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Factorization of weakly continuous holomorphic mappings

by

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Abstract. We prove a basic property of continuous multilinear mappings between topological vector spaces, from which we derive an easy proof of the fact that a multilinear mapping (and a polynomial) between topological vector spaces is weakly continuous on weakly bounded sets if and only if it is weakly *uniformly* continuous on weakly bounded sets. This result was obtained in 1983 by Aron, Hervés and Valdivia for polynomials between Banach spaces, and it also holds if the weak topology is replaced by a coarser one. However, we show that it need not be true for a stronger topology, thus answering a question raised by Aron. As an application of the first result, we prove that a holomorphic mapping f between complex Banach spaces is weakly uniformly continuous on bounded subsets if and only if it admits a factorization of the form $f = g \circ S$, where S is a compact operator and g a holomorphic mapping.

Our aim is to give characterizations of polynomials and holomorphic mappings on Banach spaces, which are weakly uniformly continuous on bounded sets. The polynomials with this property have been studied by many authors: see, for instance, [2, 3, 4]. A reason for that interest is that they are uniform limits of finite type polynomials (assuming the approximation property on the dual space) [4, Proposition 2.7]. In the case of locally convex spaces, these classes of polynomials have been analysed in several places (see, e.g., [16]). The holomorphic mappings with weakly uniformly continuous restrictions to bounded sets have also been considered in various papers [3, 8, 16].

The paper is organized in three sections. In the first one we prove a basic result (Theorem 3) on continuity of multilinear mappings between

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topological vector spaces, roughly showing that a multilinear mapping is continuous on certain subsets if and only if the Cauchy nets contained in these subsets are mapped into Cauchy nets.

As an easy consequence, we show that if a multilinear mapping between topological vector spaces is weakly continuous on weakly bounded sets, then it is also *uniformly* continuous on them. This is also true if the weak topology is replaced by a coarser one, and the proof uses the fact that weakly bounded sets are weakly precompact.

This result extends and simplifies a well known theorem proved by Aron, Hervés and Valdivia [3, Theorem 2.9] for polynomials between Banach spaces. In fact, to prove Theorem 3, we just refine what they did in [3]. The first section helps to understand what it means for a multilinear mapping to be continuous or uniformly continuous on certain classes of subsets.

In the second section, we answer negatively a question of Richard Aron's which was open for a number of years. Namely, he asked if given any vector topology τ on a Banach space X, it is true that τ-continuity on bounded sets is always equivalent to uniform τ-continuity on bounded sets, for a polynomial on X. This was known to be the case when τ is the norm topology, the weak topology, or the weak-star topology on a dual space. We show that if the unit ball is not τ-precompact, then uniform τ-continuity does not necessarily follow from τ-continuity. To this end, we consider a locally convex topology, called ckw, defined as the finest locally convex topology on a Banach space having the same convergent sequences as the weak topology. This topology has been studied in various contexts [10, 18, 20]. Since the unit ball of $L_1[0,1]$ is not ckw-precompact, and we have a nice description of the ckw-topology on this space, we are able to construct a polynomial on $L_1[0,1]$ which is ckw-continuous on bounded sets, but is not uniformly ckw-continuous on bounded sets.

Before giving the content of Section 3, we recall two easy properties of weakly uniformly continuous mappings between Banach spaces: first, every mapping whose restrictions to bounded sets are weakly uniformly continuous, takes bounded sets into relatively compact sets [4, Lemma 2.2], and second, a linear bounded operator is compact if and only if it is weakly (uniformly) continuous on bounded sets [4, Proposition 2.5].

It is then clear that if a holomorphic mapping f between complex Banach spaces admits a factorization of the form $f = g \circ S$, where S is a compact operator, and g a holomorphic mapping, then f is weakly uniformly continuous on bounded sets. In Section 3, we apply the main result of the first part (Theorem 3) to show that these easy examples are the *only* ones, i.e., every holomorphic mapping whose restrictions to bounded sets are weakly uniformly continuous admits a factorization as above. We take advantage of

work by Braunsz and Junek: namely, some ideas of Theorem 14 below are contained in [6, Proposition 2.14].

Factorizations of holomorphic mappings have already been considered, but here the factors stand in inverse order. Thus, it is proved in [5] that a holomorphic mapping f is compact (i.e., takes a neighbourhood of each point into a relatively compact set) if and only if it admits a factorization of the form $f = S \circ g$, where g is a holomorphic mapping, and S a compact operator. A similar result is proved in [19] for weakly compact holomorphic mappings.

If X, Y are topological vector spaces, then with each k-homogeneous polynomial P from X into Y, there is associated a unique symmetric k-linear mapping $\widehat{P} : X \times \overset{(k)}{\ldots} \times X \to Y$, given by the polarization formula [17, Theorem 1.10], so that $\widehat{P}(x,\ldots,x) = P(x)$ for all $x \in X$. We refer to [7, 17] for the general theory of polynomials and holomorphic mappings on infinite-dimensional spaces.

The set of natural numbers is denoted by $\mathbb{N}$. A net in a topological vector space is said to be (*weakly*) *null* if it is (weakly) convergent to zero. If X is a Banach space, then B_X stands for its closed unit ball, and X^* for its dual. By an *operator* we mean a linear continuous mapping.

1. Uniformly continuous polynomials. In this part, we give a basic property of multilinear mappings, and derive an easy proof of the equivalence between weak continuity and weak uniform continuity on weakly bounded sets.

DEFINITION 1. A family $\mathcal{B}$ of subsets of a vector space X is said to be a *bornology* if it satisfies the following conditions:

(a) $\mathcal{B}$ covers X;
(b) $A \in \mathcal{B}$, $D \subset A \Rightarrow D \in \mathcal{B}$;
(c) $A, D \in \mathcal{B} \Rightarrow A \cup D \in \mathcal{B}$;
(d) for every $A \in \mathcal{B}$, and every scalar λ, we have $\lambda A \in \mathcal{B}$;
(e) $A, D \in \mathcal{B} \Rightarrow A + D \in \mathcal{B}$.

If X is a topological vector space, examples of bornologies on X are the family of all subsets, the family of all (weakly) bounded subsets, the (weakly) compact sets, etc. Given a bornology $\mathcal{B}$ on X, we say that $A \subseteq X$ is a *$\mathcal{B}$-set* if $A \in \mathcal{B}$. We say that a net $(x_\alpha)_{\alpha\in\Gamma} \subset X$ is a *$\mathcal{B}$-net* if $\{x_\alpha : \alpha \in \Gamma\} \in \mathcal{B}$.

