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Balancing vectors and convex bodies

by

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Abstract. Let U, V be two symmetric convex bodies in $\mathbb{R}^n$ and $|U|$, $|V|$ their n-dimensional volumes. It is proved that there exist vectors $u_1, \ldots, u_n \in U$ such that, for each choice of signs $\varepsilon_1, \ldots, \varepsilon_n = \pm 1$, one has $\varepsilon_1 u_1 + \ldots + \varepsilon_n u_n \notin rV$ where $r = (2\pi e^2)^{-1/2} n^{1/2} (|U|/|V|)^{1/n}$. Hence it is deduced that if a metrizable locally convex space is not nuclear, then it contains a null sequence (u_n) such that the series $\sum_{n=1}^{\infty} \varepsilon_n u_{\pi(n)}$ is divergent for any choice of signs $\varepsilon_n = \pm 1$ and any permutation π of indices.

Let U be a convex body in $\mathbb{R}^n$. The n-dimensional volume of U will be denoted by $|U|$. We say that U is *symmetric* if $U = -U$. The family of all symmetric convex bodies in $\mathbb{R}^n$ will be denoted by $\mathcal{C}_n$.

For each pair $U, V \in \mathcal{C}_n$, we denote by $\beta(U, V)$ the smallest number r satisfying the following condition: to each system $u_1, \ldots, u_n \in U$ there correspond signs $\varepsilon_1, \ldots, \varepsilon_n = \pm 1$ such that $\varepsilon_1 u_1 + \ldots + \varepsilon_n u_n \in rV$.

Remark 1. A standard argument based on Lemma 4 shows that to each system $u_1, \ldots, u_s \in U$ (with s arbitrary) there correspond signs $\varepsilon_1, \ldots, \varepsilon_s = \pm 1$ such that $\varepsilon_1 u_1 + \ldots + \varepsilon_s u_s \in 2\beta(U, V) V$. For details, see e.g. [17], Lecture 1.

The quantities $\beta(U, V)$ for various pairs U, V were investigated in [4], [6], [7] and [15]. Combinatorial motivations are presented exhaustively in [17]; see also [14]. Perhaps the most interesting open problem here is the following. Let B_2^n and B_∞^n denote the unit balls for the l_2 and l_∞ norms on $\mathbb{R}^n$, respectively; is it true that $\beta(B_2^n, B_\infty^n)$ is bounded as $n \to \infty$? This is called the Komlós conjecture; the best reference is [16]. For an application of the quantities $\beta(U, V)$ to rearrangement of series in infinite-dimensional spaces, see [3], Remark 2 and [5], (10.18).

The aim of this paper is to prove the following result:

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THEOREM 1. *For each pair $U, V \in \mathcal{C}_n$, one has*

$$\beta(U,V) > (2\pi e^2)^{-1/2} n^{1/2} \left(\frac{|U|}{|V|}\right)^{1/n}.$$

Remark 2. Let E be an n-dimensional normed space and B_E its unit ball. Define $\beta(E) = \beta(T(B_E), T(B_E))$ where $T: E \to \mathbb{R}^n$ is a linear isomorphism. From the Milman quotient subspace theorem or from the result of Bourgain and Szarek [8], Theorem 2, it follows easily that $\beta(E) > Kn^{1/2}$ for some numerical constant K. Theorem 1 implies that $K = (2\pi e^2)^{-1/2}$ will do. If $\varphi(E)$ is the Steinitz constant of E (see e.g. [11]) then $\varphi(E) \geq \frac{1}{2}\beta(E)$ ([14], Example 2, p. 250). Hence $\varphi(E) > (8\pi e^2)^{-1/2} n^{1/2}$. Lower bounds for $\beta(l_p^n)$ and $\varphi(l_p^n)$, $1 \leq p \leq \infty$, based on Hadamard matrices, were given by Sevastyanov [14].

