7. Theorems III^m. Using the same methods as in [2], section 8, we can easily prove

7.1. Theorem. *Let the space X_α satisfy the postulate* (a_3) *and let β denote a convergence generated by norm or a strong two-norms convergence in Y. Then theorems $\mathrm{III}_1^m(X_\alpha, Y_\beta)$ and $\mathrm{III}_2^m(X_\alpha, Y_\beta)$ are true.*

Bibliography.

[1] A. Alexiewicz, *On sequences of operations, I*, Studia Mathematica 11 (1950), p. 1-30.
[2] A. Alexiewicz, *On sequences of operations, II*, ibidem, p. 200-236.
[3] S. Mazur und W. Orlicz, *Grundlegende Eigenschaften der polynomischen Operationen*, Erste Mitteilung, ibidem, 5. (1934), p. 50-68.
[4] W. Orlicz, *Sur les opérations linéaires dans l'espace des fonctions bornées*, ibidem, 10 (1949), p. 60-99.

(Reçu par la Rédaction le 2. 3. 1949).

On sequences of operations (IV)

by

A. ALEXIEWICZ (Poznań).

In this part[1]) the terminology and notations introduced in [2] will be used without any further reference.

We are concerned with linear and polynomial operations from a $\varLambda$-space to the space S of measurable functions. Because of the particular structure of this space we can obtain some more special results than in [2] and [3]. The purpose of this paper is to generalize the results of Saks ([7], [8], [9]) to the case of linear and polynomic operations in $\varLambda$-spaces.

1. The space S. Let T be any measurable set of finite measure. We will denoteby S the space of the measurable functions defined in T. Two equivalent functions being considered as one element of the space, and addition of elements and multiplication by the reals being defined as usual, if we define the norm of $x=x(t)$ as

$$\|x\| = \int_T \frac{|x(t)|}{1+|x(t)|}\,dt,$$

S becomes an F-space. The convergence generated by norm is identical with the asymptotic convergence. By π we denote the convergence almost everywhere in S. The space S_π is identical with the Kantorovitch space corresponding to the following partial ordering: $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ almost everywhere. Kantorovitch ([5], p. 155) has shown that the space S_π is regular. The convergence π^*[2]) is identical with the strong convergence in S.

---

[1]) For the first three parts see [1], [2], and [3].
[2]) [2], p. 204.

Let $U(x)$ be any operation from a linear space X to $\boldsymbol{S}$; the value of $U(x)$ is an element $y(t)$ of $\boldsymbol{S}$. We will denote this by writing $U(x)=U(x,t)$.

2. Generalization of a theorem of Banach.

2.1. Theorem[3]). *Let the space X_α satisfy the postulates* (a_1) *and* (a_2)[4]), *and let D be a set dense in X_α. If for a sequence $\{U_n(x,t)\}$ of $(X_\alpha, \boldsymbol{S})$-linear operations*

(1) $$\overline{\lim_{n\to\infty}} U_n(x,t)<\infty \text{ almost everywhere for any } x,$$

and if this sequence converges almost everywhere for any $x\epsilon D$, then it converges almost everywhere for any x.

Proof. By (1)

$$-\overline{\lim_{n\to\infty}} U_n(-x,t)=\underline{\lim_{n\to\infty}} U_n(x,t)>-\infty,$$

almost everywhere for any x, i.e. the sequence $\{|U_n(x,t)|\}$ is π-bounded. The operations $U_n(x,t)$ are $(X_\alpha, \boldsymbol{S}_{\pi^*})$-linear; hence it suffices to apply Theorem 7.4.3 of [2].