DEFINITION 2. Given topological vector spaces $X_1, \ldots, X_k, Y$, a k-linear (not necessarily continuous) mapping $A : X_1 \times \ldots \times X_k \to Y$, and a bornology $\mathcal{B}_j$ on X_j for each $1 \le j \le k$, we say that A is *continuous on $\mathcal{B}_j$-sets* if for each $B_j \in \mathcal{B}_j$, $x^j \in B_j$ $(1 \le j \le k)$, and each zero neighbourhood

V in Y, there are zero neighbourhoods U_j in X_j so that

$$A(y^1,\dots,y^k)-A(x^1,\dots,x^k)\in V$$

whenever $y^j\in B_j$ satisfy $y^j-x^j\in U_j$ for $1\le j\le k$.

Clearly, A is continuous on $\mathcal{B}_j$-sets if and only if, given a convergent $\mathcal{B}_j$-net $x^j_\alpha\to x^j$ ($\alpha\in\Gamma$) in X_j for each $1\le j\le k$, the net $(A(x^1_\alpha,\dots,x^k_\alpha))_{\alpha\in\Gamma}$ converges to $A(x^1,\dots,x^k)$ in Y.

THEOREM 3. *Let $X_1,\dots,X_k,Y$ be topological vector spaces, and $A: X_1\times\dots\times X_k\to Y$ a k-linear (not necessarily continuous) mapping. Let $\mathcal{B}_j$ be a bornology on X_j ($1\le j\le k$). Then the following assertions are equivalent*:

(a) *A is continuous on $\mathcal{B}_j$-sets*;

(b) *given Cauchy $\mathcal{B}_j$-nets $(x^j_\alpha)_{\alpha\in\Gamma}\subset X_j$ ($1\le j\le k$) such that at least one of them is null, the net $(A(x^1_\alpha,\dots,x^k_\alpha))_{\alpha\in\Gamma}$ converges to zero in Y*;

(c) *given a Cauchy $\mathcal{B}_j$-net $(x^j_\alpha)_{\alpha\in\Gamma}$ in X_j for each $1\le j\le k$, the net $(A(x^1_\alpha,\dots,x^k_\alpha))_{\alpha\in\Gamma}$ is Cauchy in Y.*

Proof. (a)⇒(b). If $k=1$, there is nothing to prove. Assume the result is true for all $(k-1)$-linear mappings and fails for the k-linear mapping A. Then we can find Cauchy $\mathcal{B}_j$-nets $(x^j_\alpha)_{\alpha\in\Gamma}\subset X_j$ ($1\le j\le k$), at least one of which is null (to simplify notation, assume $x^1_\alpha\to0$), and a zero neighbourhood V_1 in Y such that

$$A(x^1_\alpha,\dots,x^k_\alpha)\notin V_1\quad(\alpha\in\Gamma).$$

Let V_2 be a zero neighbourhood such that $V_2+V_2\subseteq V_1$. For each fixed $\alpha\in\Gamma$, the mapping Ax^k_α given by

$$Ax^k_\alpha(z^1,\dots,z^{k-1}):=A(z^1,\dots,z^{k-1},x^k_\alpha)\quad(z^j\in X_j;\ 1\le j\le k-1)$$

is $(k-1)$-linear and takes convergent $\mathcal{B}_j$-nets into convergent nets. By induction, there is $\kappa(\alpha)\in\Gamma$ so that

$$A(x^1_\beta,\dots,x^{k-1}_\beta,x^k_\alpha)=Ax^k_\alpha(x^1_\beta,\dots,x^{k-1}_\beta)\in V_2\quad(\beta\ge\kappa(\alpha)).$$

For every $\alpha\in\Gamma$, we have

$$\begin{aligned}A(x^1_{\kappa(\alpha)},\dots,&x^{k-1}_{\kappa(\alpha)},x^k_{\kappa(\alpha)}-x^k_\alpha)\\&=A(x^1_{\kappa(\alpha)},\dots,x^k_{\kappa(\alpha)})-A(x^1_{\kappa(\alpha)},\dots,x^{k-1}_{\kappa(\alpha)},x^k_\alpha)\notin V_2.\end{aligned}$$

Consider the vectors

$$y^j_\alpha:=\begin{cases}x^j_{\kappa(\alpha)} & \text{if } 1\le j\le k-1,\\ x^k_{\kappa(\alpha)}-x^k_\alpha & \text{if } j=k.\end{cases}$$

We can assume $\kappa(\alpha)\ge\alpha$. This condition ensures that the $\mathcal{B}_j$-nets $(y^j_\alpha)_{\alpha\in\Gamma}\subset X_j$ ($1\le j\le k$) are Cauchy, and at least two of them are null. By repeating the process, we obtain $\mathcal{B}_j$-nets $(z^j_\alpha)_{\alpha\in\Gamma}\subset X_j$ ($1\le j\le k$), all of them null, and a zero neighbourhood V_k in Y so that

$$A(z^1_\alpha,\dots,z^k_\alpha)\notin V_k\quad(\alpha\in\Gamma).$$

This contradicts our assumption (a).

(b)⇒(c). Let $(x^j_\alpha)_{\alpha\in\Gamma}\subset X_j$ ($1\le j\le k$) be Cauchy $\mathcal{B}_j$-nets. We have

$$\begin{aligned}&A(x^1_\alpha,\dots,x^k_\alpha)-A(x^1_\beta,\dots,x^k_\beta)\\&\quad=A(x^1_\alpha-x^1_\beta,x^2_\alpha,\dots,x^k_\alpha)+A(x^1_\beta,x^2_\alpha-x^2_\beta,x^3_\alpha,\dots,x^k_\alpha)+\dots\\&\qquad+A(x^1_\beta,\dots,x^{k-1}_\beta,x^k_\alpha-x^k_\beta).\end{aligned}$$

In each of the above terms, letting $(\alpha,\beta)\in\Gamma\times\Gamma$ increase, all the nets are Cauchy $\mathcal{B}_j$-nets, and at least one of them is null. Then

$$\lim_{\alpha,\beta}[A(x^1_\alpha,\dots,x^k_\alpha)-A(x^1_\beta,\dots,x^k_\beta)]=0,$$

and therefore $(A(x^1_\alpha,\dots,x^k_\alpha))_\alpha$ is a Cauchy net.

