

**On the ideal structure of algebras
of LMC-algebra valued functions**

by

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Abstract. Let X be a completely regular topological space and A a commutative locally m -convex algebra. We give a description of all closed and in particular closed maximal ideals of the algebra $C(X, A)$ (= all continuous A -valued functions defined on X). The topology on $C(X, A)$ is defined by a certain family of seminorms. The compact-open topology of $C(X, A)$ is a special case of this topology.

Introduction. Let A be a commutative locally m -convex algebra with identity e over the field \mathbf{C} of the complex numbers. Let $\mathcal{P} = \{p_\lambda \mid \lambda \in \Lambda\}$ be a family of seminorms which defines the topology in A denoted by $T(\mathcal{P})$. It is assumed that $T(\mathcal{P})$ is a Hausdorff topology, in other words $p_\lambda(x) = 0$ for all $\lambda \in \Lambda$ only if $x = 0$. Furthermore, we assume that the family \mathcal{P} is directed. For $\lambda \in \Lambda$ we set $N_\lambda = \{x \in A \mid p_\lambda(x) = 0\}$. For each $\lambda \in \Lambda$ the quotient algebra $A/N_\lambda = A_\lambda$ is a normed algebra with the norm $\hat{p}_\lambda(x + N_\lambda) = p_\lambda(x)$, $x + N_\lambda \in A/N_\lambda$. We shall denote the completion of A_λ by \tilde{A}_λ . General properties of locally m -convex algebras can be found for example in [4], [12] and [13]. Let $\Delta(A)$ be the set of all nontrivial continuous \mathbf{C} -homomorphisms on A . The set $\Delta(A)$ will be equipped with the relative $\sigma(A', A)$ -topology called the *Gelfand topology*. With this topology $\Delta(A)$ is called the *carrier space* of $(A, T(\mathcal{P}))$.

Let $x \in A$ be given. The \mathbf{C} -valued function \hat{x} on the carrier space $\Delta(A)$ defined by $\hat{x}(\tau) = \tau(x)$, $\tau \in \Delta(A)$, is continuous, whence $\hat{x} \in C(\Delta(A))$.

Let I be an ideal of A . The *hull* of I , denoted by $h(I)$, is then defined as $h(I) = \{\tau \in \Delta(A) \mid \hat{x}(\tau) = 0, x \in I\}$. The *kernel* $k(E)$ of a subset E of $\Delta(A)$ is defined by $k(E) = \{x \in A \mid \hat{x}(\tau) = 0, \tau \in E\}$ and for the empty set \emptyset we define $k(\emptyset) = A$. Obviously $h(I)$ is a closed subset of $\Delta(A)$ and $k(E)$ is a closed ideal of $(A, T(\mathcal{P}))$.

For a completely regular topological space X denote by $C(X, A)$ the set

of all continuous A -valued functions defined on X . Algebraic operations in $C(X, A)$ are defined pointwise. For a given element x in A we denote by f_x the constant function $f_x(t) = x, t \in X$. Thus f_e is the unit element and f_0 is the zero element of $C(X, A)$.

Let \mathcal{K} be a compact cover of X which is closed under finite unions, in other words, \mathcal{K} is a family of compact subsets of X for which

- (1)
$$\bigcup \{K \mid K \in \mathcal{K}\} = X,$$
- (2) if $K_1, K_2 \in \mathcal{K}$, then $K_1 \cup K_2 \in \mathcal{K}$.

For $K \in \mathcal{K}$ and $\lambda \in A$, we define a seminorm $p_{(K,\lambda)}$ on $C(X, A)$ by

$$p_{(K,\lambda)}(f) = \sup_{t \in K} p_\lambda(f(t)), \quad f \in C(X, A).$$

We write $\mathcal{P}(\mathcal{K}, A) = \{p_{(K,\lambda)} \mid K \in \mathcal{K}, \lambda \in A\}$. The family $\mathcal{P}(\mathcal{K}, A)$ defines a locally m -convex Hausdorff topology on $C(X, A)$ denoted from now on by $T(\mathcal{K}, A)$. Obviously $\mathcal{P}(\mathcal{K}, A)$ is a directed family of seminorms. If $\mathcal{K} = \mathcal{K}(X) =$ the set of all compact subsets of X , then $T(\mathcal{K}, A)$ is the compact-open topology of $C(X, A)$. Given $K \in \mathcal{K}$ and $\lambda \in A$, define $N_{(K,\lambda)} = \{f \in C(X, A) \mid p_{(K,\lambda)}(f) = 0\}$.

