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A characterization of maximal regular ideals in lmc algebras

by

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Abstract. A question of Warner and Whitley concerning a nonunital version of the Gleason–Kahane–Żelazko theorem is considered in the context of nonnormed topological algebras. Among other things it is shown that a closed hyperplane M of a commutative symmetric F*-algebra E with Lindelöf Gel'fand space is a maximal regular ideal iff each element of M belongs to some closed maximal regular ideal of E.

1. Introduction. In 1969 Warner and Whitley presented an example [16; p. 277] showing that for a hyperplane M in a nonunital commutative Banach algebra E, the property:

(1) each element of M belongs to some maximal regular ideal of E,

does not characterize M as a maximal regular ideal, as in the unital case (Gleason–Kahane–Żelazko theorem). Their example pointed out that, in this regard, one should consider algebras E satisfying the condition:

(2) some element of E belongs to no maximal regular ideal of E.

The convolution algebra $L^1(G)$, G an abelian metrizable locally compact group, satisfies (2) [16]. In 1988 Maltese and Wille-Fier, omitting the commutativity assumption and considering maximal regular 2-sided ideals of codimension 1, gave necessary and sufficient conditions for (2) to hold [13; Theorem 2.1]. Moreover, they showed that (2) holds for $L^1(G)$, even if G as above is not abelian (ibid., Theorem 4.1).

In this paper we investigate (2) in the setting of nonnormed topological algebras. The results obtained apply in the commutative case to symmetric

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LFQ *-algebras with second countable Gel'fand space (cf. Corollary 3.5 and Remark 3.6) as well as to $C_0(X)$, X locally compact σ-compact, endowed with the compact-open topology (Corollary 4.3). In the noncommutative case the applications concern various tensor product function algebras (see Section 4.4).

We should note that according to [9; p. 343] the Gleason–Kahane–Żelazko theorem is valid for *every unital commutative complete lmc* (locally m-convex) *algebra* E *with* f ($M = \ker(f)$) *a continuous linear form on* E. On the other hand, using [15; p. 113, Corollary] a variant of the preceding result is here stated for *unital strong spectrally bounded lmc* (resp. *unital topological* Q-) *algebras* (for the respective concepts cf. [2] and Section 2) *without assuming commutativity and completeness of* E *or continuity of* f. Note that, in general, the Gleason–Kahane–Żelazko theorem is not true without m-convexity [9; p. 343] or without continuity of the linear form involved [18; Proposition]. Furthermore (cf. [18] and [17; Theorem 2, Corollary 2]), in a unital complete lmc algebra E, a closed hyperplane M is a maximal 2-sided ideal iff each $x \in M$ belongs to some maximal 2-sided ideal of E of codimension 1.

I am indebted to Professors E. Albrecht and J. Eschmeier whose comments led to Remark 3.6(1). My special thanks are also due to the referee for his constructive comments including the indication of the redundancy of $r|_{H(E)} < \infty$ in Theorem 3.1(ii) when E is metrizable. As a consequence, we get (iii) of Theorem 3.1 and thus Theorem 4.1.

2. Preliminaries. The algebras we deal with are complex and the topological spaces Hausdorff.

An *lmc* (locally m-convex) *algebra* is a topological algebra E whose topology is defined by a family $\Gamma_E = (p_\alpha)$, $\alpha \in A$ (A a directed index set), of submultiplicative seminorms [12, 14]. A (complete) lmc algebra whose underlying locally convex space is a Fréchet space will be called an *F-algebra*. A *Q-algebra* is a topological algebra such that the set of its quasi-invertible elements is open (ibid.). Furthermore, an algebra E is called an *LF-algebra* (see [12; p. 301, Definition 9.1] and [14; Definition 15.2]) whenever there exists an increasing sequence (E_n), $n \in \mathbb{N}$, of F-algebras (subalgebras of E) such that:

(1) $E = \bigcup_n E_n$ and each E_n is a 2-sided ideal in E.
(2) $\tau_n = \tau_{n+1}|_{E_n}$, where τ_n is the given topology of E_n, $n \in \mathbb{N}$.
(3) $\tau|_{E_n} = \tau_n$, $n \in \mathbb{N}$, where τ is the final lmc topology on E induced by the natural injections $E_n \to E$, $n \in \mathbb{N}$.