(c)⇒(a). Suppose A is not continuous on $\mathcal{B}_j$-sets. Then we can find sets $B_j\in\mathcal{B}_j$, points $x^j\in B_j$, and a zero neighbourhood V in Y so that for every zero neighbourhood U_j in X_j ($1\le j\le k$) there is $y_{U_j}\in B_j$ with $y_{U_j}-x^j\in U_j$ but

$$A(y_{U_1},\dots,y_{U_k})-A(x^1,\dots,x^k)\notin V.$$

Let $\mathcal{U}_j$ be the family of all zero neighbourhoods in X_j, and $\mathcal{U}:=\mathcal{U}_1\times\dots\times\mathcal{U}_k\times\mathbb{N}$, ordered in the natural way. For each $U=(U_1,\dots,U_k,n)\in\mathcal{U}$, and $j\in\{1,\dots,k\}$, let

$$z^j_U:=\begin{cases}y_{U_j} & \text{if } n \text{ is even},\\ x^j & \text{if } n \text{ is odd}.\end{cases}$$

Then $(z^j_U)_{U\in\mathcal{U}}$ is a Cauchy $\mathcal{B}_j$-net. However, the net $(A(z^1_U,\dots,z^k_U))_{U\in\mathcal{U}}$ is not Cauchy. ■

If, for each j, we take as $\mathcal{B}_j$ the bornology of all subsets of X_j, the assertion (a) in the last theorem simply states that A is continuous.

DEFINITION 4. Let $f: X\to Y$ be a mapping between topological vector spaces, and $\mathcal{B}$ a bornology on X. We say that f is *uniformly continuous on $\mathcal{B}$-sets* if for every zero neighbourhood V in Y and every $B\in\mathcal{B}$ there is a zero neighbourhood U in X such that $f(x)-f(y)\in V$ whenever $x,y\in B$ satisfy $x-y\in U$. This definition may be adapted to multilinear mappings in an obvious way.

Recall that a subset B of a topological vector space X is *precompact* if for every zero neighbourhood U in X there is a finite set $M\subseteq B$ such that $B\subseteq M+U$. It is well known that B is precompact if and only if every net in B has a Cauchy subnet [14, Theorem 6.32]. We shall use this fact in the proof of the following result, which relates uniform continuity to the properties considered in Theorem 3.

THEOREM 5. *Let X, Y be topological vector spaces, and $\mathcal{B}$ a bornology of precompact sets in X. Then a mapping $f : X \to Y$ is uniformly continuous on $\mathcal{B}$-sets if and only if it takes Cauchy $\mathcal{B}$-nets into Cauchy nets.*

Proof. Let f be uniformly continuous on $\mathcal{B}$-sets, $(x_\alpha) \subset X$ a Cauchy $\mathcal{B}$-net, and V a zero neighbourhood in Y. There is a zero neighbourhood $U \subset X$ so that whenever $x_\alpha - x_\beta \in U$, then $f(x_\alpha) - f(x_\beta) \in V$. Now, since (x_α) is Cauchy, there is α_0 such that

$$x_\alpha - x_\beta \in U \quad (\alpha, \beta \geq \alpha_0),$$

and hence $(f(x_\alpha))$ is Cauchy.

Conversely, assume f is not uniformly continuous on $\mathcal{B}$-sets. Then we can find $B \in \mathcal{B}$ and a zero neighbourhood $V \subset Y$ so that for every zero neighbourhood $U \subset X$, there are $x_U, y_U \in B$ with $x_U - y_U \in U$ and $f(x_U) - f(y_U) \notin V$.

Let $\mathcal{U}$ be the family of all zero neighbourhoods in X. Since every $\mathcal{B}$-net has a Cauchy subnet, we can assume that the nets $(x_U)_{U\in\mathcal{U}}$ and $(y_U)_{U\in\mathcal{U}}$ are Cauchy. Consider the set $\mathcal{W} := \mathcal{U} \times \mathbb{N}$, ordered in the natural way. With each $W = (U, i) \in \mathcal{W}$ we associate

$$z_W := \begin{cases} x_U & \text{if } i \text{ is even,} \\ y_U & \text{if } i \text{ is odd.} \end{cases}$$

Then the $\mathcal{B}$-net (z_W) is Cauchy. However, $(f(z_W))$ is not Cauchy. ∎

Clearly, Theorem 5 is also valid for multilinear mappings, with obvious modifications.

Remark 6. If $f : X \to Y$ satisfies the hypotheses of Theorem 5, then $f(B)$ is precompact for each $B \in \mathcal{B}$. The converse is not true. Indeed, by modifying an example given in [15, p. 82], we now construct a real-valued function f on a Banach space with the following conditions:

(a) f is weakly continuous on bounded sets (it will even be weakly continuous on the whole space);

(b) f takes bounded sets into precompact sets, i.e., f is bounded on bounded sets;

(c) f is not weakly uniformly continuous on bounded sets.

Let X be a separable, nonreflexive Banach space. By James' theorem, we can find $\phi \in X^*$, with $\|\phi\| = 1$, which does not attain its norm on B_X. Let

$$g(x) := \frac{1}{\phi(x) - 1} \quad (x \in B_X).$$

Since X is normal for the weak topology, and g is weakly continuous, g admits an extension $\widetilde{g}$ to X which is weakly continuous. Since $\widetilde{g}$ is unbounded on B_X, it is not weakly uniformly continuous on B_X. Therefore, there is $\delta > 0$ so that for each convex, weak zero neighbourhood U in X we can find $x, y \in B_X$ with $x - y \in U$ and $|\widetilde{g}(x) - \widetilde{g}(y)| > \delta$. The segment $[x, y]$ is clearly contained in $B_X \cap (y + U)$. Choose $\lambda > \pi\delta^{-1}$, and define

$$f(x) := \sin(\lambda\widetilde{g}(x)) \quad (x \in X).$$

Clearly, f is weakly continuous and bounded. However, it is not weakly uniformly continuous on B_X since we can find $z \in [x, y]$ so that $|f(z) - f(y)| \geq 1$.