Let $t \in X$ and let I be an ideal of A . We define an ideal $J_{(t,I)}$ of $C(X, A)$ by $J_{(t,I)} = \{f \in C(X, A) \mid f(t) \in I\}$ and furthermore $C(X, I) = \{f \in C(X, A) \mid f(t) \in I, t \in X\}$. Obviously $C(X, I) = \bigcap_{t \in X} J_{(t,I)}$. If I is a closed ideal of $(A, T(\mathcal{P}))$ and $t \in X$, then it is easy to see that both the ideals $J_{(t,I)}$ and $C(X, I)$ are closed in $(C(X, A), T(\mathcal{K}, A))$.

Let $K \in \mathcal{K}, \lambda \in A$ and $\varepsilon > 0$. We denote by $V_{(K,\lambda)}(\varepsilon)$ the set $\{f \in C(X, A) \mid p_{(K,\lambda)}(f) < \varepsilon\}$. Obviously the sets $V_{(K,\lambda)}(\varepsilon), K \in \mathcal{K}, \lambda \in A, \varepsilon > 0$, form a subbase of neighbourhoods of f_0 (see [14], p. 8).

On closed ideals of $(C(X, A), T(\mathcal{K}, A))$. We give a description of the closed ideals of the algebra $(C(X, A), T(\mathcal{K}, A))$. We also prove that the carrier space $\Delta(C(X, A))$ is homeomorphic to $X \times \Delta(A)$ if $\Delta(A)$ is locally equicontinuous.

The following lemmas are easy to verify.

LEMMA 1. $N_{(K,\lambda)} = \bigcap_{t \in K} J_{(t,N_\lambda)}$ for all $K \in \mathcal{K}$ and $\lambda \in A$.

LEMMA 2. Let $\{I_\alpha \mid \alpha \in \Gamma\}$ be a family of closed ideals in $(A, T(\mathcal{P}))$. Then $\bigcap_{\alpha \in \Gamma} C(X, I_\alpha) = C(X, \bigcap_{\alpha \in \Gamma} I_\alpha)$.

Let J be a closed ideal of $(C(X, A), T(\mathcal{K}, A))$ and let $t \in X$ be given. We define $I_t = \{f(t) \mid f \in J\}$ and $I(t) = \text{cl}(I_t) =$ the closure of I_t in $(A, T(\mathcal{P}))$. It is easy to see that either $I(t)$ is a closed proper ideal of $(A, T(\mathcal{P}))$ or $I(t) = A$.

Next we shall prove a useful result.

THEOREM 3. If J is a closed proper ideal of $(C(X, A), T(\mathcal{K}, A))$, then there is t in X such that $I(t)$ is a closed proper ideal of $(A, T(\mathcal{P}))$.

Proof. Suppose that $I(t) = A$ for all $t \in X$. Let $K \in \mathcal{K}, \lambda \in A$ and $\varepsilon > 0$ be arbitrary. Then for each $t_0 \in K$ there is $f_{t_0} \in J$ such that $p_\lambda(f_{t_0}(t_0) - e) = p_\lambda(f_{t_0}(t_0) - f_e(t_0)) < \varepsilon$. By continuity, there is a neighbourhood $U(t_0)$ of t_0 such that

$$(3) \quad p_\lambda(f_{t_0}(t) - f_e(t)) < \varepsilon \quad \text{for all } t \in U(t_0).$$