If $\overline{\lim_{n\to\infty}} |U_n(x,t)|<\infty$ almost everywhere for any x, $\{U_n(x,t)\}$ being a sequence of $(X_\alpha, \boldsymbol{S})$-polynomials of degree at most m, then using the formulae of Mazur and Orlicz ([6], p. 51-56, [1], p. 27) we can easily prove that the sequence $\{|U_n(x,t)|\}$ is bounded almost everywhere for any x. Thus Theorem 6.3 of [3] yields

2.2. Theorem. *Let the space X_α satisfy the postulates* (a_1) *and* (a_2)[4]), *and let D be a set dense in X_α. If for a sequence $\{U_n(x,t)\}$ of $(X_\alpha, \boldsymbol{S})$-polynomials of degree at most m*

$$\overline{\lim_{n\to\infty}} |U_n(x,t)|<\infty \text{ almost everywhere for any } x,$$

and if this sequence converges almost everywhere for any x in D, then it converges almost everywhere for any x.

3. Theorems of Saks.

In this section X denotes an F-space. Saks ([7], [8], [9]) has proved the following theorems concerning the structure of sequences of $(X, \boldsymbol{S})$-linear operations:

[3]) This theorem was proved by Banach [4] for the case when X is a Banach space.

[4]) [2], p. 202.

3.1. Theorem. *Let $\{U_n(x,t)\}$ be a sequence of $(X, \boldsymbol{S})$-linear operations; then there exist two residual sets X_1, X_2 and decompositions $T=A_1+B_1=A_2+B_2$ such that*

(a) $\overline{\lim_{n\to\infty}} |U_n(x,t)|<\infty$ *almost everywhere in A_1 for any x,*

(b) $\overline{\lim_{n\to\infty}} |U_n(x,t)|=\infty$ *almost everywhere in B_1 for any $x\epsilon X_1$,*

(c) $\lim_{n\to\infty} U_n(x,t)$ *exists almost everywhere in A_2 for any x,*

(d) $\lim_{n\to\infty} U_n(x,t)$ *does not exist almost everywhere in B_2 for any $x\epsilon X_2$.*

This theorem implies that, if for a sequence $\{U_n(x,t)\}$ of $(X, \boldsymbol{S})$-linear operations there exists an x_0 such that $\overline{\lim_{n\to\infty}} |U_n(x_0,t)|=\infty$ (or the sequence $\{U_n(x_0,t)\}$ diverges) in a set A, then the same holds almost everywhere in A for every x belonging to a residual set X^*. From this we easily derive the following theorem on the condensation of singularities:

3.2. Theorem. *Let $\{U_{pq}(x,t)\}_{q=1,2,...}$ be a sequence of $(X, \boldsymbol{S})$-linear operations. If for every natural p there exists an element x such that $\overline{\lim_{q\to\infty}} |U_{pq}(x_p,t)|=\infty$ (such that the sequence $\{U_{pq}(x_p,t)\}_{q=1,2,...}$ is divergent) in a set T_p, then there exists a residual set X^* such that $\overline{\lim_{q\to\infty}} U_{pq}(x,t)=\infty$ (such that the sequence $\{U_{pq}(x,t)\}_{q=1,2,...}$ diverges) almost everywhere in T_p for any $x\epsilon X^*$ and any p.*

4. Theorems of Saks in Λ-spaces.

In [2], section 8, it is shown that the fulfilment of the postulate (a_3)[5]) in the space X_α enables us to transfer the problem of the condensation of singularities to the case of operations defined in a Banach space. Using the same method we can now easily establish

4.1. Theorem. *Let the space X_α satisfy the postulate* (a_3), *and let $\{U_{pq}(x,t)\}_{q=1,2,...}$ be a sequence of $(X_\alpha, \boldsymbol{S})$-linear operations. If for any p there exists an element x_p such that $\overline{\lim_{q\to\infty}} |U_{pq}(x_p,t)|=\infty$ (such that the sequence $\{U_{pq}(x_p,t)\}_{q=1,2,...}$ is divergent) in a set T_p, then there exists an element x_0 such that $\overline{\lim_{q\to\infty}} |U_{pq}(x_0,t)|=\infty$ (such*

[5]) [2], p. 203.

that the sequence $\{U_{pq}(x_0,t)\}_{q=1,2,\ldots}$ *is divergent) almost everywhere in* T_p *for* $p=1,2,\ldots$

In the sequel we shall need the following

4.2. Lemma. *Let* $\mathfrak{E}$ *be a class of measurable subsets of* T, *containing the empty set. There exists a sequence* $\{E_n\}$ *of mutually disjoint sets of* $\mathfrak{E}$ *such that no set of positive measure contained in* $T-\sum_{n=1}^{\infty}E_n$ *belongs to* $\mathfrak{E}$.

