$n \to \infty$. Let $f_1 = |h_1|$. If $k \in \mathbb{N}$, and $2^{k-1} < j \le 2^k$, let $f_j = \sqrt{2^{k-1}}|h_j|$. Define $n_1 = 1$. Since $x_1 = T(h_1 \cdot r_{n_1}) \in m^{2,\infty}(\Gamma, w)$, there is a finite subset σ_1 of Γ such that $\|x_1 \chi_{\sigma_1^c}\| < 2^{-3}$. Now suppose that numbers n_i and finite sets σ_i have been chosen for $i \le j$. Since $T(f_{j+1} \cdot r_n) \to 0$ weakly as $n \to \infty$, and $\bigcup_{i=1}^j \sigma_i$ is finite, there exists $n_{j+1} > n_j$ so that $\|x_{j+1}\chi_{\bigcup_{i=1}^j \sigma_i}\| < 2^{-j-4}$, where $x_{j+1} = T(f_{j+1} \cdot r_{n_{j+1}})$. Now we can choose a finite subset σ_{j+1} of Γ, disjoint from $\bigcup_{i=1}^j \sigma_i$, such that $\|x_{j+1}\chi_{\sigma_{j+1}^c}\| < 2^{-j-3}$. Finally, let $y_j = x_j \chi_{\sigma_j}$ for all $j \in \mathbb{N}$. Then (y_j) is pairwise disjoint sequence, and hence is a basic sequence with basis constant 1. Moreover,

$$\|y_j\| > \|x_j\| - 2^{-j-2} \ge \|f_j \cdot r_{n_j}\| - 2^{-j-2} > 1/2.$$

Also, $\sum \|x_j - y_j\| < 1/4$. By Proposition 1.a.9 in [5], (y_j) and (x_j) are equivalent. But then $(f_j \cdot r_{n_j})$ is equivalent to a pairwise disjoint sequence in $\ell^{2,\infty}(\Gamma, w)$. However, it is easy to see that $(f_j \cdot r_{n_j})$ is equivalent to $(a_j h_j)$, where $a_1 = 1$ and $a_j = \sqrt{2^{k-1}}$ if $2^{k-1} < j \le 2^k$. Hence we obtain an embedding S of $[(h_j)]$ into $\ell^{2,\infty}(\Gamma, w)$ such that (Sh_j) is a pairwise disjoint sequence. As (h_j) is a basis of $M^{2,\infty}[0,1]$, we have reached a contradiction to Proposition 8. This completes the proof of Theorem 3.

References

[1] M. I. Kadec and A. Pełczyński, *Bases, lacunary sequences and complemented subspaces in the spaces L_p*, Studia Math. 21 (1962), 161–176.

[2] H. E. Lacey, *The Isometric Theory of Classical Banach Spaces*, Springer, 1974.

[3] D. H. Leung, *Isomorphism of certain weak L^p spaces*, Studia Math. 104 (1993), 151–160.

[4] —, *Isomorphic classification of atomic weak L^p spaces*, in: Interaction between Functional Analysis, Harmonic Analysis and Probability, N. J. Kalton, E. Saab and S. J. Montgomery-Smith (eds.), Marcel Dekker, 1996, 315–330.

[5] J. Lindenstrauss and L. Tzafriri, *Classical Banach Spaces I*, Springer, 1977.

[6] —, *Classical Banach Spaces II*, Springer, 1979.

[7] H. P. Lotz, *Weak* convergence in the dual of weak L^p*, unpublished manuscript.

[8] H. H. Schaefer, *Banach Lattices and Positive Operators*, Springer, 1974.

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Distinguishing Jordan polynomials by means of a single Jordan-algebra norm

by

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Abstract. For $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ we exhibit a Jordan-algebra norm $|\cdot|$ on the simple associative algebra $M_\infty(\mathbb{K})$ with the property that Jordan polynomials over $\mathbb{K}$ are precisely those associative polynomials over $\mathbb{K}$ which act $|\cdot|$-continuously on $M_\infty(\mathbb{K})$. This analytic determination of Jordan polynomials improves the one recently obtained in [5].