Now the neighbourhoods $\{U(t_0) \mid t_0 \in K\}$ form an open cover of the compact set K . Take its finite subcovering U_1, \dots, U_n and denote by f_1, \dots, f_n those functions of J for which (3) is valid. By Lemma 2.1.1 of [6], there are $\alpha_i \in C(X), i = 1, \dots, n$, such that $0 \leq \alpha_i(t) \leq 1$ for all $t \in X$ and $i = 1, \dots, n, \text{supp } \alpha_i \subset U_i$ for all $i = 1, \dots, n$ and $\sum_{i=1}^n \alpha_i(t) = 1$ for all $t \in K$. Now, if we define a function $F_{(K,\lambda)}$ by

$$F_{(K,\lambda)}(t) = \sum_{i=1}^n (\alpha_i f_i)(t), \quad t \in X,$$

we see that $F_{(K,\lambda)} \in J$ and

$$p_{(K,\lambda)}(F_{(K,\lambda)} - f_e) < \varepsilon.$$

Thus, $F_{(K,\lambda)} \in f_e + V_{(K,\lambda)}(\varepsilon)$. Since the $V_{(K,\lambda)}(\varepsilon)$ form a subbasis of the zero neighbourhoods, the unit element f_e is in $\text{cl}(J) = J$, which is impossible.

Next we shall give a description of the carrier space $\Delta(C(X, A))$. The structure of $\Delta(C(X, A))$ has been considered in many papers under various topological assumptions on X and A (see [1], [3], [5], [6], [8]–[11], [16] and [17]). For example in [6] Dietrich assumed that X is a k -space and A is a complete locally convex algebra for which $\Delta(A)$ is locally equicontinuous. He used the tensor product representation of $C(X, A)$. Tensor product techniques have also been used in [11]. Abel has proved a corresponding result for a more general case in [1]. In this paper we consider the carrier space of the algebra $(C(X, A), T(\mathcal{K}, A))$.

Let $t \in X$ and $\tau \in \Delta(A)$ be given. We define a mapping $\phi_{(t,\tau)} : C(X, A) \rightarrow \mathbb{C}$ by

$$\phi_{(t,\tau)}(f) = \tau(f(t)), \quad f \in C(X, A).$$

Obviously $\phi_{(t,\tau)} \in \Delta(C(X, A))$ and $\ker \phi_{(t,\tau)} = J_{(t, \ker \tau)}$.

LEMMA 4. If N is a closed maximal ideal of $(C(X, A), T(\mathcal{K}, A))$, then there are unique points $t \in X$ and $\tau \in \Delta(A)$ such that $N = \ker \phi_{(t,\tau)}$.

Proof. By Theorem 3, there is $t \in X$ such that $I(t) = \text{cl}(\{f(t) \mid f \in N\})$ is a closed proper ideal of $(A, T(\mathcal{P}))$. Now there is $\tau \in \Delta(A)$ such that

$I(t) \subset \ker \tau$. If $f \in N$ is arbitrary, then $\phi_{(t,\tau)}(f) = \tau(f(t)) = 0$. Thus $N \subset \ker \phi_{(t,\tau)}$, and so $N = \ker \phi_{(t,\tau)} = J_{(t,\ker \tau)}$. It is easy to see that the points $t \in X$ and $\tau \in \Delta(A)$ are unique.

We now define a mapping $\varphi : X \times \Delta(A) \rightarrow \Delta(C(X, A))$ by

$$(4) \quad \varphi(t, \tau) = \phi_{(t,\tau)}, \quad (t, \tau) \in X \times \Delta(A).$$

THEOREM 5. *The mapping φ defined in (4) is a bijection from $X \times \Delta(A)$ onto $\Delta(C(X, A))$. The inverse mapping φ^{-1} is continuous, and φ is continuous if $\Delta(A)$ is locally equicontinuous.*

Proof. Obviously $\varphi(t, \tau) \in \Delta(C(X, A))$ if $(t, \tau) \in X \times \Delta(A)$. It is also easy to see that φ is an injection. Now, if $\phi \in \Delta(C(X, A))$, then $\ker \phi$ is a closed maximal ideal of $(C(X, A), T(\mathcal{K}, A))$. By Lemma 4 there are unique $t \in X$ and $\tau \in \Delta(A)$ such that $\ker \phi = \ker \phi_{(t,\tau)}$ and thus $\phi = \phi_{(t,\tau)}$. So φ is also a surjection.