Proof. If every set in $\mathfrak{E}$ is null, it suffices to put $E_1=E_2=\ldots=0$. In the contrary case denote by ω_1 the least upper bound of measures of the sets belonging to $\mathfrak{E}$ and choose $E_1\epsilon\mathfrak{E}$ so that $|E_1|>\omega_1/2$. Suppose we have defined the sets $E_1,E_2,\ldots,E_n$; then by ω_{n+1} we denote the least upper bound of measures of the sets of $\mathfrak{E}$ which lie in $Q_n=T-(E_1+\ldots+E_n)$. If $\omega_{n+1}=0$, we put $E_{n+1}=E_{n+2}=\ldots=0$; if $\omega_{n+1}>0$, we choose $E_{n+1}\subset Q_n$ so that $|E_{n+1}|>\omega_{n+1}/2$. No set E of $\mathfrak{E}$ of positive measure lies in $T-\sum_{n=1}^{\infty}E_n$. For in the contrary case we should have $|E|<\omega_n$ for $n=1,2,\ldots$ This is, however, impossible since, the sets E_n being disjoint, $|E_n|\to 0$ and $2|E_n|>\omega_n$.

4.3. Theorem. *Let the space* X_α *satisfy the postulate* ($\mathfrak{a}_3$). *If* $\{U_n(x,t)\}$ *is a sequence of* $(X_\alpha,\boldsymbol{S})$*-linear operations then there exists an element* x_1 *and a decomposition* $T=A_1+B_1$ *such that*

(a) $\overline{\lim_{n\to\infty}}|U_n(x,t)|<\infty$ *almost everywhere in* A_1 *for any* x,

(b) $\overline{\lim_{n\to\infty}}|U_n(x_1,t)|=\infty$ *in* B_1.

Proof. If $\overline{\lim_{n\to\infty}}|U_n(x,t)|<\infty$ almost everywhere for any x, it suffices to set $A_1=T, B_1=0$. In the contrary case there must exist elements x such that $\overline{\lim_{n\to\infty}}|U_n(x,t)|=\infty$ in a set T_x of positive measure. Denote by $\mathfrak{E}$ the class of all measurable sets Q for which there exists an x such that $Q\subset T_x$. Let $\{Q_n\}$ be the sequence the existence of which is assured by Lemma 4.2. Then $Q_k\subset T_{x_k}$; hence $\overline{\lim_{n\to\infty}}|U_n(x_k,t)|=\infty$ in Q_k. Write $B_1=\sum_{n=1}^{\infty}Q_n$, $A_1=T-B_1$; then (a) holds by 4.2. To prove (b) put

$$U_{pq}(x,t)=\begin{cases} U_q(x,t) & \text{for } t\epsilon Q_p,\\ 0 & \text{for } t\epsilon T-Q_p.\end{cases}$$

These operations are $(X_\alpha,\boldsymbol{S})$-linear and $\overline{\lim_{q\to\infty}}|U_{pq}(x_p,t)|=\infty$ for every $t\epsilon Q_p$. By Theorem 4.1 there exists an element x_1 such that $\overline{\lim_{q\to\infty}}|U_{pq}(x_1,t)|=\infty$ almost everywhere in Q_p for $p=1,2,\ldots$

In a similar manner we can prove the following

4.4. Theorem. *Let the space* X_α *satisfy the postulate* ($\mathfrak{a}_3$) *and let* $\{U_n(x,t)\}$ *be a sequence of* $(X_\alpha,\boldsymbol{S})$*-linear operations. Then there exists an element* x_2 *and a decomposition* $T=A_2+B_2$ *such that*

(c) $\lim_{n\to\infty}U_n(x,t)$ *exists almost everywhere in* A_1 *for any* x,

(d) $\lim_{n\to\infty}U_n(x_2,t)$ *does not exist in* B_1.