1. Introduction. The Jordan product of a (real or complex) associative algebra is defined as the symmetrization of the associative product. Jordan polynomials are those (non-commutative) associative polynomials which can be expressed from the indeterminates by means of a finite process of taking sums, multiplications by scalars, and Jordan products. Clearly, every Jordan polynomial acts continuously on any associative algebra endowed with a Jordan-algebra norm. The question of the continuity of the action of particular non-Jordan associative polynomials (like the associative product $\mathbf{xy}$ or the tetrad $\mathbf{xyzt} + \mathbf{tzyx}$) on suitable associative algebras endowed with Jordan-algebra norms has received special attention in the literature, mainly because of its close relation to positive results and limits in the normed treatment of the Zel'manov prime theorem [15] for Jordan algebras. In this direction the interested reader can consult [14], [10], [11], [2], [8], [6], [7], [12], [13], [3], [4] and [9]. The introduction of [5], together with that of [9] already quoted, can also be interesting for a historical view of progresses in the above mentioned question. Among these progresses, we only emphasize here that every Jordan-algebra norm on a simple associative algebra with unit makes the associative product (and hence, every associative polynomial) continuous, and that the result need not remain true if the assumption of the existence of a unit is removed [3]. In fact, a first "monster" is built in [3] by providing a Jordan-algebra norm on the simple associative algebra $M_\infty(\mathbb{K})$ (of all countably infinite matrices over $\mathbb{K}$ with a finite number of non-zero entries) and a $\mathbb{K}$-linear involution $*$ on

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$M_\infty(\mathbb{K})$ such that the action of the tetrad (hence the associative product) on $H(M_\infty(\mathbb{K}),*) := \{A \in M_\infty(\mathbb{K}) : A^* = A\}$ is discontinuous.

The question of the continuity of the action of general associative polynomials on associative algebras endowed with Jordan-algebra (semi-) norms has been first considered by R. Arens and M. Goldberg [1]. They prove that for "almost" every non-Jordan associative polynomial $\mathbf{p}$ there exists a non-simple associative algebra (depending only on the degree of $\mathbf{p}$ and the number of indeterminates involved in $\mathbf{p}$) endowed with a Jordan-algebra seminorm making the action of $\mathbf{p}$ discontinuous. Very recently, the Arens–Goldberg result has been significantly improved in [5], where it is shown that, for every non-Jordan associative polynomial $\mathbf{p}$ over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, there exists a Jordan-algebra norm $|\cdot|$ (depending only on the degree of $\mathbf{p}$ and the number of indeterminates involved by $\mathbf{p}$) on $M_\infty(\mathbb{K})$ such that the action of $\mathbf{p}$ on $M_\infty(\mathbb{K})$ is $|\cdot|$-discontinuous. Moreover, the $|\cdot|$-discontinuity of the action of $\mathbf{p}$ can be centered in $H(M_\infty(\mathbb{K}),*)$ for a suitable $\mathbb{K}$-linear involution $*$ on $M_\infty(\mathbb{K})$, which can be chosen of arbitrarily given type (hermitian or alternate).

In this paper we present the "absolute monster" for the analytical determination of Jordan polynomials. Precisely, for $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, we construct a Jordan-algebra norm on $M_\infty(\mathbb{K})$ making the action of any non-Jordan polynomial on $M_\infty(\mathbb{K})$ discontinuous. Moreover, our norm exhibits all additional pathologies of the norms built in [5]. For the most part, our arguments are more or less deep refinements of the ideas developed in [5]. However, we would like to emphasize, as a new auxiliary result of independent interest, the existence of a Jordan subalgebra J of $M_\infty(\mathbb{K})$ such that no non-Jordan associative polynomial leaves it invariant. This property of $M_\infty(\mathbb{K})$ is shared in an obvious way by the free associative algebra on a countably infinite set of indeterminates, but this last algebra is not simple. It is also worth mentioning that, for a suitable (associative) algebra-norm $\|\cdot\|$ on $M_\infty(\mathbb{K})$, the Jordan subalgebra J above becomes $\|\cdot\|$-closed.

2. The result. As we have said in the introduction, our work continues and refines the ideas developed in [5]. Therefore, in order to avoid repetition, we refer the reader to that paper for all standard concepts not explicitly explained here.