Now, φ is continuous provided so is the mapping $\hat{f} : X \times \Delta(A) \rightarrow \mathbb{C}$ defined by

$$\hat{f}(t, \tau) = \hat{f}(\phi_{(t,\tau)}) = \tau(f(t)), \quad (t, \tau) \in X \times \Delta(A).$$

But it is well-known that \hat{f} is continuous if $\Delta(A)$ is locally equicontinuous (see e.g. [10], Lemma 3). So φ is continuous if $\Delta(A)$ is locally equicontinuous. The continuity of φ^{-1} can be shown by a similar method to that used in [17] for the Nachbin algebras.

COROLLARY 6. *The carrier space $\Delta(C(X, A))$ of the algebra $(C(X, A), T(\mathcal{K}, A))$ is homeomorphic to $X \times \Delta(A)$ if $\Delta(A)$ is locally equicontinuous.*

COROLLARY 7. *If $\phi \in \Delta(C(X, A))$, then there is exactly one $t_0 \in X$ such that $I(t_0) = \text{cl}(\{f(t_0) \mid f \in \ker \phi\})$ is a closed proper ideal of $(A, T(\mathcal{P}))$, and furthermore $I(t_0) = \ker \tau$ for some $\tau \in \Delta(A)$.*

Next we give a description of closed (proper) ideals J of $(C(X, A), T(\mathcal{K}, A))$. It has been proved in [2] (Theorem 2) that in the case where X is a compact Hausdorff space and A is a locally convex topological algebra there is $E \subset X$ and a family $\{I(t) \mid t \in E\}$ of closed proper ideals of A such that $J = \bigcap_{t \in E} J_{(t, I(t))}$. We now prove this for $(C(X, A), T(\mathcal{K}, A))$.

THEOREM 8. *If J is a closed proper ideal of $(C(X, A), T(\mathcal{K}, A))$, then there is $E \subset X$ and a family $\{I(t) \mid t \in E\}$ of closed ideals of $(A, T(\mathcal{P}))$ such that $J = \bigcap_{t \in E} J_{(t, I(t))}$.*

Proof. For $t \in X$ set $I(t) = \text{cl}(\{f(t) \mid f \in J\})$. As noted earlier, either $I(t)$ is a proper closed ideal of $(A, T(\mathcal{P}))$ or $I(t) = A$. Define $E = \{t \in X \mid I(t) \text{ is a proper subset of } A\}$. By Theorem 3, E is nonempty.

It is easy to see that $J \subset \bigcap_{t \in E} J_{(t, I(t))}$. Now, if $t \in X \sim E =$ the complement of E in X , then $J_{(t, I(t))} = C(X, A)$ and therefore $\bigcap_{t \in X} J_{(t, I(t))} = \bigcap_{t \in E} J_{(t, I(t))}$. Let $f \in \bigcap_{t \in X} J_{(t, I(t))}$ and fix $K \in \mathcal{K}$, $\lambda \in A$, $t_0 \in K$ and $\varepsilon > 0$. From the definition of $I(t_0)$ it follows that there is $f_{t_0} \in J$ such that $p_\lambda(f_{t_0}(t_0) - f(t_0)) < \varepsilon$. By continuity, there is a neighbourhood $U(t_0) \subset X$ of t_0 such that

$$(5) \quad p_\lambda(f_{t_0}(t) - f(t)) < \varepsilon, \quad t \in U(t_0).$$

Now $\{U(t_0) \mid t_0 \in K\}$ is an open cover of K . Taking a finite subcover as in the proof of Theorem 3, we obtain a function $F_{(K, \lambda)} \in J$ such that

$$p_{(K, \lambda)}(F_{(K, \lambda)} - f) < \varepsilon.$$

Hence $f \in \text{cl}(J) = J$, and so $\bigcap_{t \in E} J_{(t, I(t))} \subset J$, which completes the proof.

The set E above is not necessarily closed as the following example shows.

