|T : \ell_\infty \to \ell_r\| \le 1\} = c_{r',q'}/c_{r',1}.$$

(3) (2) *holds if* ℓ_∞ *is replaced by* ℓ_1.

(4) *Let* $1 \le p \le q < r < 2$ *or*: $r = 2$ *and* $1 \le p, q < \infty$. *Then each operator* T *from an arbitrary Banach space* E *into* ℓ_r *is* p*-summing whenever its dual is* q*-summing. More precisely,*

$$\sup\{\pi_p(T) \mid \pi_q(T' : \ell_r' \to E') \le 1,\ E \text{ Banach space}\} = c_{r,q}/c_{r,p}.$$

By using different techniques the qualitative part of these results can be extended to much larger classes of spaces (see e.g. [5] or [6]); on the other hand, the formulas for the norms show that at least in an $L_p(\mu)$-setting stable measures seem to be the appropriate tool if one is interested in best constants. The upper estimates are again known: (2) and (4) are due to Kwapień [17], and (3) to Saphar [27] (see also [25, Prop. 4] and [5, 24.6]).

Proof of Corollary. (2) is clear since

$$\mathbf{M}_{q',1}(\ell_r) = \sup\{\pi_q(T) \mid \|T : \ell_\infty \to \ell_r\| \le 1\}$$

(see e.g. [5, 20.19 or 32.2(3)]). Moreover, by the remarks from Section 1 and [5, 28.5(2)],

$$\begin{aligned} k_{q',1}(r') &= \mathbf{L}_{1,q}^{\text{inj sur}}(\mathrm{id}_{\ell_{r'}}) = \mathbf{L}_{q,1}^{\text{inj sur}}(\mathrm{id}_{\ell_r}) \\ &= \pi_q^{\text{sur}}(\mathrm{id}_{\ell_r}) = \sup\{\pi_q(T) \mid \|T : \ell_1 \to \ell_r\| \le 1\}, \end{aligned}$$

which gives (3). Finally, an argument for (4): In the case of $1 \le p \le q < r < 2$ use [5, 25.9 Prop., (2)] in order to show that

$$k_{q,p}(\ell_r) = \mathbf{L}^{\mathrm{inj\,sur}}_{p,q'}(\mathrm{id}_{\ell_r}) \le \mathbf{L}^{\mathrm{inj}}_{p,q'}(\mathrm{id}_{\ell_r}) = \sup\{\pi_p(T) \mid \pi_q(T' : \ell_r' \to E') \le 1\}.$$

The other case follows if the latter equality is combined with [26, 22.1.5]. ■

5. Asymptotics for $1 \le p \le q \le \infty$. The following result gives, for every triple (p,q,r), $1 \le p \le q \le \infty$, the precise asymptotic growth of the Marcinkiewicz–Zygmund constants $k^{(n)}_{q,p}(r)$ in n.

THEOREM. *Let $p, q, r \in [1,\infty]$.*

(1) *For $1 \le p \le 2 \le q \le \infty$,*

$$k^{(n)}_{q,p}(r) \asymp n^{|1/2-1/r|}.$$

(2) *For $1 \le p \le q < 2$,*

$$k^{(n)}_{q,p}(r) \asymp \begin{cases} n^{1/r-1/q}, & r < q, & \text{(a)}\\ n^{\max(0,1/2-1/r)}, & r > q > 1, & \text{(b)}\\ 1, & 1 \le r \le \infty,\ q = 1, & \text{(c)}\\ 1, & r = q,\ p = q, & \text{(d)}\\ (1+\log n)^{1/q}, & r = q,\ p < q. & \text{(e)} \end{cases}$$

By duality the results in (2) *also cover the case $2 < p \le q \le \infty$.*

The following immediate consequence answers problem 3 from [9].

COROLLARY. *Let $1 \le p \le q \le \infty$ and $1 \le r \le \infty$. Then $k_{q,p}(\ell_r) < \infty$ if and only if the triple (p,q,r) belongs to one of the following six cases:*

- $p = q = r$,
- $p = q = 1$ *and* $1 \le r \le \infty$,
- $p = q = \infty$ *and* $1 \le r \le \infty$,
- $1 \le p \le q \le 2$ *and* $q < r \le 2$,
- $2 \le p \le q \le \infty$ *and* $2 \le r < p$,
- $1 \le p \le 2 \le q \le \infty$ *and* $r = 2$.