5. Theorems of Saks for polynomials in F**-spaces** [6]. In this section X denotes a F-space. Let $\mathfrak{N}$ be the space composed of the measurable subsets of T. The distance of the two sets $E_1,E_2\epsilon\mathfrak{N}$ being defined as $\varrho(E_1,E_2)=|E_1-E_2|+|E_2-E_1|$, $\mathfrak{N}$ is a complete and separable metric space [7].

Given any sequence $\{U_n(x,t)\}$ of operations from X to $\boldsymbol{S}$ we shall denote by Θ_x and Ω_x respectively the sets of the points t at which this sequence is bounded or convergent respectively.

5.1. Theorem. *Let* $\{U_n(x,t)\}$ *be a sequence of* $(X,\boldsymbol{S})$*-polynomials of degree at most* m, H *— any measurable set* X_0, *— a set of the second category, and let* ε *be positive. If* $|H-\Theta_x|\leqslant\varepsilon$ *for every* $x\epsilon X_0$, *then*

$$|H-\Theta_x|<(m+1)\varepsilon \quad \textit{for every } x.$$

Proof. Put

$$\Gamma_x^n=\mathop{E}_t\{\sup_{i=1,2,\ldots}|U_i(x,t)|\leqslant n\},\quad X_n=\mathop{E}_x\{|H-\Gamma_x^n|\leqslant\varepsilon\},\quad X^*=\sum_{n=1}^{\infty}X_n.$$

It is easily seen that $x\epsilon X^*$ implies $|H-\Theta_x|\leqslant\varepsilon$. The sets X_n are closed. For, let $x_k\epsilon X_n$, $x_k\to x_0$; since $\lim_{k\to\infty}U_i(x_k)=U_i(x_0)$ for $i=1,2,\ldots$ there exists a subsequence $\{x_{r_k}\}$ such that $\lim_{k\to\infty}U_i(x_{r_k},t)=U_i(x_0,t)$ for $i=1,2,\ldots$ except at a null set N. If $\Gamma_0=\overline{\lim_{k\to\infty}}\Gamma_{x_{r_k}}^n$, then $|H-\Gamma_0|=|\underline{\lim_{k\to\infty}}(H-\Gamma_{x_{r_k}}^n)|\leqslant\varepsilon$. If $t\epsilon\Gamma_0-N$, then $|U_i(x_{r_k},t)|\leqslant n$

[6]) We use here the methods due to Saks [9].

[7]) See [1], section 8.

for every i and infinitely many k; this implies $|U_i(x_0,t)|\leqslant n$ for $i=1,2,\ldots$; hence $x_0\in X_n$.

The formula $X_0\subset X^*$ implies the existence of an n_0 such that the set X_{n_0} contains a sphere $K(x_0,r)$. We can suppose that $x_0=0$ (for in the contrary case it is sufficient to consider the operations $W_n(x)=U_n(x+x_0)$). Let

(2) $$U_n(x)=U_{n_0}(x)+U_{n1}(x)+\ldots+U_{nm}(x)$$

be the canonical representation ([6], p. 51) of the polynomial $U_n(x)$. The formulae ([6], p. 51)