Let (u_n) be a null sequence in a nuclear Fréchet space. It follows easily from the results of [3] that one can find signs $\varepsilon_n = \pm 1$ and a permutation π of indices such that the series $\sum_{n=1}^{\infty} \varepsilon_n u_{\pi(n)}$ is convergent. In the second part of the paper we show, using Theorem 1, that the following statement is true:

THEOREM 2. *If a metrizable locally convex space is not nuclear, then it contains a null sequence $(u_n)_{n=1}^{\infty}$ such that the series $\sum_{n=1}^{\infty} \varepsilon_n u_{\pi(n)}$ is divergent for any choice of signs $\varepsilon_n = \pm 1$ and any permutation π of indices.*

We begin with some lemmas. By ω_n we denote the n-dimensional volume of the euclidean unit ball in $\mathbb{R}^n$.

LEMMA 1. *Each symmetric convex body U in $\mathbb{R}^n$ is contained in some n-dimensional parallelepiped P such that*

$$|U| \geq \sqrt[4]{2\pi n}\, e^{-n/2} 2^{-n} \omega_n |P|. \tag{1}$$

This result was essentially proved by Dvoretzky and Rogers [10]. See also Babenko [1], Kashin [12] and Ball [2]. A detailed analysis and a slight improvement of (1) was given by Pełczyński and Szarek [13].

Let U be a symmetric convex body in $\mathbb{R}^n$. By U^0 we denote the polar body, i.e.

$$U^0 = \{u \in \mathbb{R}^n : (u, v) \leq 1 \text{ for all } v \in U\}.$$

Let $\mathrm{Gram}(u_1, \ldots, u_n)$ denote the Gram determinant of vectors $u_1, \ldots, u_n \in \mathbb{R}^n$.

LEMMA 2. *Each symmetric convex body U in $\mathbb{R}^n$ contains some vectors $u_1, \ldots, u_n$ such that*

$$\mathrm{Gram}(u_1, \ldots, u_n) > (2\pi e^2)^{-n} n^n |U|^2. \tag{2}$$

Proof. By Lemma 1, there exists an n-dimensional parallelepiped P in $\mathbb{R}^n$ with $U^0 \subset P$ and

$$|U^0| \geq \sqrt[4]{2\pi n}\, e^{-n/2} 2^{-n} \omega_n |P|. \tag{3}$$

We may write

$$P = \{v \in \mathbb{R}^n : |(v, u_i)| \leq 1 \text{ for } i = 1, \ldots, n\}$$

for some $u_1, \ldots, u_n \in U$. Then

$$\mathrm{Gram}(u_1, \ldots, u_n) = 2^{2n} |P|^{-2}. \tag{4}$$

From (3), (4) and the Santaló inequality $|U| \cdot |U^0| \leq \omega_n^2$, after easy calculations we obtain (2). ∎

A *lattice* in $\mathbb{R}^n$ is an additive subgroup of $\mathbb{R}^n$ generated by n linearly independent vectors. The *determinant* of a lattice L, denoted by $d(L)$, is the n-dimensional volume of the parallelepiped

$$\{t_1 u_1 + \ldots + t_n u_n : 0 \leq t_1, \ldots, t_n \leq 1\}$$

where $u_1, \ldots, u_n$ is any system of free generators of L. The *covering radius* of L with respect to a given convex body U, denoted by $\mu(L, U)$, is defined by

$$\mu(L, U) = \inf\{r > 0 : L + rU = \mathbb{R}^n\}.$$

LEMMA 3. *Let U be a convex body in $\mathbb{R}^n$. For each lattice L in $\mathbb{R}^n$, one has $d(L) \leq |U| \cdot [\mu(L, U)]^n$.*

This is a standard fact; see e.g. inequality (8) in [9], Ch. XI, n° 3.