As pointed out in the introduction the "if-part" of this equivalence (up to duality and nowadays trivial cases) is due to Paley, Marcinkiewicz, Zygmund and Grothendieck.

Proof of Theorem. (1) From the remark made at the end of Section 1 we know that $k^{(n)}_{q,p}(r)$ up to a uniform constant equals the Banach–Mazur distance between ℓ_r^n and ℓ_2^n:

$$k_{q,p}(r) \asymp d(\ell_r^n, \ell_2^n) = n^{|1/2-1/r|}.$$

(2) For the proof of (2a) recall the remark in Section 2:

$$\mathbf{ST}_{q,p}(\ell_r^n) \le k_{q,p}(\ell_r^n).$$

Since $\mathbf{ST}_{q,p}$ (up to constants) equals $\mathbf{ST}_{q,r}$, and

$$n^{1/r-1/q} \prec \mathbf{ST}_{q,r}(\ell_r^n)$$

(insert the unit vectors e_k in the definition), the desired lower bound follows.

Upper bound: For $p \le r$ we follow the proposition of Section 3 and estimate $\mathbf{ST}_{q,p}(\ell_r^n)$ from above. The result is then a consequence of the following chain of inequalities (using the fact that $\mathbf{ST}_{q,p}(\ell_r^n) \asymp \mathbf{ST}_{q,r}(\ell_r^n)$, the stability of the measures μ_q^m and Hölder's inequality): For $x_1, \dots, x_m \in \ell_r^n$,

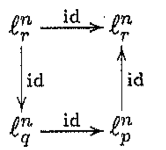
$$\Big(\int \Big\| \sum_{k=1}^m h_k x_k \Big\|_{\ell_r^n}^r d\mu_q^m\Big)^{1/r}$$

$$= \Big(\sum_{j=1}^n \int \Big| \sum_{k=1}^m h_k x_k(j)\Big|^r d\mu_q^m\Big)^{1/r} = \Big(\sum_{j=1}^n c_{q,r}^r \Big(\sum_{k=1}^m |x_k(j)|^q\Big)^{r/q}\Big)^{1/r}$$

$$\le c_{q,r} n^{1/r-1/q} \Big(\sum_{j=1}^n \sum_{k=1}^m |x_k(j)|^q\Big)^{1/q}$$

$$\le c_{q,r} n^{1/r-1/q} \Big(\sum_{k=1}^m \Big(\sum_{j=1}^n |x_k(j)|^r\Big)^{q/r}\Big)^{1/q} = c_{q,r} n^{1/r-1/q} \Big(\sum_{k=1}^m \|x_k\|_{\ell_r^n}^q\Big)^{1/q}.$$

For $r < p$ we consider the factorization



and obtain, from Corollary 2 of Section 1,

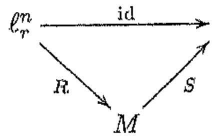
$$k_{q,p}(r) \le n^{1/p-1/q}\, n^{1/r-1/p} = n^{1/r-1/q}.$$

(2b) For $r \le 2$ the result was already stated in the Theorem of Section 4. Assume that $2 < r \le \infty$. Then the upper estimate comes from

$$k^{(n)}_{q,p}(r) \le K_G k^{(n)}_{2,2}(r) = K_G n^{1/2-1/r}$$

(see again the remark made at the end of Section 1).

Lower estimate: There is a subspace M of a quotient of some $L_q(\mu)$ and a factorization



$$\|S\| \cdot \|R\| \le 2k_{q,q}(\ell_r^n)$$

(Kwapień's characterization from Section 1). Moreover, by Pisier's factorization theorem (see e.g. [5, 31.4]) R factors through a Hilbert space with control of the norm:

$$\begin{array}{ccc} \ell_r^n & \xrightarrow{R} & M \\ & {}_U\searrow \quad \nearrow_V & \\ & H & \end{array} \qquad \|V\|\cdot\|U\| \le (2\mathbf{C}_2(\ell_{r'}^n)\mathbf{C}_2(M))^{3/2}\|R\|,$$