(3) $$U_{n\nu}(x)=a_{\nu 0}U_n(0\cdot x)+a_{\nu 1}U(1\cdot x)+\ldots+a_{\nu m}U(m\cdot x)$$

imply the existence of a constant A such that $|U_{n\nu}(x,t)|<A$ in the sphere $K(0,r/m)$ for every $t\in\Gamma_{0x}^{n_0}\cdot\Gamma_{1\cdot x}^{n_0}\cdot\ldots\cdot\Gamma_{m\cdot x}^{n_0}=\Delta_x$. Now, $\Delta_x\subset\Theta_x$ and $|H-\Theta_x|\leqslant|H-\Delta_x|\leqslant\sum_{i=0}^{m}|H-\Gamma_{ix}^{n_0}|\leqslant(m+1)\varepsilon$. The ν-homogeneity of $U_{n\nu}(x)$ yields $\Delta_{\lambda x}=\Delta_x$; hence $|H-\Theta_x|\leqslant(m+1)\varepsilon$ for every x.

5.2. Theorem. *Let X be a F-space. If $\{U_n(x,t)\}$ is a sequence of (X,S)-polynomials of degree at most m, then there exists a residual set $X_1\subset X$ and a decomposition $T=A_1+B_1$ such that*

(a) $\overline{\lim\limits_{n\to\infty}}|U_n(x,t)|<\infty$ *almost everywhere in A_1 for any x,*

(b) $\overline{\lim\limits_{n\to\infty}}|U_n(x,t)|=\infty$ *almost everywhere in B_1 for any $x\in X_1$.*

Proof. Let $\{H_i\}$ be a sequence dense in $\mathfrak{N}$. Writing $P_\gamma=\underset{x}{E}\{|\Theta_x|>\gamma\}$ let γ_0 be the least upper bound of the numbers γ for which the set P_γ is of the second category. If $\gamma_0=0$, it suffices to put $A_1=0$. If $\gamma_0>0$, choose $\varepsilon_k>0$ so that $\sum_{k=1}^{\infty}\varepsilon_k<\infty$ and put

$$X_{pq}=\underset{x}{E}\{|H_q-\Theta_x|<\frac{\varepsilon_p}{m+1}\ \text{ and }\ |H_q|>\gamma_0-\varepsilon_p\}.$$

Since $P_{\gamma_0-\varepsilon_p}\subset\sum_{q=1}^{\infty}X_{pq}$ for each p, there must exist a q_p such that the set X_{pq_p} is of the second category. By 5.1 $|H_{q_p}-\Theta_x|\leqslant\varepsilon_p$ for every x. Put $A_1=\overline{\lim\limits_{p\to\infty}}H_{q_p}$, $B_1=T-A_1$. Then $|A_1|\geqslant\overline{\lim\limits_{p\to\infty}}|H_{q_p}|\geqslant\gamma_0$, and for every s and x

$$|A-\Theta_x|\leqslant|\sum_{p=s}^{\infty}(H_{q_p}-\Theta_x)|\leqslant\sum_{p=s}^{\infty}\varepsilon_p;$$

hence $|A_1-\Theta_x|=0$. Thus (a) is satisfied. Since $|A_1|\geqslant\gamma_0$, the statement (b) must hold too.

5.3. Theorem. *Let $\{U_n(x,t)\}$ be a sequence of (X,S)-polynomials of degree at most m, H — a measurable set, X_0 — a set of the second category, and let ε be positive. If $x\in X_0$ implies $|H-\Omega_x|<\varepsilon$, then there exists a residual set X_1 such that $|H-\Omega_x|\leqslant(m+1)\varepsilon$ for every $x\in X_1$.*

Proof. Put

$$\Phi_x^{kn}=\underset{t}{E}\{\sup_{p,q\geqslant k}|U_p(x,t)-U_q(x,t)|\leqslant 1/n\},$$

$$X_{kn}=\underset{x}{E}\{|H-\Phi_x^{kn}|\leqslant\varepsilon\},\qquad X^*=\prod_{n=1}^{\infty}\sum_{k=1}^{\infty}X_{kn}.$$