COROLLARY 7. *Let $X_1, \ldots, X_k, Y$ be topological vector spaces. Let τ_j be a vector topology on X_j coarser than or equal to the weak topology, and $\mathcal{B}_j$ a bornology of* weakly bounded *sets on X_j $(1 \leq j \leq k)$. Then a k-linear mapping from $X_1 \times \ldots \times X_k$ into Y is τ_j-continuous on $\mathcal{B}_j$-sets if and only if it is uniformly τ_j-continuous on $\mathcal{B}_j$-sets.*

Proof. Since the weakly bounded sets coincide with the weakly precompact sets [13, Corollary 8.1.6], every $\mathcal{B}_j$-set is τ_j-precompact. Therefore, it is enough to apply Theorems 3 and 5. ∎

Using the polarization formula, it is clear that a polynomial P between topological vector spaces takes convergent $\mathcal{B}$-nets into convergent nets if and only if so does $\widehat{P}$, and that P takes Cauchy $\mathcal{B}$-nets into Cauchy nets if and only if so does $\widehat{P}$. Therefore, we obtain:

COROLLARY 8. *Let X, Y be topological vector spaces. Let τ be a vector topology on X coarser than or equal to the weak topology, and $\mathcal{B}$ a bornology on X consisting of* weakly bounded *sets. Then a homogeneous polynomial from X into Y is τ-continuous on $\mathcal{B}$-sets if and only if it is uniformly τ-continuous on $\mathcal{B}$-sets.*

The particular case when $\mathcal{B}$ is the bornology of bounded sets in a Banach space, and τ the weak topology, was proved in [3, Theorem 2.9]. The result for $\mathcal{B}$ being the bornology of Rosenthal sets, or Dunford–Pettis sets in a Banach space, and τ the weak topology, was obtained in [11, Proposition 3.6 and the comment after it].

2. A counterexample. In this section, we show that the rôle of precompactness in Corollaries 7 and 8 is essential. We shall consider a topology on $L_1[0,1]$, compatible with the dual pairing $\langle L_1[0,1], L_\infty[0,1]\rangle$, for which the unit ball is not precompact, and give an example of a polynomial not satisfying the conclusion of Corollary 8 for this topology.

If X and Y are Banach spaces, the space of k-homogeneous (continuous) polynomials from X into Y is denoted by $\mathcal{P}({}^kX; Y)$, and that of k-linear (continuous) mappings from $X^k = X\times \overset{(k)}{\cdots} \times X$ into Y by $\mathcal{L}({}^kX;Y)$. If Y is omitted, it is understood to be the scalar field.

The *ckw* topology [10] on a Banach space X is the finest locally convex topology having the same convergent sequences as the weak topology. On an infinite-dimensional space, it is strictly finer than the weak topology, and on a Banach space without the Schur property, it is strictly coarser than the norm topology. It is therefore a topology compatible with the pairing $\langle X, X^*\rangle$, and so its bounded sets are the norm bounded sets.

Given a Banach space X, a subset K of its dual X^* is an (L)-*set* if, for every weakly null sequence $(x_n)\subset X$, we have

$$\lim_n \sup_{\phi\in K} |\langle x_n, \phi\rangle| = 0.$$

The *ckw* topology on a Banach space turns out to be the topology of uniform convergence on (L)-subsets of the dual [10, Theorem 3.1]. A subset is *ckw*-precompact if and only if each sequence in it has a weak Cauchy subsequence [10, Theorem 4.4]. An operator between Banach spaces is *ckw*-to-norm continuous if and only if it is *completely continuous*, i.e., it takes weakly null sequences into norm null sequences [10, Proposition 3.2].

If a Banach space contains a copy of ℓ_1, then its unit ball will not be *ckw*-precompact. This is the case, for instance, of $L_1[0,1]$. Moreover, it is proved in [10, Theorem 3.7] that a bounded subset K of $L_\infty[0,1] = L_1[0,1]^*$ is an (L)-set if and only if it is relatively compact as a subset of $L_1[0,1]$, when we consider $L_\infty[0,1]$ embedded in $L_1[0,1]$ by means of the identity map.

We first give a result whose proof uses standard techniques. With each polynomial $P\in\mathcal{P}({}^kX;Y)$, we associate an operator

$$T_P : X \to \mathcal{L}({}^{k-1}X,Y)$$

given by

$$T_P(x)(x_1,\ldots,x_{k-1}) := \widehat{P}(x,x_1,\ldots,x_{k-1})$$

for $x, x_1,\ldots,x_{k-1}\in X$.

PROPOSITION 9. *Let τ be a vector topology on a Banach space X, and $P\in\mathcal{P}({}^kX;Y)$. Then P is uniformly τ-continuous on bounded sets if and only if so is T_P.*

Proof. Suppose P is uniformly τ-continuous on B_X. Given $\varepsilon>0$, we can find a balanced τ zero neighbourhood U in X so that $\|Px-Py\|<\varepsilon$ whenever $x,y\in B_X$ satisfy $x-y\in U$.

Assume x,y satisfy the above conditions, and let $z_2,\ldots,z_k\in B_X$. By the polarization formula,

$$\begin{aligned}(T_P(x) - T_P(y))(z_2,\ldots,z_k) &= \widehat{P}(x,z_2,\ldots,z_k) - \widehat{P}(y,z_2,\ldots,z_k)\\ &= \frac{k^k}{2^k k!}\sum_{\varepsilon_j=\pm1} \varepsilon_1\ldots\varepsilon_k\\ &\quad\times\Big[P\Big(\frac{\varepsilon_1 x+\varepsilon_2 z_2+\ldots+\varepsilon_k z_k}{k}\Big) - P\Big(\frac{\varepsilon_1 y+\varepsilon_2 z_2+\ldots+\varepsilon_k z_k}{k}\Big)\Big].\end{aligned}$$

We easily conclude that

$$\|T_P(x)-T_P(y)\| \le \varepsilon\,\frac{k^k}{k!},$$

and T_P is uniformly τ-continuous on B_X.