An LF-algebra which is also a Q-algebra will be called an *LFQ-algebra*. Let now E be an lmc algebra and $p_\alpha \in \Gamma_E$. Set $N_\alpha := \ker(p_\alpha)$ and endow E/N_α with the norm $\|\cdot\|_\alpha$ induced by p_α, i.e., $\|x_\alpha\|_\alpha := p_\alpha(x)$, $x_\alpha \equiv x + N_\alpha$, $x \in E$. The Banach algebra completion of $(E/N_\alpha, \|\cdot\|_\alpha)$ will be denoted by E_α, $\alpha \in A$. If E' (resp. E'_s) stands for the *topological* (resp. *weak topological*) *dual* of E, the space

$$\mathcal{M}(E) := \{f \in E' : f \neq 0 \text{ and } f(xy) = f(x)f(y),\ \forall x, y \in E\},$$

endowed with the relative topology from E'_s is called the *Gel'fand space* of E [12; p. 139]. When E is commutative, $\mathcal{M}(E)$ is identified with the set of all closed maximal regular ideals of E. If E is noncommutative, $\mathcal{M}(E)$ is identified with all closed maximal regular 2-sided ideals of codimension 1. In either case one might have $\mathcal{M}(E) = \emptyset$. Throughout this paper, given an lmc algebra E, we shall always assume that $\mathcal{M}(E) \neq \emptyset$. In this regard, see comments after 4.4(3). It is easily seen that each $f \in E'$ (resp. $f \in \mathcal{M}(E)$) defines, for some $\alpha \in A$, an element $f_\alpha \in E'_\alpha$ (resp. $f_\alpha \in \mathcal{M}(E_\alpha)$) with

$$f_\alpha(x_\alpha) := f(x), \quad x \in E; \tag{2.1}$$

f_α is called the element of E'_α (resp. $\mathcal{M}(E_\alpha)$) *associated with* f. In particular,

$$\mathcal{M}(E) = \bigcup_\alpha \mathcal{M}(E_\alpha) \tag{2.2}$$

[12; p. 172, Lemma 6.3]. Furthermore, denote by r the spectral radius of E. Then, if E is moreover commutative and complete, r is given by (ibid., p. 104, (6.16))

$$r(x) = \sup\{|f(x)| : f \in \mathcal{M}(E)\}, \quad x \in E. \tag{2.3}$$

Suppose now that E is an involutive algebra. Then $H(E) := \{x \in E : x^* = x\}$, and E will be called *symmetric* if

$$f(x^*) = \overline{f(x)} \Leftrightarrow (x^*)^\wedge(f) = (\widehat{x})^-(f), \quad \forall f \in \mathcal{M}(E),\ x \in E, \tag{2.4}$$

where $\widehat{x}$ is the *Gel'fand transform* of x. An lmc algebra E with a continuous involution (i.e., $p_\alpha(x^*) = p_\alpha(x)$, for any $x \in E$, $\alpha \in A$), will be called an *lmc *-algebra*. By a *σ-compact space* we will simply mean a topological space which is a countable union of compact subsets [10; p. 172].

3. Characterization of maximal regular ideals in certain commutative lmc algebras. Given an lmc algebra E, condition (2) of Section 1 can also be stated as follows:

(e) there exists $x \in E$ with $h(x) \neq 0$, $\forall h \in \mathcal{M}(E)$.

In this connection, the following extends [13; Theorem 2.1].

THEOREM 3.1. (i) *Every metrizable lmc algebra E with property* (e) *has a Lindelöf Gel'fand space.*

(ii) *Every commutative symmetric complete lmc *-algebra E with $\mathcal{M}(E)$ Lindelöf and $r|_{H(E)} < \infty$ possesses property* (e).

(iii) *If E in* (ii) *is metrizable, commutativity of E, continuity of the involution and $r|_{H(E)} < \infty$ are redundant.*

Proof. (i) $\Gamma_E = (p_n)$, $n \in \mathbb{N}$, and each E_n, $n \in \mathbb{N}$, satisfies (e); hence the assertion follows by (2.2) and the respective Banach algebra result [13; Theorem 2.1].

