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The functor $\sigma^2 X$

by

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Abstract. We disprove the existence of a universal object in several classes of spaces including the class of weakly Lindelöf Banach spaces.

It is well known that the Tikhonov cube is an injectively universal compact space in a given weight. Surjectively universal compact spaces can also be constructed for certain weights (see [5]) but frequently one would like to know whether there can be universal objects in some more restrictive classes of compacta such as, for example, the class of first countable compacta or one of the classes of compacta which naturally occur in functional analysis (see [14; p. 620]). The purpose of this note is to answer a number of questions of this sort by introducing a new topological functor which might be of independent interest. The following is an example of a result which can be obtained by the new method.

THEOREM. *For every compact countably tight space X of weight continuum there is a first countable retractive* (1) *Corson compact space Y which is not a continuous image of any closed subspace of X.*

It follows that a number of natural classes of compact spaces mentioned in Question 10.6 of [14] have neither injectively nor surjectively universal objects. Similarly, this shows that there are neither injectively nor surjectively universal objects in the class of Corson compacta of weight continuum. In particular, the class of first countable compacta does not have such universal objects. The fact that there is no injectively universal first countable space follows from an earlier result of Filippov [6] (see also [20]) who showed that there exist more separable perfectly normal compacta than closed separable subsets of a given first countable space.

To state the dual form of our result let us recall that a Banach space E

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(1) A space X is *retractive* if every closed subset of X is a retract of X.

is said to have the *property* $(\mathcal{C})$ of Corson if every family of convex subsets of E with empty intersection contains a countable subfamily with empty intersection ([4], [13]). So, the property $(\mathcal{C})$ can be thought of as a convex analogue of the Lindelöf property since clearly every weakly Lindelöf Banach space has the property $(\mathcal{C})$.

THEOREM*. *For every Banach space E of size continuum and with property $(\mathcal{C})$ there is a weakly Lindelöf Banach space F of size continuum which is not isomorphic to a subspace of any quotient of E.*

In fact, the space F is a member of the smaller class of so-called *weakly Lindelöf determined* spaces ([18], [1]) for which a substantial theory has been developed similar to that of the better known class of *weakly countably determined spaces* ([9], [19]). Thus we are going to show that none of the three well-known classes of Banach spaces contain universal objects.

1. Free sequences of regular pairs. In this section we describe a functor which associates a tree σX with every space X, while in the next section we describe a version of its converse. If X is a topological space, let $\mathrm{rp}(X)$ be the set of all *regular pairs* in X, i.e., the pairs $\langle F, G\rangle$, where F is closed, G is open and $F \subseteq G \subseteq X$. If $\mathcal{B}$ is a given basis of X, which we always assume to be closed under finite unions and intersections, let $\mathrm{rp}(\mathcal{B})$ denote the set of all regular pairs of the form $\langle \overline{U}, V\rangle$, where U and V are elements of $\mathcal{B}$. We shall always assume that X is at least a regular space so there will be many regular pairs in $\mathrm{rp}(\mathcal{B})$. A sequence $\langle F_\xi, G_\xi\rangle$ $(\xi < \alpha)$ of regular pairs is *free* if for any two finite sets Γ and Δ of indices from α such that $\xi < \eta$ for every $\xi \in \Gamma$ and $\eta \in \Delta$ (written as $\Gamma < \Delta$), the intersection

$$\Big(\bigcap_{\xi\in\Gamma} F_\xi\Big) \cap \Big(\bigcap_{\eta\in\Delta} X \setminus G_\eta\Big)$$

is not empty. Let σX denote the set of all free sequences of regular pairs of X. If $\mathcal{B}$ is a given basis of X closed under finite unions and intersections let $\sigma_{\mathcal{B}} X$ denote the set of all free sequences of regular pairs from $\mathrm{rp}(\mathcal{B})$ rather than $\mathrm{rp}(X)$.