Given a field $\mathbb{F}$, a natural number n, and $\varepsilon = \pm 1$, we consider the involution $*$ on $M_{2n}(\mathbb{F})$ defined by $a^* := s^{-1}a^t s$, where a^t denotes the transpose of a and $s := \mathrm{diag}\{q, \overset{n}{\ldots}, q\}$ with $q := \begin{pmatrix} 0 & 1 \\ \varepsilon 1 & 0 \end{pmatrix}$. If $\varepsilon = 1$, then $*$ will be called the *symmetric involution* on $M_{2n}(\mathbb{F})$. In the case $\varepsilon = -1$ we obtain the familiar *symplectic involution*. Both the symmetric and the symplectic involutions pass from matrix algebras of the form $M_{2n}(\mathbb{F})$ $(n \in \mathbb{N})$ to the algebra $M_\infty(\mathbb{F})$

(of all countably infinite matrices over $\mathbb{F}$ with a finite number of non-zero entries) by regarding $M_\infty(\mathbb{F})$ as $\bigcup_{n\in\mathbb{N}} M_{2n}(\mathbb{F})$ in the most natural way.

For a real or complex associative algebra A, a *Jordan-algebra norm* on A is a norm $|\cdot|$ on the vector space of A satisfying $|a.b| \le |a||b|$ for all a, b in A, where $a.b := \frac{1}{2}(ab + ba)$ is the *Jordan product* of A.

Now, we can state our main result:

THEOREM. *Let $\mathbb{K}$ be either $\mathbb{R}$ or $\mathbb{C}$, and denote by $*$ either the symmetric or the symplectic involution on $M_\infty(\mathbb{K})$. Then there exists a Jordan-algebra norm on $M_\infty(\mathbb{K})$ making discontinuous the action on $H(M_\infty(\mathbb{K}),*)$ of every non-Jordan associative polynomial.*

As in [5], the proof of the theorem relies on two results of independent interest (Propositions 1 and 2 below) refining the corresponding Propositions 1 and 2 of that paper.

Given an algebra B, we denote by $M_\infty(B)$ the algebra of all countably infinite matrices over B with a finite number of non-zero entries. In the proof of the next proposition, for n in $\mathbb{N}$, we will identify the algebra $M_n(B)$ of all $n \times n$ matrices over B with the subalgebra of $M_\infty(B)$ of those matrices $(b_{ij})_{(i,j)\in\mathbb{N}\times\mathbb{N}}$ in $M_\infty(B)$ satisfying $b_{ij} = 0$ whenever either $i > n$ or $j > n$. If B has an involution $*$, then $M_\infty(B)$ has a "standard" involution (also denoted by $*$) consisting in transposing a given matrix and applying the original involution to each entry.

PROPOSITION 1. *Let $(B, \|\cdot\|)$ be an associative normed algebra over $\mathbb{K}$ $(= \mathbb{R}$ or $\mathbb{C})$, and J be a closed Jordan subalgebra of B. Then there exists a Jordan-algebra norm $|\cdot|$ on $M_\infty(B)$ making discontinuous the action on $M_\infty(B)$ of every associative polynomial $\mathbf{p}$ such that J is not invariant under $\mathbf{p}$. Moreover, if B has an involution $*$, and if J is contained in $H(B,*)$, then the norm $|\cdot|$ can be chosen in such a way that the action on $H(M_\infty(B),*)$ of every polynomial $\mathbf{p}$ as above is $|\cdot|$-discontinuous.*

Proof. The proof of this proposition involves only minor changes on that of [5, Proposition 1], hence we limit ourselves to provide a sketch of it, emphasizing only the required changes.

We consider the algebra norm $\|\cdot\|$ on $M_\infty(B)$ defined by

$$\|(b_{ij})\| := \sum_{(i,j)\in\mathbb{N}\times\mathbb{N}} \|b_{ij}\|$$

for all (b_{ij}) in $M_\infty(B)$. Given a subspace S of $M_\infty(B)$ and an element α in $M_\infty(B)$, we write $\|\alpha + S\| := \inf\{\|\alpha + \beta\| : \beta \in S\}$. Also, we consider the identification $M_\infty(B) = M_\infty(\mathbb{K}) \otimes_{\mathbb{K}} B$. For k in $\mathbb{N}$, we denote by $\mathcal{J}_k$ the Jordan subalgebra of $M_\infty(B)$ given by $\mathcal{J}_k := M_{k-1}(\mathbb{K}) \otimes B + e_k \otimes J$, where $M_0(\mathbb{K}) := 0$, and we denote by e_k the element $(\lambda_{ij})_{(i,j)\in\mathbb{N}\times\mathbb{N}}$ in $M_\infty(\mathbb{K})$ given by $\lambda_{ij} = 0$ whenever $(i,j) \neq (k,k)$, and $\lambda_{kk} = 1$.