EXAMPLE. Let X be the real line \mathbb{R} with the usual topology and consider the open interval $(0, 1) \subset \mathbb{R}$. Let $A = C(0, 1)$ with the topology $T(\mathbb{N})$ given by the sequence of seminorms

$$p_n(x) = \sup_{t \in [1/(n+1), 1-1/(n+1)]} |x(t)|, \quad x \in A, n \in \mathbb{N},$$

denoted by $\mathcal{P}(\mathbb{N})$. For a natural number m and a seminorm $p_n \in \mathcal{P}(\mathbb{N})$ let $p_{(m, n)}$ be the seminorm on $C(X, A)$ defined by

$$p_{(m, n)}(f) = \sup_{t \in [-m, m]} p_n(f(t)), \quad f \in C(X, A).$$

Now $\{p_{(m, n)} \mid m, n \in \mathbb{N}\}$ is a directed family of seminorms on $C(X, A)$ which defines a locally m -convex topology on $C(X, A)$ denoted by $T(\mathbb{N}, \mathbb{N})$.

Let $k \in \mathbb{N}$ be given. If $t \in X$, we define

$$E_t = \begin{cases} \{s \in (0, 1) \mid -t/k \leq s < 1\} & \text{if } t < 0, \\ \{s \in (0, 1) \mid t/k \leq s < 1\} & \text{if } t \geq 0. \end{cases}$$

Obviously E_t is a closed subset of $(0, 1)$ for all $t \in \mathbb{R}$ and $E_t = \emptyset$ if $t \in X \sim (-k, k)$. If we define $k(E_t) = \{x \in A \mid x(s) = 0, s \in E_t\}$, then obviously $k(E_t)$ is a closed ideal of $(A, T(\mathbb{N}))$ for each $t \in \mathbb{R}$ and $k(E_t) = A$ if $t \in X \sim (-k, k)$.

If we choose $J = \bigcap_{t \in \mathbb{R}} J_{(t, k(E_t))}$, then J is a closed ideal of $(C(X, A), T(\mathbb{N}, \mathbb{N}))$ and clearly $J = \bigcap_{t \in (-k, k)} J_{(t, k(E_t))}$. Thus

$$E = \{t \in X \mid I(t) \text{ is a proper subset of } A\} = (-k, k),$$

whence E is an open subset of X .

Next we shall give some conditions which guarantee that the set E in Theorem 8 is closed. First we recall that $(A, T(\mathcal{P}))$ is a Q -algebra if the set of regular elements is open.

THEOREM 9. *Let J be a closed ideal of $(C(X, A), T(\mathcal{K}, A))$ and let $E = \{t \in X \mid I(t) \neq A\}$ where $I(t) = \text{cl}(\{f(t) \mid f \in J\})$. Then E is closed in each of the following cases:*

1° $(A, T(\mathcal{P}))$ is a Q -algebra.

2° $\Delta(A)$ is locally equicontinuous and $\{\tau \mid \phi_{(t,\tau)} \in h(J)\} \subset h(N_\lambda)$ for some $\lambda \in \Lambda$.

3° For each boundary point t in E there is $f \in J$ such that $f(t) \in \ker \tau$ for some $\tau \in \Delta(A)$.

Proof. 1° and 3° are obvious. We only prove case 2°. Suppose that $\Delta(A)$ is locally equicontinuous. Now $\varphi^{-1}(h(J)) = \{(t, \tau) \mid \phi_{(t,\tau)} \in h(J)\}$ is a closed subset of $X \times \Delta(A)$. If $\{\tau \in \Delta(A) \mid \phi_{(t,\tau)} \in h(J) \text{ for all } t \in X\} \subset h(N_\lambda)$ for some $\lambda \in \Lambda$, then $\varphi^{-1}(h(J)) \subset X \times h(N_\lambda)$, and so E is just the projection of the closed set $\varphi^{-1}(h(J)) \subset X \times h(N_\lambda)$ into X . By Theorem 4.1 and Corollary 2.1 of [12], $h(N_\lambda)$ is homeomorphic to $\Delta(\tilde{A}_\lambda)$. So $h(N_\lambda)$ is compact, since $\Delta(\tilde{A}_\lambda)$ is. The closedness of E now follows from Theorem 2.5 of [7].

COROLLARY 10. *If $K \in \mathcal{K}$ and $\lambda \in \Lambda$, then*

$$\{(t, \tau) \in X \times \Delta(A) \mid \phi_{(t,\tau)} \in h(N_{(K,\lambda)})\} = K \times h(N_\lambda).$$

In particular, if $\Delta(A)$ is locally equicontinuous, then $h(N_{(K,\lambda)})$ is homeomorphic to $K \times h(N_\lambda)$.