LEMMA 1. *If X is a compact countably tight space and if $\mathcal{B}$ is its basis of size continuum then $\sigma_{\mathcal{B}} X$ is a tree of size $\mathfrak{c}$ without uncountable chains.*

Proof. By using the compactness of X, an uncountable free sequence $\langle F_\xi, G_\xi\rangle$ $(\xi < \omega_1)$ of regular pairs would give us a sequence x_α $(\alpha < \omega_1)$ of points of X by simply choosing x_α to be an arbitrary element of the intersection

$$\Big(\bigcap_{\xi<\alpha} F_\xi\Big) \cap \Big(\bigcap_{\eta\geq\alpha} X \setminus G_\eta\Big).$$

This sequence has the property that

$$\overline{\{x_\xi : \xi < \alpha\}} \subseteq X \setminus G_\alpha \quad \text{and} \quad \overline{\{x_\xi : \xi \geq \alpha\}} \subseteq F_\alpha$$

for every $\alpha < \omega_1$. Thus if x is a complete accumulation point of x_ξ $(\xi < \omega_1)$ then no countable subset of $\{x_\xi : \xi < \omega_1\}$ would accumulate to x, and this would contradict the countable tightness of X.

2. The space of paths. In this section we shall describe a version of the functor sigma which for every tree T gives us a compact space σT with very strong topological properties. At first sight, order-theoretically σT does not look as it should if we are to follow the idea of §1. In §4, however, we shall show that the difference is unessential (from the order-theoretic point of view).

If T is a tree, let σT denote the set of all *paths* of T, i.e., downward closed chains of T. We shall consider σT as a topological space with the natural Tikhonov topology induced by the sets of the form

$$[t] = \{p \in \sigma T : t \in p\} \quad (t \in T)$$

as clopen subbasis. Since σT can naturally be identified with a closed subset of $\{0,1\}^T$ the topology of σT is always compact.

LEMMA 2. *Every closed subset F of σT is its retract.*

Proof. We may assume that $\emptyset \in F$. Then the retraction $r : \sigma T \to F$ is defined by simply letting $r(x)$ be equal to the maximal path $p \in F$ such that $p \subseteq x$.

From this one can easily deduce that σT is a hereditarily normal and in fact monotonically normal compact space. It can also be shown that σT is a continuous image of a compact ordered space, giving us further explanations for the strong topological properties of σT. On several previous occasions we have shown that spaces of the form σT can be relevant to some problems from function space theory. For example, note the following straightforward fact.

LEMMA 3. *The space σT is a Corson compactum iff T has no uncountable chains.*

Another interesting property of compact spaces of the form σT is given in the following proposition which essentially appears in [8, §3].

LEMMA 4. *If T has no uncountable chains then every positive Radon measure on σT has separable support.*

Proof. Given a positive Radon measure μ on σT, let $S = \{s \in T : \mu([s]) > 0\}$ and prove that S must be countable. From this the conclusion follows easily.

This shows that if T has no uncountable chains then the Banach space $\mathcal{C}(\sigma T)$ of all continuous real-valued functions on σT is weakly Lindelöf—a property of considerable interest (see [1]). To describe some stronger properties of $\mathcal{C}(\sigma T)$ in terms of properties of T, for t in T let χ_t be the characteristic function of the subbasic clopen set $[t]$. Then

$$T^* = \{\chi_t : t \in T\} \cup \{0\}$$

is a closed subset of $\mathcal{C}_{\mathrm{p}}(\sigma T)$ with the single nonisolated point 0 which generates the subspace of all continuous $\{0,1\}$-valued functions on T. [The index p here stands to indicate that we are looking at $\mathcal{C}(\sigma T)$ with the topology of pointwise convergence.] Standard restrictions on a compact space X in that subject start with the strongest one stating that X is a *uniform Eberlein compactum*, i.e., a weakly compact subset of a Hilbert space ([3]); then there is the restriction that $\mathcal{C}(X)$ is $\mathcal{K}$-*analytic* in the weak topology (see [16]), and then the restriction of being *countably determined*, i.e., a continuous image of a closed subset of some product of a separable metric space and a compact space (see [9] and [19]). Since this last property is preserved by closed subspaces, countable products and continuous images it follows that $\mathcal{C}(\sigma T)$ is countably determined iff T^* is countably determined, which in turn is easily seen to be equivalent to its σ-compactness. But clearly T^* is σ-compact iff T is the union of countably many antichains (see [12]). On the other hand, if $\{A_n\}$ is an antichain decomposition of T, then $[t]$ $(t \in A_n,\ n \in \mathbb{N})$ is a σ-disjoint T_0-separating family of clopen subsets of σT, which means that σT is a uniform Eberlein compactum (see [3]). This shows that all these restrictions are equivalent in the class of path spaces σT:

LEMMA 5. *A tree T is the union of countably many antichains iff σT is homeomorphic to a weakly compact subset of a Hilbert space iff the Banach space $\mathcal{C}(\sigma T)$ is countably determined.*

3. The functor $\sigma^2 X$. Recall that if X is a topological space, σX is the tree of all free sequences of regular pairs of X. So we can consider its path space $\sigma(\sigma X)$ which we shall denote simply by $\sigma^2 X$. The following is a basic property of the functor $\sigma^2 X$, making it very relevant to many questions about the existence of universal objects.

LEMMA 6. *$\sigma^2 X$ is not a continuous image of any closed subspace of X.*

Proof. Suppose Y is a closed subspace of X, that $f : Y \to \sigma^2 X$ is a continuous surjection, and let us look for a contradiction. For every t in σX, let

$$F_t = f^{-1}[t] \quad \text{and} \quad G_t = X \setminus (Y \setminus F_t).$$

(Recall that $[t] = \{p \in \sigma(\sigma X) : t \in p\}$ is the subbasic clopen subset of $\sigma^2 X$.) Then F_t is closed, G_t is open in X, and $F_t \subseteq G_t$, i.e., $\langle F_t, G_t \rangle$ is a regular pair in X. Define

$$s : \mathrm{Ord} \to \mathrm{rp}(X)$$

by the recursive formula

$$s(\xi) = \langle F_{s \restriction \xi}, G_{s \restriction \xi} \rangle.$$

It is easily checked that s is well defined, i.e., that $s \restriction \xi \in \sigma X$ for all ξ. The existence of such an s is the desired contradiction.

A similar fact can be proved for the relativized functor $\sigma^2_{\mathcal{B}} X = \sigma(\sigma_{\mathcal{B}} X)$, where $\mathcal{B}$ is a fixed basis of X. To make the proof of Lemma 6 go through we need to assume that X is a compact space in order to find for every t in $\sigma_{\mathcal{B}} X$ a regular pair $\langle \overline{U}_t, V_t \rangle$ from $\mathrm{rp}(\mathcal{B})$ such that $U_t \cap Y = V_t \cap Y = f^{-1}[t]$. Then the recursive formula $s(\xi) = \langle \overline{U}_{s \restriction \xi}, V_{s \restriction \xi} \rangle$ works as in the proof of Lemma 6. So we have the following relativization of Lemma 6.

LEMMA 7. *Let X be a compact space and let $\mathcal{B}$ be a basis of X closed under finite unions. Then $\sigma^2_{\mathcal{B}} X$ is not a continuous image of any closed subspace of X.*

To prove the Theorem, let X be a given compact countably tight space of weight $\mathfrak{c}$ and let $\mathcal{B}$ be a fixed basis of X of size $\mathfrak{c}$ which is closed under finite unions. Then $\sigma_{\mathcal{B}} X$ is a tree of size $\mathfrak{c}$ without uncountable chains (Lemma 1), so $\sigma^2_{\mathcal{B}} X$ is a Corson compact space (Lemma 3). By Lemmas 2 and 7, $Y = \sigma^2_{\mathcal{B}} X$ satisfies all the conclusions of the Theorem except of being first countable. To get the first countability we modify the tree $\sigma_{\mathcal{B}} X$ by inserting a binary tree of height ω between every node of $\sigma_{\mathcal{B}} X$ and its immediate successors, i.e., we find a binary tree $\widetilde{\sigma}_{\mathcal{B}} X$ such that $\sigma_{\mathcal{B}} X$ is equal to the set at limit nodes of $\widetilde{\sigma}_{\mathcal{B}} X$. It should be clear that for the tree so modified the path space $\sigma(\widetilde{\sigma}_{\mathcal{B}} X)$ satisfies the conclusion of Lemma 7. But now the space $Y = \sigma(\widetilde{\sigma}_{\mathcal{B}} X)$ is first countable and we are done.