Now the norm $|\!|\!|\cdot|\!|\!|$ on the vector space of $M_\infty(B)$ defined by

$$|\!|\!|\alpha|\!|\!| := \|\alpha\| + \sum_{i=1}^{\infty} 2^{i^i}\|\alpha + \mathcal{J}_i\|$$

is a Jordan-algebra norm satisfying

$$|\!|\!|e_k \otimes b|\!|\!| = (1 + 2^{1^1} + 2^{2^2} + \ldots + 2^{(k-1)^{k-1}})\|b\| + 2^{k^k}\|b + J\|$$

for all k in $\mathbb{N}$ and b in B.

Let $\mathbf{q} = \mathbf{q}(\mathbf{x}_1, \ldots, \mathbf{x}_s)$ be a homogeneous associative polynomial such that J is not invariant under $\mathbf{q}$, and let m denote the degree of $\mathbf{q}$. Then there exist $x_1, \ldots, x_s$ in J satisfying $\|\mathbf{q}(x_1, \ldots, x_s) + J\| > 0$, and we easily obtain

$$\frac{|\!|\!|\mathbf{q}(e_k \otimes x_1, e_k \otimes x_2, \ldots, e_k \otimes x_s)|\!|\!|}{\max\{|\!|\!|e_k \otimes x_1|\!|\!|, |\!|\!|e_k \otimes x_2|\!|\!|, \ldots, |\!|\!|e_k \otimes x_s|\!|\!|\}^m} \geq \frac{2^{k^k}}{k^m\ [2^{(k-1)^{k-1}}]^m}\ \frac{\|\mathbf{q}(x_1, x_2, \ldots, x_s) + J\|}{\max\{\|x_1\|^m, \ldots, \|x_s\|^m\}}.$$

Therefore, for $k > m$, we have

$$\frac{|\!|\!|\mathbf{q}(e_k \otimes x_1, e_k \otimes x_2, \ldots, e_k \otimes x_s)|\!|\!|}{\max\{|\!|\!|e_k \otimes x_1|\!|\!|, |\!|\!|e_k \otimes x_2|\!|\!|, \ldots, |\!|\!|e_k \otimes x_s|\!|\!|\}^m} \geq \frac{2^{(k-1)^k}}{k^m}\ \frac{\|\mathbf{q}(x_1, x_2, \ldots, x_s) + J\|}{\max\{\|x_1\|^m, \ldots, \|x_s\|^m\}} \underset{k\to\infty}{\longrightarrow} \infty.$$

From [5, Lemma 1] we deduce that the action of $\mathbf{q}$ on $M_\infty(B)$ is not $|\!|\!|\cdot|\!|\!|$-continuous at zero. The passing from homogeneous polynomials to general ones, as well as the remaining part of the proof, follow without changes the corresponding arguments in [5, Proposition 1]. ∎

PROPOSITION 2. *Let $\mathbb{F}$ be a field of characteristic not two, and let $*$ denote either the symmetric or the symplectic involution on $M_\infty(\mathbb{F})$. Then there exists a Jordan subalgebra J of $M_\infty(\mathbb{F})$ contained in $H(M_\infty(\mathbb{F}), *)$ such that J is not invariant under any non-Jordan associative polynomial. Moreover, if $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$, and if we consider the algebra norm $\|(\mu_{ij})\| := \sum_{(i,j)\in\mathbb{N}\times\mathbb{N}} |\mu_{ij}|$ on $M_\infty(\mathbb{F})$, then the Jordan subalgebra J above can be chosen $\|\cdot\|$-closed.*

Proof. For p in $2\mathbb{N} \cup \{\infty\}$, let $*$ denote the symmetric involution on $M_p(\mathbb{F})$ (the argument for symplectic involutions is the same). According to [5, Proposition 2], for every natural number n there exists an even number d_n and a Jordan subalgebra J_n of $M_{d_n}(\mathbb{F})$ contained in $H(M_{d_n}(\mathbb{F}), *)$ such that J_n is not invariant under any non-Jordan associative polynomial involving at most n indeterminates and of degree $\leq n$.