(ii) By the symmetry of E and the fact that $\mathcal{M}(E)$ is Lindelöf, one gets

$$\mathcal{M}(E) = \bigcup_{k=1}^{\infty}\{U_{h_k} : \widehat{z}_k(U_{h_k}) > 0,\ z_k := y_k^* y_k\}\,, \tag{3.1}$$

where U_{h_k} is a neighborhood of h_k in $\mathcal{M}(E)$ and $y_k \in E$ with $h_k(y_k) \neq 0$. Now, since $z_k \in H(E)$ we may consider the element

$$w_k := \frac{z_k}{r(z_k)+1} \in E$$

so that [12; p. 99, (6.1)] $r_{E_\alpha}(w_{k,\alpha}) < 1$, for all α's. Thus, applying a result of G. Lumer [11; p. 136] for $F_\alpha := E_\alpha \oplus \mathbb{C}$ (unitization of E_α) (with Γ (ibid.) being the finite group of norm-preserving transformations of F_α given by the involution), we conclude that there is a Banach *-algebra norm $\|\cdot\|'_\alpha$ on E_α, equivalent to $\|\cdot\|_\alpha$ and such that $\|w_{k,\alpha}\|'_\alpha \le 1$, $\alpha \in A$. Hence, if

$$w_\alpha := \sum_{k=1}^{\infty}\frac{1}{2^k}w_{k,\alpha} \in E_\alpha, \quad \alpha \in A\,,$$

the element $w := (w_\alpha) \in E = \varprojlim E_\alpha$ satisfies (e) (cf. also (3.1)).

(iii) In this case w is given by the series $\sum_{k=1}^{\infty} \alpha_k z_k$ where the z_k are as in (ii) and the α_k are any positive scalars such that the series is convergent. ■

A pertinent question at this point would be to find a counterexample showing that the assumption $r|_{H(E)} < \infty$ in Theorem 3.1(ii) is essential (I am indebted to Professor W. Żelazko for raising this question).

Remark 3.2. Regarding (ii) of Theorem 3.1, we note:

(1) When E is not symmetric, (e) may fail according to an example given by Maltese and Wille-Fier [13; p. 136].

(2) Commutativity and continuity of the involution can be omitted from the hypotheses by replacing $r|_{H(E)} < \infty$ with $\sup_\alpha(p_\alpha|_{H(E)}) < \infty$ (examples of lmc algebras with the latter property can be found in [2; pp. 51, 52]). In fact, $\|x\|_b := \sup_\alpha p_\alpha(x)$, $x \in E$, is a norm on E, thus w_α's can be given by $\sum_{k=1}^{\infty} z_{k,\alpha}/(2^k\|z_k\|_b)$.

PROPOSITION 3.3. *Every metrizable lmc algebra with a countable left (or right) a.u. (approximate unit), as well as every commutative LFQ-algebra E with $\mathcal{M}(E_n)$ second countable for each n* (cf. Section 2), *has a Lindelöf Gel'fand space.*

Proof. The first assertion follows by (2.2) and [13; Theorem 2.2]. For the second, one has (cf. [14; Proposition 15.9])

$$\mathcal{M}(E) = \bigcup_n \mathcal{M}(E_n)\,. \tag{3.2}$$

We show that each $\mathcal{M}(E_n)$ is locally compact. In virtue of [12; p. 75, Proposition 7.1 and p. 143, Theorem 1.1] it suffices to show that each E_n, $n \in \mathbb{N}$, is a Q-algebra. This follows from [12; p. 105, Lemma 6.1] since E is a Q-algebra and $r_{E_n}(x) = r_E(x)$, $x \in E_n$, $n \in \mathbb{N}$ (cf. (3.2), (2.3)). Thus, $\mathcal{M}(E_n)$, $n \in \mathbb{N}$, being second countable, becomes σ-compact. The assertion now results from (3.2) (see also [14; Proposition C.3.]). ■

If in the second case of Proposition 3.3 we suppose that $\mathcal{M}(E)$ is second countable, then commutativity of E is redundant, since E, being a Q-algebra, has $\mathcal{M}(E)$ locally compact.