Conversely, let T_P be uniformly τ-continuous on B_X. For $0<\varepsilon<1$, there is a τ zero neighbourhood $U\subset X$ so that $\|T_P(x)-T_P(y)\|<\varepsilon$ whenever $x,y\in B_X$ satisfy $x-y\in U$. For such x,y, we have

$$\begin{aligned}\|Px-Py\| &\le \|\widehat{P}(x,\ldots,x) - \widehat{P}(x,y,x,\ldots,x)\|\\ &\quad + \|\widehat{P}(x,y,x,\ldots,x) - \widehat{P}(x,y,y,x,\ldots,x)\| + \ldots\\ &\quad + \|\widehat{P}(x,y,\ldots,y) - \widehat{P}(y,\ldots,y)\|\\ &= \|(T_P(x)-T_P(y))(x,\ldots,x)\|\\ &\quad + \|(T_P(x)-T_P(y))(x,y,x,\ldots,x)\| + \ldots\\ &< k\varepsilon,\end{aligned}$$

and P is uniformly τ-continuous on B_X, which completes the proof. ■

Since T_P is linear, T_P is uniformly τ-continuous on bounded sets if and only if it is τ-continuous on bounded sets. Therefore, we easily obtain:

COROLLARY 10. *A polynomial $P\in\mathcal{P}({}^kX;Y)$ is uniformly ckw-continuous on bounded sets if and only if the associated operator T_P is completely continuous.*

We are now ready to construct our polynomial, which is a modification of an example given in [1]. To this end, we divide the interval $(0,1)$ into subintervals

$$I_j := (1/2^j, 1/2^{j-1}) \quad (j\in\mathbb{N}),$$

and denote by χ_j the characteristic function of I_j. Let (r_n) be the Rademacher functions on $[0,1]$, given by

$$r_n(t) := \operatorname{sign}\sin 2^n\pi t \quad (t\in[0,1]).$$

We easily obtain

$$\langle r_n, \chi_j\rangle = \begin{cases} 0, & n > j, \\ -2^{-j}, & n = j, \\ 2^{-j}, & n < j. \end{cases}$$

Note that for each $f \in L_1[0,1]$ we have

$$\sum_{j=1}^{\infty} |\langle f, \chi_j\rangle| \le \|f\|. \tag{1}$$

For $f, g, h \in L_1[0,1]$, define

$$A(f,g,h) := \sum_{j=1}^{\infty} \langle f, \chi_j\rangle\langle g, \chi_j\rangle\langle h, r_j\rangle.$$

Since

$$|A(f,g,h)| \le \|g\|\cdot\|h\|\cdot \sum_{j=1}^{\infty} |\langle f, \chi_j\rangle| \le \|f\|\cdot\|g\|\cdot\|h\|,$$

we see that A is a 3-linear continuous form on $L_1[0,1]$. Then the function

$$P(f) := A(f,f,f) \quad (f \in L_1[0,1])$$

is a 3-homogeneous continuous polynomial on $L_1[0,1]$.

PROPOSITION 11. *The polynomial P is ckw-continuous on bounded sets of $L_1[0,1]$, but is not uniformly ckw-continuous on bounded sets.*

Proof. We first show that the associated operator T_P is not completely continuous. By Corollary 10 this will imply that P is not uniformly *ckw*-continuous on bounded sets. Since

$$\widehat{P}(f,g,h) = \tfrac{1}{3}[A(f,g,h) + A(h,f,g) + A(g,h,f)],$$

the operator $T_P : L_1[0,1] \to \mathcal{L}(^2L_1[0,1])$ is given by

$$T_P(f) = \tfrac{1}{3}[A(\cdot,\cdot,f) + A(f,\cdot,\cdot) + A(\cdot,f,\cdot)].$$

Then

$$3T_P(r_n) = \sum_{j=1}^{\infty} [\langle\cdot,\chi_j\rangle^2\langle r_n, r_j\rangle + 2\langle\cdot,\chi_j\rangle\langle\cdot,r_j\rangle\langle r_n,\chi_j\rangle]$$
$$= \langle\cdot,\chi_n\rangle^2 - 2\cdot 2^{-n}\langle\cdot,\chi_n\rangle\langle\cdot,r_n\rangle + 2\sum_{j=n+1}^{\infty} 2^{-j}\langle\cdot,\chi_j\rangle\langle\cdot,r_j\rangle.$$

Since $\|\langle\cdot,\chi_n\rangle^2\| = 1$, and

$$\Big\| -2\cdot 2^{-n}\langle\cdot,\chi_n\rangle\langle\cdot,r_n\rangle + 2\sum_{j=n+1}^{\infty} 2^{-j}\langle\cdot,\chi_j\rangle\langle\cdot,r_j\rangle\Big\| \le 4\cdot 2^{-n},$$

we see that $\|T_P(r_n)\|$ does not converge to zero. Since (r_n) is a weakly null sequence in $L_1[0,1]$, this shows that T_P is not completely continuous.

Let us now prove that P is *ckw*-continuous on bounded sets, in other words, given $f_0 \in L_1[0,1]$, $\varepsilon > 0$ and $r \ge \|f_0\|$, there exists an (L)-set $K \subset L_1[0,1]^*$ so that $|P(f) - P(f_0)| < \varepsilon$ whenever $f \in L_1[0,1]$ satisfies $\|f\| \le r$ and $f - f_0 \in {}^\circ K$, where

$${}^\circ K := \{g \in L_1[0,1] : |\langle g, h\rangle| \le 1 \text{ for all } h \in K\}.$$

Indeed, choose $\delta > 0$ with

$$5r^2\delta < \varepsilon. \tag{2}$$

Thanks to (1), we can find $j_0 \in \mathbb{N}$ so that

$$|\langle f_0, \chi_j\rangle| < \delta \quad \text{for every } j > j_0. \tag{3}$$

The set

$$K := \{\delta^{-1}\chi_j : j \in \mathbf{N}\} \cup \{\delta^{-1}r_j : 1 \le j \le j_0\}$$

is an (L)-set in $L_1[0,1]^*$. Moreover, for $f \in L_1[0,1]$, we can write

$$P(f) - P(f_0)$$
$$= A(f, f - f_0, f) + A(f, f_0, f - f_0) + A(f - f_0, f_0, f_0)$$
$$= \sum_{j=1}^{\infty}\langle f,\chi_j\rangle\langle f - f_0,\chi_j\rangle\langle f, r_j\rangle + \sum_{j=1}^{j_0}\langle f,\chi_j\rangle\langle f_0,\chi_j\rangle\langle f - f_0, r_j\rangle$$
$$+ \sum_{j=j_0+1}^{\infty}\langle f,\chi_j\rangle\langle f_0,\chi_j\rangle\langle f - f_0, r_j\rangle + \sum_{j=1}^{\infty}\langle f - f_0,\chi_j\rangle\langle f_0,\chi_j\rangle\langle f_0, r_j\rangle.$$