Let P be an n-dimensional parallelepiped in $\mathbb{R}^n$. By P^0 we denote the set of all vertices of P. For each $k = 0, 1, \ldots$, let P^k denote the set of all points of the form $t_1 v_1 + \ldots + t_s v_s$ where $v_1, \ldots, v_s \in P^0$, $t_1 + \ldots + t_s = 1$ and $t_1, \ldots, t_s = 0/2^k, 1/2^k, \ldots, 2^k/2^k$. In other words, if

$$P = a + \{t_1 u_1 + \ldots + t_n u_n : 0 \leq t_1, \ldots, t_n \leq 1\},$$

then

$$P^k = a + \left\{t_1 u_1 + \ldots + t_n u_n : t_1, \ldots, t_n = \frac{0}{2^k}, \frac{1}{2^k}, \ldots, \frac{2^k}{2^k}\right\}.$$

LEMMA 4. *Let P be an n-dimensional parallelepiped and U a convex body in $\mathbb{R}^n$. If $P^1 \subset P^0 + U$, then $P \subset P^0 + 2U$.*

Proof. Suppose that $P^1 \subset P^0 + U$. Then, clearly, $P^{k+1} \subset P^k + 2^{-k} U$ for each $k = 0, 1, 2, \ldots$ Hence, by induction, we get

$$P^{k+1} \subset 2^{-k} U + 2^{-k+1} U + \ldots + 2^{-1} U + U + P^0 \subset 2U + P^0$$

for every k. Since P is contained in the closure of $\bigcup_{k=1}^{\infty} P^k$, it follows that $P \subset 2U + P^0$. ∎

Proof of Theorem 1. By Lemma 2, we can find some $u_1, \ldots, u_n \in U$ satisfying (2). Let L be the lattice generated by $u_1, \ldots, u_n$. Then

$$d(L) = [\mathrm{Gram}(u_1, \ldots, u_n)]^{1/2}. \tag{5}$$

From (2), (5) and Lemma 3 we obtain

$$\mu(L, U) > \varrho := (2\pi e^2)^{-1/2} n^{1/2} \left(\frac{|U|}{|V|}\right)^{1/n}. \tag{6}$$

Let us write

$$P = \{t_1 u_1 + \ldots + t_n u_n : 0 \le t_1, \ldots, t_n \le 1\}.$$

Due to (6), there exists some $u \in P \setminus (L + \varrho V)$. Since $P^0 \subset L$, it follows that $u \notin P^0 + \varrho V$. Thus, by Lemma 4, there exists some $w \in P^1 \setminus (P^0 + \frac{1}{2}\varrho V)$. We may write $w = \frac{1}{2}\sum_{i \in I} u_i + \sum_{i \in J} u_i$ for some $I, J \subset \{1, \ldots, n\}$ with $I \cap J = \emptyset$.

Let us choose an arbitrary system of signs $\varepsilon_i = \pm 1$ for $i \in I$. Then we have $w - \frac{1}{2}\sum_{i \in I} \varepsilon_i u_i \in P^0$, so that

$$\frac{1}{2}\sum_{i \in I} \varepsilon_i u_i = w - \left(w - \frac{1}{2}\sum_{i \in I} \varepsilon_i u_i\right) \notin \frac{1}{2}\varrho V.$$

This means that $\beta(U, V) > \varrho$. ∎

Let U, V be two symmetric convex bodies in an n-dimensional real vector space N. Their volume ratio will be denoted by $|U|/|V|$. In other words, $|U|/|V| = |T(U)|/|T(V)|$ where $T : N \to \mathbb{R}^n$ is any linear isomorphism.

Let E, F be real normed spaces with unit balls B_E, B_F, respectively, and let $T : E \to F$ be a bounded linear operator. For each $k = 1, 2, \ldots$, define

$$v_n(T) = \sup_N \left(\frac{|T(B_E \cap N)|}{|B_F \cap T(N)|}\right)^{1/n}$$

where the supremum is taken over all linear subspaces N of E with $\dim N = \dim T(N) = n$. If $\dim T(N) < n$, we define $v_n(T) = 0$.