THEOREM 3.4. *Let E be a commutative symmetric complete lmc *-algebra with $\mathcal{M}(E)$ Lindelöf and $r|_{H(E)} < \infty$. Let also M be a closed hyperplane of E each element of which is contained in some closed maximal regular ideal of E. Then M itself is a maximal regular ideal.*

Proof. Clearly $M = \ker(f)$ with $f \in E'$. Hence by Theorem 3.1(ii) and the symmetry of E we may assume that $f(x) = 1$, $f(x^2) \neq 0$, for some $x \in H(E)$ with $h(x) \neq 0$ for all $h \in \mathcal{M}(E)$. Then following the proof of [13; Theorem 2.3], which is purely algebraic, we conclude that the function $g(y) := f(x^2)f(y)$, $y \in E$, is an element of $\mathcal{M}(E)$ with $M = \ker(g)$. ■

COROLLARY 3.5. *Let E be a commutative symmetric LFQ *-algebra with $\mathcal{M}(E_n)$, $n \in \mathbb{N}$, second countable. Then a (closed) hyperplane $\mathcal{M}$ of E, each element of which belongs to a maximal regular ideal of E, is itself a maximal regular ideal.*

Proof. Apply Theorem 3.4 using the property Q of E and Proposition 3.3. ■

It is easily seen that Theorem 3.4 also applies to the generalized group algebra $L^1(G,E)$ [12; p. 402] with G an abelian metrizable locally compact group and E a commutative symmetric Banach *-algebra with either a unit or a countable a.u. Symmetry of $L^1(G,E)$ and the property of $\mathcal{M}(L^1(E,G))$ being Lindelöf are shown as in 4.4(3).

Now let X be a σ-compact finite-dimensional C^∞-manifold and $D(X)$ the algebra of test functions on X, i.e.,

$$D(X) := \{f \in C^\infty(X) : \operatorname{supp}(f) \text{ compact}\} = \varinjlim D_{K_n}(X)\,,$$

where K_n, $n \in \mathbb{N}$, are compact subsets exhausting X and $D_{K_n}(X) = \{f \in D(X) : \operatorname{supp}(f) \subseteq K_n\}$, an FQ-algebra endowed with the relative

C^∞-topology from $C^\infty(K_n)$, $n \in \mathbb{N}$ (cf. [12; p. 132] and proof of Proposition 3.3). $D(X)$ equipped with the inductive limit topology, say τ, is an LFQ-algebra (cf. [12; p. 133, (4.25)] and [4; proof of Proposition 3.3]), symmetric under the continuous involution induced by the complex conjugate. In fact, $r(f) = \sup\{|f(x)| : x \in X\}$, $f \in D(X)$ [4; proof of Proposition 3.3], so that $r(f^*f) = r(f)^2$, $f \in D(X)$, which yields symmetry for $D(X)$ [3; Theorem 2.4]. In this connection, if $\mathcal{M}_\tau := \mathcal{M}(D(X), \tau)$ and $\mathcal{M}_n := \mathcal{M}(D_{K_n}(X))$, $n \in \mathbb{N}$, we have the following.

Remark 3.6. (1) *$\mathcal{M}_\tau$ is not Lindelöf* (and a fortiori not second countable or σ-compact). Indeed, if τ_∞ is the relative C^∞-topology on $D(X)$ and $\mathcal{M}_\infty := \mathcal{M}(D(X), \tau_\infty)$, one has

$$\mathcal{M}_\infty = X \subseteq \mathcal{M}_\tau$$

(cf. [4; proof of Proposition 3.3]); therefore if $\mathcal{M}_\tau$ is Lindelöf, Theorem 3.1(ii) leads to the contradiction that X is compact.