Then $x\in X^*$ implies $|H-\Omega_x|\leqslant\varepsilon$. For, let $x\in X^*$; then, given any n, there exists a k_n such that $x\in X_{k_nn}$, i.e. $|H-\Phi_x^{k_nn}|\leqslant\varepsilon$. Write

$$\Psi_x=\overline{\lim_{n\to\infty}}\,\Phi_x^{k_nn};$$

it is obvious that $\Psi_x\subset\Omega_x$; hence

$$|H-\Omega_x|\leqslant|H-\Psi_x|\leqslant\underline{\lim_{n\to\infty}}|H-\Phi_x^{k_nn}|\leqslant\varepsilon.$$

We can prove similarly as in proof of 5.1 that the sets X_{kn} are closed. It is easy to prove the formula

$$X_0\subset\prod_{n=1}^{\infty}\sum_{k=1}^{\infty}X_{kn}=X^*;$$

this enables us to replace the set X_0 by the set X^*. The set X^* being measurable (B), it is residual in a sphere $K(x_0,r)$. We can suppose without loss of generality that $x_0=0$. Given any number α denote by αV the set of the elements αx with $x\in V$. Since the sets $V=K(0,r/m)-X^*$ are of the first category for $m=1,2,\ldots$, the same holds for the set $V^*=0V+1V+\ldots+mV$. The formula (2) implies the convergence of the sequences $\{U_{n\nu}(x,t)\}_{n=1,2,\ldots}$ for every $x\in X_2=K(0,r/m)X^*$ and $t\in\Delta_x=\Omega_{0x}\Omega_{1x}\ldots\Omega_{mx}$. Thus $|H-\Omega_x|\leqslant|H-\Delta_x|\leqslant(m+1)\varepsilon$ for every $x\in X_2$ and in virtue of the ν-homogeneity of $U_{n\nu}(x)$, we get $|H-\Omega_x|\leqslant(m+1)\varepsilon$ for every x belonging to the set X_1 of the elements tx with $x\in X_2$; this set is obviously residual.

5.4. Theorem. *Let X be a F-space. Given any sequence* $\{U_n(x,t)\}$ of (X,S)*-polynomials of degree at most m there exists a residual set X_2 and a decomposition $T=A_2+B_2$ such that*

(c) $\lim\limits_{n\to\infty} U_n(x,t)$ *exists almost everywhere in A_2 for every x,*

(d) $\lim\limits_{n\to\infty} U_n(x,t)$ *does not exist for almost every t in B_2 for every $x\epsilon X_2$.*

Proof. By Theorem 5.2 we can suppose that for each the x inequality $\overline{\lim\limits_{n\to\infty}}|U_n(x,t)|<\infty$ holds almost everywhere in T. Put $R_\delta=\underset{x}{E}\{|\Omega_x|\geqslant\delta\}$. If, given any $\delta>0$, the set R_δ is of the first category it suffices to write $A_2=0$. In the contrary case denote by δ_0 the least upper bound of the numbers δ for which the set R_δ is of the second category. Choose $\varepsilon_k>0$ so that $\sum\limits_{k=1}^{\infty}\varepsilon_k<\infty$ and put

$$X_{pq}=\underset{x}{E}\{|H_q-\Omega_x|<\frac{\varepsilon}{m+1}\quad\text{and}\quad |H_q|\geqslant\delta_0-\varepsilon_p\}.$$

Since $\sum\limits_{q=1}^{\infty}X_{pq}\supset R_{\delta_0-\varepsilon_p}$, there exists a q_p such that the set X_{pq_p} is of the second category. By 5.3 there exists a residual set X_p^* such that $|H_{q_p}-\Omega_x|<\varepsilon_p$ for every $x\epsilon X_p$. Write $Z=\prod\limits_{p=1}^{\infty}X_p^*$, $A_2=\overline{\lim\limits_{p\to\infty}}H_{q_p}$, $B_2=T-A_2$. The set Z is residual, $|A_2|\geqslant\delta_0$, and $x\epsilon Z$ implies $|A_2-\Omega_x|=0$, i.e. $\lim\limits_{n\to\infty}U_n(x,t)$ exists almost everywhere in A_2 for any $x\epsilon Z$. Applying now Theorem 2.2 we get (c); $|A_2|\geqslant\delta_0$ yields $|A_2|=\delta_0$ in virtue of the definition of δ_0; hence (d) holds too.