If $f - f_0 \in {}^\circ K$, we have $|\langle f - f_0, r_j\rangle| \le \delta$ for $1 \le j \le j_0$, and $|\langle f - f_0, \chi_j\rangle| \le \delta$ for all $j \in \mathbb{N}$. Using these two inequalities along with (1)–(3), we get

$$|P(f) - P(f_0)|$$
$$\le \|f\|\cdot\delta\cdot\|f\| + \|f\|\cdot\|f_0\|\cdot\delta + \|f\|\cdot\delta\cdot\|f - f_0\| + \delta\cdot\|f_0\|^2$$
$$\le 5r^2\delta < \varepsilon,$$

and the proof is complete. ■

When we consider the norm topology, we do conclude that if a polynomial is norm continuous (on bounded sets), it is also uniformly continuous on bounded sets, and however the unit ball is not norm precompact in infinite-dimensional Banach spaces. This is due to the fact that the balls centered at the origin constitute a base of zero neighbourhoods, and a polynomial on a locally convex space is continuous if and only if it is uniformly continuous on some zero neighbourhood [9].

3. Factorization results. In this part, we apply Theorem 3 to prove that a holomorphic mapping f is weakly uniformly continuous on bounded sets if and only if it may be written in the form $f = g \circ S$, where S is a compact operator, and g a holomorphic mapping.

For Banach spaces $X_1, \ldots, X_k$ and Y, we use $\mathcal{L}(X_1, \ldots, X_k; Y)$ to represent the space of k-linear (continuous) mappings from $X_1 \times \ldots \times X_k$ into Y. The space of compact operators from X into Y is denoted by $\mathcal{C}o(X; Y)$.

We denote by $\mathcal{L}_{\text{wb}}(X_1, \ldots, X_k; Y)$ the space of k-linear mappings which are weakly continuous on bounded sets, in the sense of Definition 2. We define the spaces $\mathcal{L}_{\text{wb}}({}^kX; Y)$ and $\mathcal{P}_{\text{wb}}({}^kX; Y)$ in an analogous way.

For each $1 \le i \le k$, consider the mapping

$$\xi_i : \mathcal{L}(X_1, \ldots, X_k; Y) \to \mathcal{L}(X_1, \ldots, X_i; \mathcal{L}(X_{i+1}, \ldots, X_k; Y))$$

taking A into $\overline{A}$ given by

$$\overline{A}(x_1, \ldots, x_i)(x_{i+1}, \ldots, x_k) := A(x_1, \ldots, x_k)$$

for $x_j \in X_j$ $(1 \le j \le k)$. It is well known that ξ_i is a linear surjective isometry.

PROPOSITION 12. *Given Banach spaces $X_1, \ldots, X_k, Y$, the isometry ξ_i maps the space $\mathcal{L}_{\text{wb}}(X_1, \ldots, X_k; Y)$ onto the space*

$$\mathcal{L}_{\text{wb}}(X_1, \ldots, X_i; \mathcal{L}_{\text{wb}}(X_{i+1}, \ldots, X_k; Y)).$$

Proof. If A is weakly continuous on bounded sets, it is clear that the range of $\overline{A}$ lies in $\mathcal{L}_{\text{wb}}(X_{i+1}, \ldots, X_k; Y)$. Now, let bounded, weak Cauchy nets $(x_\alpha^j) \subset X_j$ be given for $1 \le j \le i$, at least one of which being weakly null. Suppose we have

$$\|\overline{A}(x_\alpha^1, \ldots, x_\alpha^i)\| > \delta$$

for some $\delta > 0$. Then we can find nets $(x_\alpha^j) \subset B_{X_j}$ $(i+1 \le j \le k)$, that can be assumed to be weak Cauchy, so that

$$\|A(x_\alpha^1, \ldots, x_\alpha^k)\| = \|\overline{A}(x_\alpha^1, \ldots, x_\alpha^i)(x_\alpha^{i+1}, \ldots, x_\alpha^k)\| > \delta,$$

a contradiction (Theorem 3). Applying again Theorem 3, we conclude that $\overline{A}$ is weakly continuous on bounded sets.

Conversely, if $\overline{A} \in \mathcal{L}_{\text{wb}}(X_1, \ldots, X_i; \mathcal{L}_{\text{wb}}(X_{i+1}, \ldots, X_k; Y))$ and, for each $1 \le j \le k$, a bounded net $(x_\alpha^j) \subset X_j$ is given which converges weakly to x^j, we easily see that $A \in \mathcal{L}_{\text{wb}}(X_1, \ldots, X_k; Y)$ by writing

$$\begin{aligned}&\|A(x_\alpha^1, \ldots, x_\alpha^k) - A(x^1, \ldots, x^k)\| \\ &\quad \le \|\overline{A}(x_\alpha^1, \ldots, x_\alpha^i)(x_\alpha^{i+1}, \ldots, x_\alpha^k) - \overline{A}(x^1, \ldots, x^i)(x_\alpha^{i+1}, \ldots, x_\alpha^k)\| \\ &\quad\quad + \|\overline{A}(x^1, \ldots, x^i)(x_\alpha^{i+1}, \ldots, x_\alpha^k) - \overline{A}(x^1, \ldots, x^i)(x^{i+1}, \ldots, x^k)\|.\end{aligned}$$

This completes the proof. ∎

Our next result shows that P factorizes if and only if so does $\widehat{P}$.

PROPOSITION 13. *Let $\mathcal{U}$ be an operator ideal, and $P \in \mathcal{P}({}^kX; Y)$ for Banach spaces X, Y. The following assertions are equivalent:*

(a) *there are a Banach space Z, an operator $S \in \mathcal{U}(X; Z)$, and a polynomial $Q \in \mathcal{P}({}^kZ; Y)$ so that $P = Q \circ S$;*

(b) *there are Banach spaces Z_i, operators $S_i \in \mathcal{U}(X; Z_i)$ $(1 \le i \le n)$, and a k-linear mapping $B \in \mathcal{L}(Z_1, \ldots, Z_k; Y)$ so that $\widehat{P} = B \circ (S_1, \ldots, S_k)$.*

Moreover, if (b) *is satisfied, then we can choose S and Q in* (a) *so that $\|S\| = \max \|S_i\|$ and $\|Q\| = \|B\|$.*

Proof. (a)⇒(b). Take $Z_i = Z$, $S_i = S$ $(1 \le i \le k)$, and $B = \widehat{Q}$.