Proof. Just note that $N_{(K,\lambda)} = \bigcap_{t \in K} J_{(t,N_\lambda)}$.

We shall say that $(A, T(\mathcal{P}))$ has the *property of spectral synthesis* if $k(h(I)) = I$ for each closed ideal I of $(A, T(\mathcal{P}))$.

COROLLARY 11. *The algebra $(C(X, A), T(\mathcal{K}, A))$ has the property of spectral synthesis if and only if $(A, T(\mathcal{P}))$ has this property.*

Proof. Suppose $k(h(I)) = I$ for each closed ideal of $(A, T(\mathcal{P}))$. Now, if J is a closed ideal of $(C(X, A), T(\mathcal{K}, A))$, then by Theorem 8, we have $J = \bigcap_{t \in E} J_{(t,I(t))}$ for some $E \subset X$ and a family $\{I(t) \mid t \in E\}$ of closed ideals of $(A, T(\mathcal{P}))$. It is easy to see that $h(J) = \{\phi_{(t,\tau)} \mid t \in E, \tau \in h(I(t))\}$. Thus the following equivalences are valid:

$$\begin{aligned} f \in k(h(J)) &\Leftrightarrow \phi(f) = 0 \text{ for all } \phi \in h(J) \\ &\Leftrightarrow \phi_{(t,\tau)}(f) = 0 \text{ for all } t \in E \text{ and } \tau \in h(I(t)) \\ &\Leftrightarrow \tau(f(t)) = 0 \text{ for all } t \in E \text{ and } \tau \in h(I(t)) \\ &\Leftrightarrow f(t) \in k(h(I(t))) \text{ for all } t \in E \end{aligned}$$

$$\Leftrightarrow f(t) \in I(t) \text{ for all } t \in E \Leftrightarrow f \in \bigcap_{t \in E} J_{(t,I(t))} = J.$$

Thus $k(h(J)) = J$.

Conversely, suppose that $k(h(J)) = J$ for each closed ideal J of $(C(X, A), T(\mathcal{K}, A))$. Let I be an arbitrary closed ideal of $(A, T(\mathcal{P}))$. Now, if $x \in k(h(I))$, then $f_x \in J_{(t,k(h(I)))}$ for each $t \in X$. It is easy to see that $J_{(t,k(h(I)))} = k(h(J_{(t,I)}))$. But by the hypotheses $k(h(J_{(t,I)})) = J_{(t,I)}$. So $f_x \in J_{(t,I)}$ for any $t \in X$ and therefore $x \in I$. Thus $k(h(I)) \subset I$, whence $k(h(I)) = I$, which completes the proof.

The result of Corollary 11 is a generalization of the corresponding result of [3]. The structure of closed ideals of $C(X, A)$ with the compact-open topology has been considered for example in [15].

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Corrigendum to the paper
“On the reproducing kernel for harmonic functions and
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On page 24 we got a formula for the projector P :

$$Pu = c(n) \Delta \left[(|x|^2 - 1) \int_B \frac{(1 - |x|^2|y|^2)u(y) dV_y}{[|x - y|^2 + (1 - |x|^2)(1 - |y|^2)]^{n/2}} \right].$$

In the next step we made a mistake in calculating the above laplacian. The next formula should read

$$Pu = c(n) \int_B \left(\frac{2n(1 - |x|^2|y|^2)^2}{[|x - y|^2 + (1 - |x|^2)(1 - |y|^2)]^{n/2+1}} - \frac{8|x|^2|y|^2}{[|x - y|^2 + (1 - |x|^2)(1 - |y|^2)]^{n/2}} \right) u(y) dV_y,$$

and the formula on the top of page 25 should read

$$K(x, y) = c(n) \left(\frac{2n(1 - |x|^2|y|^2)^2}{[|x - y|^2 + (1 - |x|^2)(1 - |y|^2)]^{n/2+1}} - \frac{8|x|^2|y|^2}{[|x - y|^2 + (1 - |x|^2)(1 - |y|^2)]^{n/2}} \right).$$

The kernel $K(x, y)$ in its correct form satisfies the estimates of page 26 and therefore the other results of the paper remain valid.

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