Let p be a seminorm on a real vector space E. Let us define

$$B_p = \{u \in E : p(u) \le 1\}.$$

The quotient space $E/p^{-1}(0)$ endowed with its canonical norm will be denoted by E_p and the canonical projection of E onto E_p by ψ_p. Then $\|\psi_p(u)\| = p(u)$ for $u \in E$, which implies that $\psi_p(B_p) = B_{E_p}$. Let $q \le p$ be another seminorm on E. The canonical operator from E_p to E_q will be denoted by T_{pq}. We have the canonical commutative diagram

$$\begin{array}{ccc} E & \xrightarrow{\mathrm{id}} & E \\ \downarrow \psi_p & & \downarrow \psi_q \\ E_p & \xrightarrow{T_{pq}} & E_q \end{array}$$

Define $v_n(p, q) = v_n(T_{pq})$ for $n = 1, 2, \ldots$

LEMMA 5. *Let $p \ge q$ be two seminorms on a vector space E. Suppose that*

$$\limsup_{n \to \infty} n^{1/2} v_n(p, q) = \infty. \tag{7}$$

(a) *Let s be an arbitrary positive number. Then we can find a finite system of vectors $u_1, \ldots, u_n \in B_p$ such that*

$$q\Big(\sum_{k=1}^{n} \varepsilon_k u_k\Big) > s \quad \textit{for any system } \varepsilon_1, \ldots, \varepsilon_n = \pm 1. \tag{8}$$

(b) *If F is a subspace of E with* $\operatorname{codim} F < \infty$, *then*

$$\limsup_{n \to \infty} n^{1/2} v_n(p_{|F}, q_{|F}) = \infty.$$

Proof. By (7), we can find an index n such that $n^{1/2} v_n(p, q) > (2\pi e^2)^{1/2} s$. Consequently, there is some n-dimensional subspace N of E_p such that $\dim T_{pq}(N) = n$ and

$$n^{1/2}\left(\frac{|T_{pq}(B_{E_p} \cap N)|}{|B_{E_q} \cap T_{pq}(N)|}\right)^{1/n} > (2\pi e^2)^{1/2} s.$$

Hence, by Theorem 1,

$$\beta(T_{pq}(B_{E_p} \cap N), B_{E_q} \cap T_{pq}(N)) > s.$$

This means that we can find some vectors $v_1, \ldots, v_n \in B_{E_p} \cap N$ such that

$$\Big\|\sum_{k=1}^{n} \varepsilon_k T_{pq}(v_k)\Big\| > s \quad \text{for each system } \varepsilon_1, \ldots, \varepsilon_n = \pm 1. \tag{9}$$

For each $k = 1, \ldots, n$, let us choose some $u_k \in B_p$ with $\psi_p(u_k) = v_k$. Then, by (9), we have

$$q\Big(\sum_{k=1}^{n} \varepsilon_k u_k\Big) = \Big\|\psi_q\Big(\sum_{k=1}^{n} \varepsilon_k u_k\Big)\Big\| = \Big\|\sum_{k=1}^{n} \varepsilon_k \psi_q(u_k)\Big\|$$
$$= \Big\|\sum_{k=1}^{n} \varepsilon_k T_{pq}\psi_p(u_k)\Big\| = \Big\|\sum_{k=1}^{n} \varepsilon_k T_{pq}(v_k)\Big\| > s$$

for each system $\varepsilon_1, \ldots, \varepsilon_n = \pm 1$. This proves (8).

The proof of (b) is the same as that of Lemma (6.8) in [5] (the assumption that T_{pq} is injective is not essential). ■

Proof of Theorem 2. Let E be a metrizable locally convex space. We can find a sequence $p_0 \le p_1 \le p_2 \le \ldots$ of continuous seminorms on E such that $\{B_{p_k}\}_{k=0}^{\infty}$ is a basis of neighbourhoods of zero in E. Suppose that E is not nuclear. Then there is an index k_0 such that

$$\limsup_{n\to\infty} n^{1/2}\upsilon_n(p_k, p_{k_0}) = \infty$$

for all $k \ge k_0$ (see [5], Lemma (6.5)). We may assume that $k_0 = 0$.