(b)⇒(a). Take $Z = Z_1 \times \ldots \times Z_k$, $Sx := (S_1x, \ldots, S_kx)$ for all $x \in X$, and

$$Q((z_1, \ldots, z_k)) := B(z_1, \ldots, z_k).$$

To see that Q is a polynomial, note that its associated symmetric k-linear mapping is

$$\widehat{Q}((z_1^1, \ldots, z_k^1), \ldots, (z_1^k, \ldots, z_k^k)) := \frac{1}{k!} \sum B(z_1^{i_1}, \ldots, z_k^{i_k}),$$

where the sum is taken over all permutations $(i_1, \ldots, i_k)$ of $(1, \ldots, k)$. Endowing Z with the supremum norm, we obtain $\|S\| = \max \|S_i\|$ and $\|Q\| = \|B\|$. ∎

In the proof of the next theorem, we shall use the well known fact that for every compact operator $T : X \to Y$ between Banach spaces, we can find a space Z and compact operators $S : X \to Z$ and $R : Z \to Y$ so that $T = R \circ S$ [13, Theorem 17.1.4]. We are indebted to H. Junek who pointed out that, as shown in [12, Lemma 1.2], we can renorm Z so that $\|R\| \cdot \|S\| = \|T\|$. As usual, we denote by $c_0(X)$ the Banach space of all null sequences in X, endowed with the supremum norm.

THEOREM 14. *Given Banach spaces $X_1, \ldots, X_k, Y$ and a number $\varepsilon > 0$, for each $A \in \mathcal{L}_{\text{wb}}(X_1, \ldots, X_k; Y)$ there are Banach spaces $Z_1, \ldots, Z_k$, operators $S_i \in \mathcal{C}o(X_i; Z_i)$ and a mapping $B \in \mathcal{L}_{\text{wb}}(Z_1, \ldots, Z_k; Y)$ so that*

$$A(x_1, \ldots, x_k) = B(S_1x_1, \ldots, S_kx_k) \quad (x_i \in X_i),$$

and $\|B\| \cdot \|S_1\| \cdot \ldots \cdot \|S_k\| \le (1+\varepsilon)\|A\|$.

Proof. For $k = 1$, we have the above mentioned result for linear operators. Assume the theorem is true for $(k-1)$-linear mappings $(k > 1)$, and take $\lambda := (1+\varepsilon)^{1/k}$. Given $A \in \mathcal{L}_{\text{wb}}(X_1, \ldots, X_k; Y)$, Proposition 12 provides an associated

$$\overline{A} \in \mathcal{L}_{\text{wb}}(X_2, \ldots, X_k; \mathcal{C}o(X_1; Y)).$$

By induction, we can write $\overline{A} = \overline{D} \circ (S_2, \ldots, S_k)$ with $S_i \in \mathcal{C}o(X_i; Z_i)$ $(2 \le i \le k)$, $\overline{D} \in \mathcal{L}_{\mathrm{wb}}(Z_2, \ldots, Z_k; \mathcal{C}o(X_1; Y))$, and

$$\|\overline{D}\| \cdot \|S_2\| \cdot \ldots \cdot \|S_k\| \le \lambda^{k-1} \|\overline{A}\|.$$

By Proposition 12, we associate with $\overline{D}$ an operator

$$D \in \mathcal{C}o(Z_2; \mathcal{C}o(X_1; \mathcal{L}_{\mathrm{wb}}(Z_3, \ldots, Z_k; Y))).$$

Since D is compact, there is a sequence $(D_n) \subset \mathcal{C}o(X_1; \mathcal{L}_{\mathrm{wb}}(Z_3, \ldots, Z_k; Y))$ with $\|D_n\| \to 0$ so that $D(B_{Z_2})$ is contained in the absolutely convex, closed hull of $\{D_n\}$, and

$$\lambda^{-1} \le \|D\| / \sup \|D_n\|.$$

Define

$$T : X_1 \to c_0(\mathcal{L}_{\mathrm{wb}}(Z_3, \ldots, Z_k; Y))$$

by $Tx_1 := (D_n x_1)_{n=1}^{\infty}$. Clearly, T is compact and so we can find a space Z_1 and compact operators $S_1 : X_1 \to Z_1$ and $R : Z_1 \to c_0(\mathcal{L}_{\mathrm{wb}}(Z_3, \ldots, Z_k; Y))$ with $S_1(X_1)$ dense in Z_1, such that $T = R \circ S_1$ and $\|R\| \cdot \|S_1\| = \|T\|$. Define a linear mapping

$$U : T(X_1) \to \mathcal{L}_{\mathrm{wb}}(Z_2, \ldots, Z_k; Y)$$

by

$$U(Tx_1)(z_2, \ldots, z_k) := (Dz_2)(x_1)(z_3, \ldots, z_k)$$

for $x_1 \in X_1$, $z_2 \in Z_2, \ldots, z_k \in Z_k$. Clearly, U is well defined. If $\|z_2\| = 1$, we have $Dz_2 = \sum_{n=1}^{\infty} \lambda_n D_n$ with $\sum_{n=1}^{\infty} |\lambda_n| \le 1$. Then

$$\begin{aligned}
\|U(Tx_1)(z_2, \ldots, z_k)\| &= \Big\| \sum_{n=1}^{\infty} \lambda_n (D_n x_1)(z_3, \ldots, z_k) \Big\| \\
&\le \Big(\sum_{n=1}^{\infty} |\lambda_n| \Big) (\sup_n \|D_n x_1\|) \cdot \|z_3\| \cdot \ldots \cdot \|z_k\| \\
&\le \|Tx_1\| \cdot \|z_2\| \cdot \ldots \cdot \|z_k\|.
\end{aligned}$$