(2) An immediate consequence of Theorem 3.1 is that *an involutive symmetric F-algebra has property* (e) *iff it has a Lindelöf Gel'fand space*. Thus, by (1) (cf. also (3.2)), we conclude that, at least, one of the $\mathcal{M}_n$'s, say $\mathcal{M}_{n_0}$, is not σ-compact, therefore

$$\{\delta_x : x \in K_{n_0}\} \subsetneq \mathcal{M}_{n_0}$$

(δ_x being the point evaluation on $D_{K_{n_0}}(X)$), hence

$$\mathcal{M}_\infty = \{\delta_x : x \in X\} \subsetneq \mathcal{M}_\tau \, .$$

(3) Consider $U = \mathcal{M}_\tau \setminus \mathcal{M}_\infty$. Then from (1) and Theorem 3.1(ii) one is naturally led to the following question: *Is U a Lindelöf space*? If the answer is positive one gets the existence of an element $f \in D(X)$ with $h(f) \neq 0$ for every $h \in U$. Thus applying the argument of Theorem 3.4 we can prove the following: If $M := \ker(F)$, $F \in (D(X), \tau)'$, is a hyperplane in $D(X)$ with the property "every $f \in M$ is contained in some $\ker(h_f)$ with $h_f \in \mathcal{M}_\tau$ such that $h_f \neq \delta_x$ for every $x \notin \operatorname{supp}(f)$", then M is a maximal regular ideal of $D(X)$. So it would be of interest to know the topological properties of U.

One has exactly the same situation as before for the algebra $K(X)$ of all $\mathbb{C}$-valued continuous functions on X with compact support, X being a locally compact σ-compact space (cf. [12; p. 127, 4(1)]).

4. Characterization of maximal regular 2-sided ideals of codimension 1 in some lmc algebras. In this section we deal with nonunital noncommutative lmc algebras. Examples of such algebras with nonempty Gel'fand space are discussed after 4.4(3).

THEOREM 4.1. *Let E be an involutive symmetric F-algebra with $\mathcal{M}(E)$ Lindelöf. Let also M be a closed hyperplane of E each element of which is contained in some closed maximal regular 2-sided ideal of E of codimension 1. Then M itself is a maximal regular 2-sided ideal.*

Proof. Using Theorem 3.1(iii) we argue as in the proof of Theorem 3.4. ∎

A variant of Theorem 4.1 is now obtained by passing to the Banach algebra factors of the lmc algebra involved and using the respective Banach algebra result [13; Theorem 2.3] formulated in terms of multiplicative linear forms. Here one does not need completeness for the given algebra, at the cost, however, of requiring a continuous involution. More precisely, we get the following.

THEOREM 4.2. *Let $(E, (p_n))$, $n \in \mathbb{N}$, be a metrizable symmetric lmc *-algebra with either a countable left (or right) a.u. or $\mathcal{M}(E)$ second countable. Let also $f \in E'$ and let f_n be the element of E_n' associated with f* (cf. (2.1)). *Then the following are equivalent*:

(1) *There exists $0 \neq \lambda \in \mathbb{C}$ with $\lambda f \in \mathcal{M}(E)$.*

(2) *$f_n(z) = 0$, $z \in E_n$, implies $h(z) = 0$ for some $h \in \mathcal{M}(E_n)$.*

Proof. We show (2)⇒(1). Each E_n is symmetric with $\mathcal{M}(E_n)$ Lindelöf (cf. (2.4), (2.2) and Proposition 3.3). Hence [13; Theorem 2.3], there exists $0 \neq \lambda \in \mathbb{C}$ with $\lambda f_n \in \mathcal{M}(E_n)$, which in turn yields $\lambda f \in \mathcal{M}(E)$. ∎

Suppose now that X is a locally compact σ-compact space and let $C_c(X)$ be the F-*-algebra of all $\mathbb{C}$-valued continuous functions on X with the compact-open topology c. Denote by E_c the algebra $C_0(X)$ of all $f \in C_c(X)$ which vanish at infinity, endowed with the relative topology from $C_c(X)$. Then we have

COROLLARY 4.3. *If $F \in E_c'$ and F_n is the element of $(E_c)_n'$ associated with F, then $F = \lambda \delta_{x_0}$ for some $0 \neq \lambda \in \mathbb{C}$, $x_0 \in X$, iff each $f_n \in \ker(F_n)$ vanishes at some $x \in X$.*

