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Received November 22, 1991
Revised version June 19, 1992 (2860)

STUDIA MATHEMATICA 103 (2) (1992)

Erratum to the paper "On the reflexivity of pairs of isometries and of tensor products of some reflexive algebras"

(Studia Math. 83 (1986), 47–55)

by

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Abstract. A gap in the proof of [4, Theorem 1] is removed.

1. Introduction. Our purpose is to remove a gap in the proof of [4, Theorem 1]. All the notations are taken from [4]. Let us recall this theorem:

THEOREM 1. *Every pair $\{V_1, V_2\}$ of doubly commuting (V_1, V_2 commute and V_1, V_2^* commute) isometries on a Hilbert space H is reflexive.*

The main idea of the proof was to use the Wold-type decomposition (it exists for the above pair by [6, Theorem 3]): there are subspaces H_{uu}, H_{us}, H_{su}, H_{ss} such that

(1) $H = H_{uu} \oplus H_{us} \oplus H_{su} \oplus H_{ss}$, where all summands reduce V_1 and V_2,
(2) $V_1|_{H_{uu}}$ and $V_2|_{H_{uu}}$ are unitary operators,
(3) $V_1|_{H_{us}}$ is a unitary operator, $V_2|_{H_{us}}$ is a shift,
(4) $V_1|_{H_{su}}$ is a shift, $V_2|_{H_{su}}$ is a unitary operator,
(5) $V_1|_{H_{ss}}$, $V_2|_{H_{ss}}$ are shifts.

After proving the reflexivity of each component, the last step was to sum them up. This requires some extra property for each component besides the reflexivity. Property C was used (for definition see [4]). The gap was in the proof of this property for cases (3), (4), (5). For cases (3), (4), the idea of the proof is correct but the details are not straightforward. These are given in Section 2. In fact, to sum up reflexive components we need (see [2, Theorem 3.8]) to prove a weaker (see [2, Proposition 2.5, (2)]) property than C, namely property D(1) (introduced in [2] and now known as property

1991 *Mathematics Subject Classification*: Primary 47D25; Secondary 47B20.
Key words and phrases: pairs of isometries, reflexive operators.

$\mathbb{A}_1(1)$). An algebra of operators $\mathcal{A}$ has property D(1) if for any functional ϕ on $\mathcal{A}$ and $\varepsilon > 0$ there are $a, b \in H$ such that $\|a\| \cdot \|b\| \le (1+\varepsilon)\|\phi\|$ and $\phi(A) = (Aa, b)$ for all $A \in \mathcal{A}$. The proof of property D(1) for (5) is given in Section 2.

2. The proofs of Lemmas. Let us denote by S the unilateral shift on the Hardy space H^2. To prove property C for cases (3) and (4), by [4, Proposition 9] it is enough to show

LEMMA 2. *Let $A \subset L(H)$ be an algebra of normal operators. Then $\mathcal{A}(S \otimes I, I \otimes \mathcal{A})$ has property* C.

Proof. Since $\mathcal{A}(S\otimes I, I\otimes \mathcal{A}) \subset \mathcal{A}(S\otimes I, I\otimes\mathcal{A}, I\otimes\mathcal{A}^*)$, by [2, Proposition 2.1(2)], it is enough to prove property C for the latter algebra.

Let $T = (S\otimes I)^{(n)}$ and $\mathcal{A}_1 = \{(I\otimes A)^{(n)} : A \in \mathcal{A}\}$. Let $x \in (H^2 \otimes H)^{(n)}$ and $\mathcal{M} = [\mathcal{A}(T, \mathcal{A}_1, \mathcal{A}_1^*)x]$. Then $T|_{\mathcal{M}}$ is a shift; let $\mathcal{M}_0$ denote the wandering subspace for $T|_{\mathcal{M}}$. Since $\mathcal{A}_1|_{\mathcal{M}}$ is an algebra of normal operators, by [4, Proposition 9] $\mathcal{A}_1|_{\mathcal{M}}\ \mathcal{M}_0 \subset \mathcal{M}_0$ and $\mathcal{A}_1^*|_{\mathcal{M}}\ \mathcal{M}_0 \subset \mathcal{M}_0$.