Therefore, U is continuous and admits an extension V to the closure of $T(X_1)$, with $\|V\| = \|U\| \le 1$. Since R is compact, the operator

$$V \circ R : Z_1 \to \mathcal{L}_{\mathrm{wb}}(Z_2, \ldots, Z_k; Y)$$

is compact. Let $B \in \mathcal{L}_{\mathrm{wb}}(Z_1, \ldots, Z_k; Y)$ be the k-linear mapping associated with $V \circ R$ by Proposition 12. We have $\|B\| = \|V \circ R\| \le \|R\|$. For $x_1 \in X_1$, $\ldots, x_k \in X_k$, we obtain

$$\begin{aligned}
B(S_1 x_1, \ldots, S_k x_k) &= (V \circ R(S_1 x_1))(S_2 x_2, \ldots, S_k x_k) \\
&= U(Tx_1)(S_2 x_2, \ldots, S_k x_k) \\
&= (DS_2 x_2)(x_1)(S_3 x_3, \ldots, S_k x_k) \\
&= \overline{D}(S_2 x_2, \ldots, S_k x_k)(x_1) \\
&= \overline{A}(x_2, \ldots, x_k)(x_1) = A(x_1, \ldots, x_k).
\end{aligned}$$

Moreover, since $\|\overline{A}\| = \|A\|$ and

$$\|\overline{D}\| = \|D\| \ge \lambda^{-1} \sup \|D_n\| = \lambda^{-1} \|T\| = \lambda^{-1} \|R\| \cdot \|S_1\| \ge \lambda^{-1} \|B\| \cdot \|S_1\|,$$

we get

$$\|B\| \cdot \|S_1\| \cdot \|S_2\| \cdot \ldots \cdot \|S_k\| \le \lambda \|\overline{D}\| \cdot \|S_2\| \cdot \ldots \cdot \|S_k\| \le \lambda^k \|A\|,$$

and the proof is complete. ∎

COROLLARY 15. *Given Banach spaces X, Y and a polynomial $P \in \mathcal{P}(^k X; Y)$, we have $P \in \mathcal{P}_{\mathrm{wb}}(^k X; Y)$ if and only if there are a Banach space Z, an operator $S \in \mathcal{C}o(X; Z)$, and a polynomial $Q \in \mathcal{P}_{\mathrm{wb}}(^k Z; Y)$ such that $P = Q \circ S$. Moreover, given $\varepsilon > 0$, we can obtain $\|Q\| \cdot \|S\|^k \le (1 + \varepsilon) \|\widehat{P}\|$.*

Proof. Suppose $P \in \mathcal{P}_{\mathrm{wb}}(^k X; Y)$. Then $\widehat{P} \in \mathcal{L}_{\mathrm{wb}}(^k X; Y)$. Given $\varepsilon > 0$, by Theorem 14, we can write $\widehat{P} = B \circ (S_1, \ldots, S_k)$ with $S_i \in \mathcal{C}o(X; Z_i)$, $B \in \mathcal{L}_{\mathrm{wb}}(Z_1, \ldots, Z_k; Y)$, and

$$\|B\| \cdot \|S_1\| \cdot \ldots \cdot \|S_k\| \le (1 + \varepsilon) \|\widehat{P}\|.$$

We can assume that $\|S_1\| = \ldots = \|S_k\|$. By Proposition 13, we have $P = Q \circ S$ with $\|Q\| \cdot \|S\|^k \le (1 + \varepsilon) \|\widehat{P}\|$. It is easily shown that Q is weakly continuous on bounded sets. The converse is clear. ∎

For complex Banach spaces X, Y, let $\mathcal{H}(X; Y)$ denote the space of all holomorphic mappings from X into Y, and $\mathcal{H}_{\mathrm{wbu}}(X; Y)$ the subspace of all mappings in $\mathcal{H}(X; Y)$ whose restrictions to bounded subsets are weakly uniformly continuous. We obtain:

THEOREM 16. *Let X, Y be complex Banach spaces, and $f \in \mathcal{H}(X; Y)$. Then $f \in \mathcal{H}_{\mathrm{wbu}}(X; Y)$ if and only if there are a space Z, an operator $S \in \mathcal{C}o(X; Z)$, and a mapping $g \in \mathcal{H}_{\mathrm{wbu}}(Z; Y)$ so that $f = g \circ S$.*

Proof. Let $f = \sum_{k=1}^{\infty} P_k$ be the Taylor series expansion of f at the origin, and suppose $f \in \mathcal{H}_{\mathrm{wbu}}(X; Y)$. By the Cauchy–Hadamard formula, we have $\lim \|P_k\|^{1/k} = 0$. By Corollary 15, there are spaces Z_k, operators $S_k \in \mathcal{C}o(X; Z_k)$ and polynomials $Q_k \in \mathcal{P}_{\mathrm{wb}}(^k Z_k; Y)$ such that $P_k = Q_k \circ S_k$ with

$$\|Q_k\| \cdot \|S_k\|^k \le 2 \|\widehat{P}_k\| \le 2 \frac{k^k}{k!} \|P_k\|$$

(the last inequality is well known and may be seen in [17, Theorem 2.2]). Then, using the Stirling formula,

$$\lim_k \|Q_k\|^{1/k} \cdot \|S_k\| \le \lim_k \frac{2^{1/k}e}{(2\pi k)^{1/2k}} \cdot \|P_k\|^{1/k} = 0.$$

We can assume therefore that $\|S_k\| \to 0$ and $\|Q_k\|^{1/k} \to 0$. Define $S : X \to Z := c_0(Z_k)$ by $Sx = (S_k x)_k$. Clearly, S is compact. Setting $\pi_k : Z \ni (y_i) \mapsto y_k \in Z_k$, we define $g : Z \to Y$ by $g(y) := \sum_{k=1}^{\infty} Q_k \circ \pi_k(y)$. Since $\lim \|Q_k \circ \pi_k\|^{1/k} = \lim \|Q_k\|^{1/k} = 0$, we find that g is a holomorphic mapping, bounded on bounded sets. Moreover, $Q_k \circ \pi_k$ is weakly continuous on bounded sets, for all k. Therefore, $g \in \mathcal{H}_{\mathrm{wbu}}(Z;Y)$. The converse is clear. ∎

We recall that it remains unknown whether or not a holomorphic mapping between Banach spaces which is weakly continuous on bounded sets is automatically weakly uniformly continuous on bounded sets [3], i.e., whether or not a holomorphic function on a Banach space can satisfy the conditions (a), (b) and (c) given after the proof of Theorem 5.

We thank Professor S. Dineen for pointing out a gap in a first version of Proposition 12. Trying to fill in this gap led us to find Theorem 3.

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