Question 10.6 of [14] also asks about the existence of universal objects in some subclasses of the class of Corson compacta; for example, the class of Eberlein compacta, or the class of compacta X for which $\mathcal{C}(X)$ is $\mathcal{K}$-analytic in the weak topology, or even the wider class of compacta X for which $\mathcal{C}(X)$ is countably determined. So it is natural to ask whether the functor $\sigma^2 X$ can be pushed down to work also for some of these smaller classes. It turns out that the answer to this question is negative. To see this recall the result of [2] which says that every uniform Eberlein compactum X of a given weight θ is a continuous image of a closed subset of $A(\theta)^{\mathbb{N}}$, where $A(\theta)$ is the one-point compactification of the discrete space of size θ. So, for $X = A(\theta)^{\mathbb{N}}$ the space $\sigma^2 X$ (or one of its relativizations) cannot be a uniform Eberlein compactum since by Lemma 6 (or 7) it is not a continuous image of any closed subset of X. In fact, by Lemma 5, not only is $\sigma^2 X$ not a uniform

Eberlein compactum but even its function space $C(\sigma^2 X)$ is not countably determined. This does not mean, however, that the universality questions for these classes of spaces cannot be answered by some other methods leading perhaps to some weaker disproofs of the existence of universal objects. In fact, this has already been done for the class of Eberlein compacta in [0] using a very interesting argument which involves a variation on the Szlenk index of Banach spaces ([15]).

4. Sigma of a Banach space. A sequence x_ξ ($\xi < \alpha$) of elements of a Banach space E is *free* if for any two finite sets Γ and Δ of indices from α such that $\Gamma < \Delta$ there is an x^* in E^* such that $\|x^*\| \le 1$ and

(1) $\langle x^*, x_\xi \rangle \ge 1$ for all $\xi \in \Gamma$,

(2) $\langle x^*, x_\zeta \rangle \le 0$ for all $\zeta \in \Delta$.

Let σE be the set of all free sequences of elements of E considered as a tree ordered by inclusion.

LEMMA 8. *If E has the property (C), then there are no uncountable free sequences of elements of E.*

Proof. Otherwise, fix an uncountable free sequence x_ξ ($\xi < \omega_1$) of elements of E. For $\alpha < \omega_1$, set

$$K_\alpha = \overline{\mathrm{conv}}\{x_\xi : \xi < \alpha\} \quad \text{and} \quad L_\alpha = \overline{\mathrm{conv}}\{x_\xi : \xi \ge \alpha\}.$$

CLAIM. *$K_\alpha \cap L_\alpha = \emptyset$ for all α.*

Proof. It suffices to show that for every $\xi_1, \ldots, \xi_m < \alpha$ and $0 \le \lambda_1, \ldots, \lambda_m \le 1$ with $\sum_{i=1}^m \lambda_1 = 1$ and for every $\eta_1, \ldots, \eta_n \ge \alpha$ and $0 \le \nu_1, \ldots, \nu_n \le 1$ with $\sum_{i=1}^n \nu_i = 1$, if

$$x = \sum_{i=1}^m \lambda_i x_{\xi_i} \quad \text{and} \quad y = \sum_{i=1}^n \nu_i x_{\eta_i},$$

then $\|x - y\| \ge 1$. Let $\Gamma = \{\xi_1, \ldots, \xi_m\}$ and $\Delta = \{\eta_1, \ldots, \eta_n\}$ and choose an x^* in E^* with $\|x^*\| \le 1$ such that (1) and (2) above are satisfied. It follows that $\langle x^*, x \rangle \ge 1$ and $\langle x^*, y \rangle \le 0$, so that

$$1 \le \langle x^*, x - y \rangle \le \|x^*\| \cdot \|x - y\| \le \|x - y\|,$$

and so we are done.