Let $x = y + Tz$, where $y \in \mathcal{M}_0$, $z \in \mathcal{M}$. We show that $\mathcal{M}_0 = [\mathcal{A}(\mathcal{A}_1, \mathcal{A}_1^*)y]$. Let $h \in \mathcal{M}$ such that $h \perp T\mathcal{M}$ and $h \perp [\mathcal{A}(\mathcal{A}_1, \mathcal{A}_1^*)y]$. Then, for $B \in \mathcal{A}(\mathcal{A}_1, \mathcal{A}_1^*)$, we have $h \perp By + TBz = B(y+Tz) = Bx$, also $h \perp T^k\mathcal{M}$ for $k \ge 1$. Thus $h \perp [\mathcal{A}(T, \mathcal{A}_1, \mathcal{A}_1^*)x] = \mathcal{M}$. So $h = 0$. Hence $\mathcal{M}_0 = [\mathcal{A}(\mathcal{A}_1, \mathcal{A}_1^*)y]$.

The algebra $\mathcal{A}(\mathcal{A}_1|_{\mathcal{M}_0}, \mathcal{A}_1^*|_{\mathcal{M}_0})$ is an algebra of normal operators, thus it has property C (see the proof of [5, Theorem 9.21]). Hence there is $\mathcal{N}_0 = [\mathcal{A}(\mathcal{A}, \mathcal{A}^*)a]$, for some $a \in H$, and a unitary operator $U_0 : \mathcal{M}_0 \to \mathcal{N}_0$ such that $U_0(I\otimes A)^{(n)}|_{\mathcal{M}_0}U_0^* = A|_{\mathcal{N}_0}$, for all $A \in \mathcal{A}(\mathcal{A}, \mathcal{A}^*)$. Set $\mathcal{N} = \bigoplus_{n=0}^{\infty}\mathcal{N}_0$ and define $U : \mathcal{M} \to \mathcal{N}$ as follows: if $z \in \mathcal{M}_0$ and $m = 1, 2, \ldots$ then $UT^m z = (S\otimes I)^m U_0 z$, and next extend U by linearity and continuity. It is clear that U is unitary and

$$U(I\otimes A)^{(n)}|_{\mathcal{M}}U^* = (I\otimes A)|_{\mathcal{N}} \quad \text{for all } A \in \mathcal{A}(\mathcal{A}, \mathcal{A}^*),$$
$$UT|_{\mathcal{M}}U^* = (S\otimes I)|_{\mathcal{N}}.$$

Hence $\mathcal{A}(S\otimes I, I\otimes\mathcal{A}, I\otimes\mathcal{A}^*)$ has property C.

Denote by Γ the unit circle and by $H^2(\Gamma^2)$, $H^\infty(\Gamma^2)$, $H^2(\Gamma^2, H)$ the appropriate Hardy spaces. To prove property D(1) for case (5), we recall [1, Theorem 2]:

LEMMA 3. *Given $h \in L^1(\Gamma^2)$ and $\varepsilon > 0$, there are $f \in H^2(\Gamma^2)$ and $g \in L^2(\Gamma^2)$ such that $\|f\|_2, \|g\|_2 \le (1+\varepsilon)\|h\|_1^{1/2}$ and $h = fg$.*

Since each functional on $H^\infty(\Gamma^2)$ is represented by a function in $L^1(\Gamma^2)$ and there is an isometry between $H^\infty(\Gamma^2)$ and the w^*-closed algebra generated by the multiplication operators by the independent variables on $H^2(\Gamma^2)$, it is enough to define $a = f$ and $b = P_{H^2(\Gamma^2)}\overline{g}$, where f, g are given by Lemma 3, to obtain property D(1) for the latter algebra. On the other hand, if property D(1) is satisfied then w^*-closure and WOT-closure coincide by the same arguments used in the proof of [3, Theorem 2]. Thus we have property D(1) for the WOT-closed algebra generated by the multiplication operators by the independent variables on $H^2(\Gamma^2)$ and by [2, Theorem 3.8] also for the WOT-closed algebra generated by the multiplication operators by the independent variables on $H^2(\Gamma^2, H)$. By [6, Theorem 1] it is a model for a pair of doubly commuting shifts. Thus we have proved

LEMMA 4. *A pair of doubly commuting shifts has property* D(1).

Hence property D(1) is proved for case (5) and the gap in the proof of [4, Theorem 1] is removed.

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Received January 21, 1992
Revised version June 5, 1992 (2893)