For p, q in $\mathbb{N}$, denote by $M_{p,q}(\mathbb{F})$ the vector space of all $p \times q$-matrices over $\mathbb{F}$ (so that $M_p(\mathbb{F}) = M_{p,p}(\mathbb{F})$), and consider the algebra whose vector space is the abstract direct sum of the family $\{M_{d_n,d_m}(\mathbb{F})\}_{(n,m)\in\mathbb{N}\times\mathbb{N}}$ and whose product is determined, for elements $A_{n,m} \in M_{d_n,d_m}(\mathbb{F})$ and $B_{n',m'} \in M_{d_{n'},d_{m'}}(\mathbb{F})$, by

$$A_{n,m}B_{n',m'} = \begin{cases} A_{n,m}B_{n',m'} \in M_{d_n,d_{m'}} \text{ (the usual product)} & \text{if } m = n', \\ 0 & \text{otherwise.} \end{cases}$$

Then the algebra presented above is a copy of $M_\infty(\mathbb{F})$, via the mapping

$$\bigoplus_{n,m\in\mathbb{N}} A_{n,m} \to \left(\begin{array}{c|c|c} A_{1,1} & A_{1,2} & \ldots \\ \hline A_{2,1} & A_{2,2} & \ldots \\ \hline \vdots & \vdots & \ddots \end{array}\right).$$

Moreover, in that identification, the restriction of the symmetric involution on $M_\infty(\mathbb{F})$ to each diagonal summand $M_{d_n,d_n}(\mathbb{F})$ is nothing but the symmetric involution on that summand. Putting $J := \bigoplus_{n,m\in\mathbb{N}} K_{n,m}$ with $K_{n,n} = J_n$ and $K_{n,m} = 0$ if $n \neq m$, it follows that J is a Jordan subalgebra of $M_\infty(\mathbb{F})$ contained in $H(M_\infty(\mathbb{F}), *)$. Also J is not invariant under any non-Jordan associative polynomial. Indeed, if $\mathbf{p}(\mathbf{x}_1, \ldots, \mathbf{x}_s)$ is a non-Jordan associative polynomial of degree g, then, for $n := \max\{s, g\}$, we have $\mathbf{p}(J_n) \subseteq M_{d_n}(\mathbb{F})$ and $\mathbf{p}(J_n) \not\subseteq J_n$, and therefore $\mathbf{p}(J) \not\subseteq J$.

Now assume $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ and let $\|\cdot\|$ be the norm on $M_\infty(\mathbb{F})$ given in the statement of the proposition. Let $\{X_k\}$ be a sequence in J convergent to some element A in $M_\infty(\mathbb{F})$. Then there exists $N \in \mathbb{N}$ such that $A \in \bigoplus_{n,m\leq N} M_{d_n,d_m}(\mathbb{F})$. Since the natural projection Π from $M_\infty(\mathbb{F})$ onto $\bigoplus_{n,m\leq N} M_{d_n,d_m}(\mathbb{F})$ is $\|\cdot\|$-continuous, $\{\Pi(X_k)\}$ converges to A. Since $\Pi(J)$ is finite-dimensional, it follows that $A \in \Pi(J) \subset J$. Therefore J is $\|\cdot\|$-closed in $M_\infty(\mathbb{F})$. ∎

Now we are ready to conclude the proof of our main result.