where $\mathbf{C}_2(\cdot)$ stands for the Gauss cotype 2 constant. Then $(\ldots)^{3/2}$ can be estimated by a constant c depending on r and q only (and not on n) (see e.g. [6, Sections 11 and 13, in particular Corollary 13.18]), and hence as desired

$$\begin{aligned} n^{1/2-1/r} = k_{2,2}(\mathrm{id}_{\ell_r^n}) &\le \|SV\|\cdot\|U\| \\ &\le \|S\|c\|R\| \le 2ck_{q,q}(\ell_r^n) \le 2ck_{q,p}(\ell_r^n). \end{aligned}$$

Since (2c) and (2d) are trivial (see Section 1), it remains to prove (2e): According to the Proposition of Section 3 we estimate $\mathbf{M}_{q,p}(\ell_{r'}^n)$. From Proposition 3 of [2] we know that

$$\mathbf{M}_{q,p}(\ell_{q'}^n) \le c(1+\log n)^{1/q}\pi_{s,p}(\mathrm{id}_{\ell_{q'}^n}),$$

where $c \ge 0$ is a universal constant, $1/q+1/s = 1/p$ and $\pi_{s,p}$ stands for the (s,p)-summing norm. Moreover,

$$\pi_{s,p}(\mathrm{id}_{\ell_{q'}^n}) = \pi_{q',1}(\mathrm{id}_{\ell_{q'}^n}) = 1$$

(see e.g. [5, Ex. 11.21 and 24.7] or [6]), which gives

$$\mathbf{M}_{q,p}(\ell_{q'}^n) \prec (1+\log n)^{1/q}.$$

For the converse of this inequality, see e.g. [26, p. 306] or [2]. This completes the proof of the Theorem. ∎

Part (2e) shows that for $p < q < 2$ and $r = q$ a Marcinkiewicz–Zygmund inequality "almost" holds: For $1 \le p < q < 2$ there is a constant $c \ge 0$ such that for all operators $T : L_q(\mu) \to L_p(\nu)$ and n functions $f_1, \ldots, f_n \in L_q(\mu)$,

$$\Big(\int\Big(\sum_{k=1}^n |Tf_k|^q\Big)^{p/q} d\nu\Big)^{1/p} \le c(1+\log n)^{1/q}\|T\|\Big(\int\sum_{k=1}^n |f_k|^q\,d\mu\Big)^{1/q},$$

and it is not possible to avoid the log-term.

According to what was said in Section 1 this fact can also be formulated in a discrete way: For $1 \le p < q < 2$,

$$\sup_{\substack{m\in\mathbb{N} \\ \|A:\ell_q^m\to\ell_p^m\|\le 1}} \|A\otimes \mathrm{id} : \ell_q^m \otimes_{\Delta_q} \ell_q^n \to \ell_p^m \otimes_{\Delta_p} \ell_q^n\| \asymp (1+\log n)^{1/q},$$

an estimate formally stronger than the positive answer to a conjecture of Rosenthal and Szarek from [2].

6. The case $1 \le q \le p \le \infty$. Here the results are very much different from the case $1 \le p \le q \le \infty$: it turns out, for example, that $k_{q,p}(r)$ is either 1 or ∞.

Note first that (as remarked in Section 1) $k_{q,p}(r) = 1$ whenever $q = 1$ or $p = \infty$; hence we exclude these cases in the following

THEOREM. *Let* $1 < q \le p < \infty$. *Then* $k_{q,p}(r) < \infty$ *if and only if*

$$\min(q,2) \le r \le \max(p,2).$$

More precisely,

(1) $k_{q,p}(r) = 1$ *for* $\min(q,2) \le r \le \max(p,2)$.
(2) $k_{q,p}^{(n)}(r) \asymp n^{\max(0,1/r-1/\min(q,2),1/\max(p,2)-1/r)}$ *for all* r.