According to Lemma 5(a), we can find some vectors $u_1^1, \ldots, u_{n_1}^1 \in B_{p_1}$ such that

$$p_0\Big(\sum_{k=1}^{n_1} \varepsilon_k u_k^1\Big) > 1 \quad \text{for any system } \varepsilon_1, \ldots, \varepsilon_{n_1} = \pm 1 .$$

Let $M_1 = \operatorname{span}\{u_k^1\}_{k=1}^{n_1}$. A standard argument allows us to find a subspace F_1 of E with $\operatorname{codim} F_1 < \infty$ such that

$$p_0(u+v) \ge \frac{1}{3}\max(p_0(u), p_0(v)) \quad (u \in M_1,\ v \in F_1).$$

By Lemma 5(b), we have

$$\limsup_{n\to\infty} n^{1/2}\upsilon_n(p_{2|F_1}, p_{0|F_1}) = \infty .$$

So, again by Lemma 5(a), we can find some $u_1^2, \ldots, u_{n_2}^2 \in F_1 \cap B_{p_2}$ such that

$$p_0\Big(\sum_{k=1}^{n_2} \varepsilon_k u_k^2\Big) > 2 \quad \text{for any system } \varepsilon_1, \ldots, \varepsilon_{n_2} = \pm 1 .$$

Let $M_2 = \operatorname{span}(\{u_k^1\}_{k=1}^{n_1} \cup \{u_k^2\}_{k=1}^{n_2})$. There is a subspace F_2 of F_1 with $\operatorname{codim} F_2 < \infty$ such that

$$p_0(u+v) \ge \frac{1}{3}\max(p_0(u), p_0(v)) \quad (u \in M_2,\ v \in F_2).$$

Then we proceed by induction. In this way we construct a sequence of subspaces $F_1 \supset F_2 \supset \ldots$ and, for each m, a finite sequence $u_1^m, \ldots, u_{n_m}^m \in B_{p_m} \cap F_{m-1}$ such that, writing

$$M_m = \operatorname{span} \bigcup_{i=1}^{m} \{u_k^i\}_{k=1}^{n_i} \quad (m = 1, 2, \ldots),$$

we have

$$p_0\Big(\sum_{k=1}^{n_m} \varepsilon_k u_k^m\Big) > m \quad \text{for any system } \varepsilon_1, \ldots, \varepsilon_{n_m} = \pm 1, \tag{10}$$

$$p_0(u+v) \ge \frac{1}{3}\max(p_0(u), p_0(v)) \quad (u \in M_m,\ v \in F_m). \tag{11}$$

Let us arrange all the vectors u_k^m in a sequence $(u_n)_{n=1}^{\infty}$. It follows directly from our construction that $u_n \to 0$ as $n \to \infty$. Take an arbitrary permutation π of the positive integers and a sequence of signs $(\varepsilon_n)_{n=1}^{\infty}$. We shall prove that the partial sums of the series $\sum_{n=1}^{\infty} \varepsilon_n u_{\pi(n)}$ are not bounded with respect to p_0.

Take an arbitrary positive number s. Then choose an integer $m > 9s$. We can find an index r such that the sequence $(u_{\pi(n)})_{n=1}^{r}$ contains all vectors $u_1^m, \ldots, u_{n_m}^m$. Now, it is not hard to see that we may write

$$\sum_{n=1}^{r} \varepsilon_n u_{\pi(n)} = a + \sum_{k=1}^{n_m} \varepsilon'_k u_k^m + b$$

for some $a \in M_{m-1}$, $b \in F_m$ and $\varepsilon'_1, \ldots, \varepsilon'_{n_m} = \pm 1$. Taking into account that $u_1^m, \ldots, u_{n_m}^m \in M_m \cap F_{m-1}$, from (11) and (10) we obtain

$$p_0\Big(\sum_{n=1}^{r} \varepsilon_n u_{\pi(n)}\Big) = p_0\Big(a + \sum_{k=1}^{n_m} \varepsilon'_k u_k^m + b\Big) \ge \frac{1}{3} p_0\Big(a + \sum_{k=1}^{n_m} \varepsilon'_k u_k^m\Big)$$
$$\ge \frac{1}{9} p_0\Big(\sum_{k=1}^{n_m} \varepsilon'_k u_k^m\Big) > \frac{1}{9} m > s . \ ■$$

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