Proof of the theorem. Applying Proposition 1 with $B = M_\infty(\mathbb{K})$, $\|\cdot\|$ equal to the algebra norm on $M_\infty(\mathbb{K})$ given in the statement of Proposition 2, and J equal to the closed Jordan subalgebra of $M_\infty(\mathbb{K})$ provided also by Proposition 2, we obtain a Jordan-algebra norm $|\!|\!|\cdot|\!|\!|$ on $M_\infty(M_\infty(\mathbb{K}))$ making the action on $H(M_\infty(M_\infty(\mathbb{K})), *)$ of every non-Jordan associative polynomial discontinuous. Now, the proof is concluded by realizing that the algebras with involution $(M_\infty(M_\infty(\mathbb{K})), *)$ and $(M_\infty(\mathbb{K}), *)$ are isomorphic. Indeed, regarding $M_\infty(M_\infty(\mathbb{K}))$ as $M_\infty(\mathbb{K}) \otimes_{\mathbb{K}} M_\infty(\mathbb{K})$, the standard involution on $M_\infty(M_\infty(\mathbb{K}))$ relative to either the symmetric or the symplectic involution $*$ on $M_\infty(\mathbb{K})$ becomes $t \otimes *$, where t denotes transposition. In other words,

$$(M_\infty(M_\infty(\mathbb{K})), *) \simeq (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_\infty(\mathbb{K}), *).$$

But it is easy to find isomorphisms

$$(M_\infty(\mathbb{K}), *) \simeq (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_2(\mathbb{K}), *)$$

and

$$(M_\infty(\mathbb{K}), t) \simeq (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_\infty(\mathbb{K}), t).$$

It follows that

$$\begin{aligned}(M_\infty(M_\infty(\mathbb{K})), *) &\simeq (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_\infty(\mathbb{K}), *)\\ &\simeq (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_2(\mathbb{K}), *)\\ &\simeq (M_\infty(\mathbb{K}), t) \otimes_{\mathbb{K}} (M_2(\mathbb{K}), *) \simeq (M_\infty(\mathbb{K}), *). \ \blacksquare\end{aligned}$$

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References

[1] R. Arens and M. Goldberg, *Quadrative seminorms and Jordan structures on algebras*, Linear Algebra Appl. 181 (1993), 269–278.

[2] R. Arens, M. Goldberg and W. A. J. Luxemburg, *Multiplicativity factors for seminorms II*, J. Math. Anal. Appl. 170 (1992), 401–413.

[3] M. Cabrera, A. Moreno and A. Rodríguez, *On the behaviour of Jordan-algebra norms on associative algebras*, Studia Math. 113 (1995), 81–100.

[4] —, —, —, *Zel'manov's theorem for primitive Jordan–Banach algebras*, J. London Math. Soc., to appear.

[5] M. Cabrera, A. Moreno, A. Rodríguez and E. Zel'manov, *Jordan polynomials can be analytically recognized*, Studia Math. 117 (1996), 137–147.

[6] M. Cabrera and A. Rodríguez, *Zel'manov's theorem for normed simple Jordan algebras with a unit*, Bull. London Math. Soc. 25 (1993), 59–63.

[7] —, —, *Nondegenerately ultraprime Jordan–Banach algebras: a zel'manovian treatment*, Proc. London Math. Soc. 69 (1994), 576–604.

[8] A. Fernández, E. García and A. Rodríguez, *A Zel'manov prime theorem for JB^*-algebras*, J. London Math. Soc. 46 (1992), 319–335.

[9] A. Moreno and A. Rodríguez, *Algebra norms on tensor products of algebras and the norm extension problem*, preprint, Universidad de Granada, 1995.

[10] A. Rodríguez, *La continuidad del producto de Jordan implica la del ordinario en el caso completo semiprimo*, in: Contribuciones en Probabilidad, Estadística Matemática, Enseñanza de la Matemática y Análisis, Secretariado de Publicaciones de la Universidad de Granada, Granada, 1979, 280–288.

[11] —, *Jordan axioms for C^*-algebras*, Manuscripta Math. 61 (1988), 297–314.

[12] —, *Jordan structures in Analysis*, in: Jordan Algebras: Proc. Conf. Oberwolfach, August 9-15, 1992, W. Kaup, K. McCrimmon, and H. Petersson (eds.), Walter de Gruyter, Berlin, 1994, 97–186.

[13] A. Rodríguez, A. Slin'ko and E. Zel'manov, *Extending the norm from Jordan–Banach algebras of hermitian elements to their associative envelopes*, Comm. Algebra 22 (1994), 1435–1455.

[14] S. Shirali, *On the Jordan structure of complex Banach $*$-algebras*, Pacific J. Math. 27 (1968), 397–404.

[15] E. Zel'manov, *On prime Jordan algebras II*, Siberian Math. J. 24 (1983), 89–104.

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