This answers Problem 2 of [9]. The special case $p = q$ is due to [12], and the fact that $k_{q,p}(r) = 1$ for $\min(q,2) \le r \le \max(p,2)$ was stated independently in [9, Theorem 2] and [5, 26.3, Remark 1]; for the sake of completeness we give a proof (which is now almost trivial). Consider the following three cases:

(a) $q \le 2 \le p$, $q \le r \le p$,
(b) $q \le p \le 2$, $q \le r \le 2$,
(c) $2 \le q \le p$, $2 \le r \le p$.

By monotonicity and the results from Section 4,

$$\begin{aligned} 1 \le k_{q,p}(r) &\le k_{r,r}(r) = 1 \quad \text{for } (p,q,r) \text{ as in (a)}, \\ 1 \le k_{q,p}(r) &\le k_{q,q}(r) = 1 \quad \text{for } (p,q,r) \text{ as in (b)}. \end{aligned}$$

Finally, (b) implies (c) by duality. ∎

Hence it remains to prove part (2) of the Theorem. We will check the following three estimates:

(2a) $k_{q,p}^{(n)}(r) \asymp n^{1/r-1/q}$ for $q \le 2 \le p$, $r < q$,
(2b) $k_{q,p}^{(n)}(r) \asymp n^{1/r-1/q}$ for $q \le p \le 2$, $r < q$,
(2c) $k_{q,p}^{(n)}(r) \asymp n^{1/2-1/r}$ for $q \le p \le 2$, $2 < r$;

the three remaining cases then follow by duality. For the proofs of (2abc) we will need the following facts which can be found in [26, pp. 312, 313]:

(I) $\pi_{q'}(\mathrm{id} : \ell_2^n \to \ell_{r'}^n) \asymp n^{1/q'}$ for $1 \le r \le q \le 2$,
$\pi_{q'}(\mathrm{id} : \ell_2^n \to \ell_{r'}^n) \asymp n^{1/r'}$ for $1 < q \le 2 \le r \le \infty$,

(II) $i_{p'}(\mathrm{id} : \ell_r^n \to \ell_2^n) \asymp n^{1/r'}$ for $1 \le r \le 2$, $1 < p < \infty$,
$i_{p'}(\mathrm{id} : \ell_r^n \to \ell_2^n) \asymp n^{1/2}$ for $1 < p \le 2 \le r \le \infty$.

Let us start with (2a) and (2b): By (2a) of the preceding theorem,

$$k_{q,p}(\ell_r^n) \le k_{q,q}(\ell_r^n) \asymp n^{1/r-1/q},$$

and by the Proposition from Section 1,

$$n^{1/r-1/q} = \frac{n^{1/q'}}{n^{1/r'}} \asymp \frac{\pi_{q'}(\mathrm{id} : \ell_2^n \to \ell_{r'}^n)}{i_{p'}(\mathrm{id} : \ell_r^n \to \ell_2^n)} \le k_{q,p}(\ell_r^n).$$

Finally, the proof of (2c): Again by the Proposition from Section 1 and the Theorem from Section 5,

$$n^{1/2-1/r} = \frac{n^{1/r'}}{n^{1/2}} \asymp \frac{\pi_{q'}(\mathrm{id} : \ell_2^n \to \ell_{r'}^n)}{i_{p'}(\mathrm{id} : \ell_r^n \to \ell_2^n)}$$
$$\le k_{q,p}(\ell_r^n) \le k_{q,q}(\ell_r^n) \asymp n^{1/2-1/r}.$$

This completes the proof of the theorem. ■

References

[1] G. Baumbach and W. Linde, *Asymptotic behaviour of p-summing norms of identity operators*, Math. Nachr. 78 (1977), 193–196.

[2] B. Carl and A. Defant, *An inequality between the p- and (p, 1)-summing norm of finite rank operators from C(K)-spaces*, Israel J. Math. 74 (1991), 323–335.

[3] —, —, *Tensor products and Grothendieck type inequalities of operators in L_p-spaces*, Trans. Amer. Math. Soc. 331 (1992), 55–76.

[4] A. Defant, *Best constants for the norm of the complexification of operators between L_p-spaces*, in: K. D. Bierstedt, A. Pietsch, W. M. Ruess and D. Vogt (eds.), Functional Analysis, Proc. Essen Conf., 1991, Lecture Notes in Pure and Appl. Math. 150, Dekker, 1993, 173–180.