Note that $\bigcup_\alpha K_\alpha = L_0$ so by the Claim we have $\bigcap_\alpha L_\alpha = \emptyset$ and this clearly contradicts the property (C) since the sequence $\{L_\alpha\}$ is decreasing. This finishes the proof of Lemma 8.

It follows that the tree σE has no uncountable branches so its path space $\sigma(\sigma E) = \sigma^2 E$ is a Corson compactum whose weight is equal to the size of σE, which in turn is equal to the size of E. Moreover, $\sigma^2 E$ has all

the pleasant properties of §2 shared by all path spaces. In particular, its function space $C(\sigma^2 E)$ is weakly Lindelöf. So Theorem* will follow from the following analogue of Lemma 6.

LEMMA 9. *The Banach space $F = C(\sigma^2 E)$ is not isomorphic to a subspace of any quotient of E.*

Proof. Otherwise, we can find a Banach space M and a surjective linear operator $T : E \to M$ of norm 1 such that F is a subspace of M. For $t \in \sigma E$ let $x_t \in F \subseteq M$ be the characteristic function of the clopen subset

$$C_t = \{p \in \sigma^2 E : t \in p\}$$

of $\sigma^2 E$. Let $y_t \in E$ be an arbitrary element such that $x_t = T(y_t)$. Define $s : \mathrm{Ord} \to E$ recursively by the formula

$$s(\alpha) = y_{s \restriction \alpha}.$$

Clearly, this will be the desired contradiction once we show that this is indeed well defined. In other words, we have to show by induction on α that $y_{s\restriction\xi}$ ($\xi < \alpha$) is a free sequence of elements of E. Note that since $\|T\| = 1$ it would be sufficient to show that

$$x_{s\restriction\xi} = T(y_{s\restriction\xi}) \quad (\xi < \alpha)$$

is a free sequence in the range space M, or equivalently in its subspace F. Let Γ and Δ be two given finite subsets of α such that $\Gamma < \Delta$. Let p be the minimal path of σE containing $s\restriction\xi$ ($\xi \in \Gamma$). Note that, by our inductive assumption, $s\restriction\xi$ ($\xi < \alpha$) is a chain of σE enumerated increasingly so it follows that $p \in C_{s\restriction\xi}$ for all $\xi \in \Gamma$ and $p \notin C_{s\restriction\eta}$ for all $\eta \in \Delta$. Hence if $\delta_p \in F^*$ is the point mass measure at p, then

$$\langle \delta_p, x_{s\restriction\xi} \rangle = 1 \quad \text{and} \quad \langle \delta_p, x_{s\restriction\eta} \rangle = 0$$

for all $\xi \in \Gamma$ and $\eta \in \Delta$. So $x^* = \delta_p$ satisfies the conditions (1) and (2) from the definition of a free sequence. This shows that $x_{s\restriction\xi}$ ($\xi < \alpha$) is a free sequence in F and finishes the proof.

5. Concluding remarks. The functors sigma of this paper are members of a rather long list of similar functors in various other categories. For example, in the category of sets they correspond to the Hartogs number ([10]), which in turn is very closely tied to Zermelo's proof of the well-ordering theorem ([21]). In the category of posets they correspond to the functor wP considered by Kurepa [11] or some of the weaker forms considered by Gleason and Dilworth [7]. Note that if we take the category of posets where the embeddings are the strictly increasing functions, then σP should be the set of all strictly increasing maps from the ordinals into P. (Indeed, this is exactly the functor wP of Kurepa [11].) If one applies this definition to a tree T then one obtains σT which looks differently than the set of paths of T considered in §2. However, these two objects are only unessentially different (from the point of view of this particular category) since they are easily seen to be embeddable in each other. This explains the reason why we have kept the notation σT also for the path space of T. A fuller explanation of the operator sigma in the categories of sets and posets is given in [17]. Finding analogues of this functor in other categories may be a rewarding task. For example, what should the functor sigma be in the category of adequate families of sets (see [16])? A right answer to this question might give us another (perhaps stronger?) proof of the result of [0] as well as answers to the remaining questions about the existence of universal objects in classes of compacta occurring in functional analysis ([14;Q.10.6]).

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