6. Theorems of Saks for polynomials in Λ-spaces. Using the method of [2], section 8, we can easily prove that Theorem 4.1 remains true if we suppose that the operations $U_{pq}(x,t)$ are (X_α,S)-polynomials of degree at most m. From this we deduce as in section 3 the following

6.1. Theorem. *Let the space X_α satisfy the postulate* (a_3) *and let $\{U_n(x,t)\}$ be a sequnce of (X_α,S)-polynomials of degree at most m. Then there exist elements x_1, x_2 and decompositions $T=A_1+B_1=$ $=A_2+B_2$ such that*

(a) $\overline{\lim\limits_{n\to\infty}}|U_n(x,t)|<\infty$ *almost everywhere in A_1 for any x,*

(b) $\overline{\lim\limits_{n\to\infty}}|U_n(x_1,t)|=\infty$ *in B_1,*

(c) $\lim\limits_{n\to\infty}U_n(x,t)$ *exists almost everywhere in A_2 for any x,*

(d) $\lim\limits_{n\to\infty}U_n(x_2,t)$ *does not exist in B_2.*

The following example shows that Theorem 6.2 may be false if we replace the hypothesis "X_α satisfies the postulate (a_3)" by "X_α satisfies the postulates (a_1) and (a_2)". Let X_α be the space $\mathfrak{S}_\nu$ ([3]), p. 220), let $T=[0,1]$ and let $\{I_n\}$ be a sequence of non-overlapping intervals such that $[0,1)=\sum\limits_{n=1}^{\infty}I_n$; $\zeta=\{\zeta_n^*\}$ being any element of $\mathfrak{S}_\nu$, put

$$U_n(\zeta,t)=\begin{cases} n\zeta_p^* & \text{for } t\epsilon I_p,\quad p=1,2,\ldots,\\ 0 & \text{elsewhere.}\end{cases}$$

The operations $U_n(\zeta,t)$ are $(\mathfrak{S}_\nu,S)$-linear, however neither (a) nor (b) holds for the sequence $\{U_n(\zeta,t)\}$. In fact, suppose that such a decomposition exists; then evidently $|A|=0$. There does not exist any element ζ for which (b) would hold with $B_2=T$, since $\zeta=\{\zeta_1^*,\ldots,\zeta_q^*,0,0,\ldots\}$ implies $U_n(\zeta,t)=0$ for $t\epsilon I_{q+1}+I_{q+2}+\ldots$

Bibliography.

[1] A. Alexiewicz, *On sequences of operations, (I)*, Studia Mathematica 11 (1950), p. 1-30.

[2] A. Alexiewicz, *On sequences of operations (II)*, ibidem, p. 200-236.

[3] A. Alexiewicz, *On sequences of operations (III)*, ibidem 12 (1951), p. 84-92.

[4] S. Banach, *Sur la convergence presque partout des fonctionnelles linéaires*, Bulletin des Sciences Mathématiques 50 (1926), p. 27-32 and 36-43.

[5] L. Kantorovitch, *Lineare halbgeordnete Räume*, Recueil Mathématique de Moscou (1937), p. 121-168.

[6] S. Mazur und W. Orlicz, *Grundlegende Eigenschaften der polynomischen Operationen*, Erste Mitteilung, Studia Mathematica 5 (1934), p. 50-68.

[7] S. Saks, *Sur les fonctionnelles de M. Banach et leurs applications aux développements des fonctions*, Fundamenta Mathematicae 10 (1928). p. 189-196.

[8] S. Saks, *On some functionals*, Transactions of the American Mathematical Society 35 (1932), p. 549-556.

[9] S. Saks, *On some functionals II*, ibidem 41 (1937), p. 160-170.

(Reçu par la Rédaction le 2. 3. 1949).