[5] A. Defant and K. Floret, *Tensor Norms and Operator Ideals*, North-Holland Math. Stud. 176, North-Holland, 1993.

[6] J. Diestel, H. Jarchow and A. Tonge, *Absolutely Summing Operators*, Cambridge Stud. Adv. Math. 43, Cambridge Univ. Press, 1995.

[7] J. García-Cuerva and J. L. Rubio de Francia, *Weighted Norm Inequalities and Related Topics*, North-Holland Math. Stud. 104, North-Holland, 1985.

[8] D. J. H. Garling, *Absolutely p-summing operators in Hilbert space*, Studia Math. 38 (1970), 319–331.

[9] J. Gasch and L. Maligranda, *On vector-valued inequalities of Marcinkiewicz–Zygmund, Herz and Krivine type*, Math. Nachr. 167 (1994), 95–129.

[10] E. Gené, M. B. Marcus and J. Zinn, *A version of Chevet's theorem for stable processes*, J. Funct. Anal. 63 (1985), 47–73.

[11] A. Grothendieck, *Résumé de la théorie métrique des produits tensoriels topologiques*, Bol. Soc. Mat. São Paulo 8 (1956), 1–79.

[12] C. Herz, *The theory of p-spaces with application to convolution operators*, Trans. Amer. Math. Soc. 154 (1971), 69–82.

[13] J. Hoffmann-Jørgensen, *Sums of independent Banach space valued random variables*, Studia Math. 52 (1974), 159–186.

[14] M. Junge, *Geometric applications of the Gordon–Lewis property*, Forum Math. 6 (1994), 617–635.

[15] H. König, *On the complex Grothendieck constant in the n-dimensional case*, in: P. F. X. Müller and W. Schachermeyer (eds.), Proc. Strobl Conference on "Geometry of Banach spaces", London Math. Soc. Lecture Note Ser. 158, Cambridge Univ. Press, 1990, 181–199.

[16] J. L. Krivine, *Constantes de Grothendieck et fonctions de type positif sur les sphères*, Adv. Math. 31 (1979), 16–30.

[17] S. Kwapień, *On a theorem of L. Schwartz and its applications to absolutely summing operators*, Studia Math. 38 (1970), 193–201.

[18] —, *On operators factoring through L_p-space*, Bull. Soc. Math. France Mém. 31–32 (1972), 215–225.

[19] M. Ledoux and M. Talagrand, *Probability in Banach Spaces*, Ergeb. Math. Grenzgeb. 23, Springer, 1991.

[20] J. Lindenstrauss and A. Pełczyński, *Absolutely summing operators in $\mathscr{L}_p$-spaces and applications*, Studia Math. 29 (1968), 275–326.

[21] J. Marcinkiewicz et A. Zygmund, *Quelques inégalités pour les opérations linéaires*, Fund. Math. 32 (1939), 113–121.

[22] B. Maurey, *Théorèmes de factorisation pour les opérateurs linéaires à valeurs dans les espaces L^p*, Astérisque 11 (1974).

[23] B. Maurey et G. Pisier, *Séries de variables aléatoires vectorielles indépendantes et propriétés géométriques des espaces de Banach*, Studia Math. 58 (1976), 45–90.

[24] R. E. A. C. Paley, *On a remarkable series of orthogonal functions*, Proc. London Math. Soc. 34 (1932), 241–264.

[25] A. Pietsch, *Absolutely p-summing operators in L_r-spaces*, Bull. Soc. Math. France Mém. 31–32 (1972), 285–315.

[26] —, *Operator Ideals*, North-Holland, 1980.

[27] P. Saphar, *Applications p-décomposantes et p-absolument sommantes*, Israel J. Math. 11 (1972), 164–179.

[28] H. Vogt, *Komplexifizierung von Operatoren zwischen L_p-Räumen*, Diplomarbeit, Oldenburg, 1995.

Fachbereich Mathematik
Universität Oldenburg
D-26111 Oldenburg, Germany
E-mail: defant@math.uni-oldenburg.de

Mathematisches Seminar
Universität Kiel
D-24098 Kiel, Germany
E-mail: nms06@rz.uni-kiel.d400.de

Received November 12, 1996 (3775)