Proof. Clearly E_c is a metrizable lmc *-algebra and $c \le \|\cdot\|$, where $\|\cdot\|$ is the usual supnorm on $C_0(X)$. Thus,

$$X \subseteq \mathcal{M}(E_c) \subseteq \mathcal{M}(C_0(X), \|\cdot\|) = X \, .$$

Hence (cf. also (2.2), (2.4)), $\mathcal{M}((E_c)_n) \subseteq X$ for each $n \in \mathbb{N}$ and E_c is symmetric. Moreover, E_c has a countable a.u. since this is the case for $(C_0(X), \|\cdot\|)$ (cf. [1; Proposition (12.2)] and/or [13; Theorem 3.2]). The assertion now follows from Theorem 4.2. ∎

4.4. *Applications of Theorems* 4.1, 4.2 *to some topological tensor product function algebras*. (1) Let E be a symmetric F-*-algebra with a countable a.u. and with $\mathcal{M}(E)$ locally compact. Let also X be locally compact, σ-compact and let $B_1 := C_c(X, E)$ be the algebra of all E-valued continuous functions on X with the topology of compact convergence [12; p. 387]. Then (cf. [8; p. 352, Corollary 3] and [12; p. 391, Theorem 1.1]),

$$B_1 = \varprojlim C_u(K_n, E_m), \quad \text{where } K_n \subseteq X, \text{ compact, with } X = \bigcup_n K_n ,$$

and u is the topology of uniform convergence. In particular,

$$\mathcal{M}(B_1) = \mathcal{M}(C_c(X)) \times \mathcal{M}(E) = \{\delta_x \otimes h : h \in \mathcal{M}(E)\}$$

[12; p. 143, Proposition 1.1, Theorem 1.1; p. 411, Theorem 1.2; and p. 223, Theorem 1.2], where $C_c(X)$ is symmetric by [3; Corollary 2.2] and $\mathcal{M}(E)$ is σ-compact by Proposition 3.3. Thus, B_1 is a symmetric F-*-algebra with σ-compact Gel'fand space.

(2) Let E be as in (1) and X a compact (thus metrizable) n-dimensional C^∞-manifold. Let also $B_2 := C^\infty(X, E) = \varprojlim C^\infty(U_n, E_m)$ ((U_n), $n \in \mathbb{N}$, a basis for the topology of X) be the algebra of all E-valued C^∞-maps on X [12; p. 392, 2]. Arguing as in (1) and using [12; p. 394, (2.7); p. 227, Theorem 2.1], as well as the fact that $C^\infty(X)$ is symmetric [3; 2.5(i)], we conclude that B_2 satisfies the assumptions of either of Theorems 4.1, 4.2.

(3) Let E be as in (1), G a metrizable locally compact group and $B_3 := L^1(G, E) = \varprojlim L^1(G, E_m)$ the generalized group algebra of G [12; p. 402, 5]. Then

$$\mathcal{M}(B_3) = \mathcal{M}(L^1(G)) \times \mathcal{M}(E) = \widehat{G} \times \mathcal{M}(E) ,$$

where $\widehat{G}$ is the character group of $L^1(G)$, the latter being symmetric with a countable a.u. [13; proof of Theorem 4.1]. Thus, as before we find that B_3 satisfies the assumptions of Theorems 4.1, 4.2.

B_i, $i = 1, 2, 3$, provide examples of lmc algebras with a countable a.u. Moreover, if G in (3) is abelian and $E := L^1(G)$, the respective F-algebras B_1, B_2 as above give examples of nonunital commutative lmc algebras with nonempty Gel'fand space. The same is also true for B_3 with G as before and E either of the algebras $C_c(X)$, $C^\infty(X)$. On the other hand, taking X, G as in (1), (2), (3) respectively, one sees that the nonunital noncommutative F-algebras $C_c(X, E)$, $C^\infty(X, E)$ with $E := L^1(G)$, as well as $L^1(G, E)$ with $E := C^\infty(X)$, have a nonempty Gel'fand space; for the latter case see also [6; (23.3), Corollary 23.7].

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