tych kwadratur, to można ich nie wykonywać. Aby to okazać, zestawmy następujące równości:

$$C^{(3)} - C = \frac{u_{01}}{u}; \quad C' - C'^{(3)} = \frac{u_{10}}{u},$$

$$C^{(4)} - C = \frac{v_{01}}{v}; \quad C' - C'^{(4)} = \frac{v_{10}}{v},$$

$$C^{(2)} - C = \frac{\alpha u_{01} + \beta v_{01}}{\alpha u + \beta v + \gamma}; \quad C' - C'^{(2)} = \frac{\alpha u_{10} + \beta v_{10}}{\alpha u + \beta v + \gamma}.$$

Ostatnią równość otrzymaliśmy dzięki niezmiennikom (19) grupy $W^{(2)}f$. Ponieważ $C^{(2)}$ i $C'^{(2)}$ spełniają równania (25), przeto zawsze możemy podać wyrażenia na nie, w których będą figurowały dwie stałe dowolne. Znajdujemy w ten sposób układ sześciu równań liniowych względem u, v, u_{10}, u_{01}, v_{10}, v_{01}, na podstawie którego możemy wogóle za pomocą li tylko operacyj algebraicznych podać wyrażenia na u i v, zawierające ośm stałych dowolnych.

G. A. MILLER.

Groups in which the subgroup which involves all the substitutions omitting a given letter is regular.

(GRUPY, W KTÓRYCH PODGRUPA, OBEJMUJĄCA W SOBIE WSZYSTKIE PODSTAWIENIA Z WYŁĄCZENIEM DANEJ LITERY, JEST REGULARNA).

Many of the transitive substitution groups of low degrees have been determined by means of their subgroups involving all the substitutions which omit a given letter. It is customary to represent such a subgroup by G_1 when the entire group is represented by G. If G could be directly determined from G_1 we would have a method of constructing all the transitive groups of degree n whenever all those of the lower degrees are known. A number of theorems along this line have been published, especially for the case when G is a primitive group.[1]) In the present paper we consider the case when G_1 is a regular group.

If the degree of G_1 is $n-1$, n being the degree of G, it follows that G is a doubly transitive group of the smallest possible order. These groups have received considerable attention and have been extended along different lines. In particular, it has been proved that such a group cannot exist unless n is a power of a single prime and that G contains an abelian invariant subgroup of type (1, 1, 1, ...).[2]) Moreover, there is at least one

[1]) Cf. Quarterly Journal of Mathematics, vol. 28 (1896), p. 215.
[2]) Jordan, Liouville's Journal, vol. 17 (1872), p. 355.

such group for every value of n which is a power of α prime, in which G_1 is cyclic. That other groups may exist follows from the fact that when $n=9$, G_1 may be the quaternion group, as is proved by the well known five-fold transitive group of degree 12.

When the degree of G_1 is $n-\alpha$, $\alpha>1$, it is known that G contains n/α systems of imprimitivity each of degree α and that each of the substitutions of G is commutative with $\alpha-1$ regular substitutions of degree n. These $\alpha-1$ substitutions together with the identity constitute a group of degree n whose n/α systems of intransitivity are the systems of imprimitivity of G in question. These systems are transformed according to a primitive group by the substitutions of G since they cannot be combined into larger systems of imprimitivity. When this primitive group is regular it is of order α and G may be constructed by extending the direct product of two simply isomorphic regular groups, represented on distinct letters, by means of a substitution of order 2 which interchanges its systems of intransitivity.

When the primitive group in question is not of order 2 it is multiply transitivy since G_1 is regular and hence permutes the $n/\alpha-1$ systems of intransitivity of G according to a transitive group. When G_1 is abelian it must involve α substitutions which transform each of these systems of imprimitivity into itself. As this is also true of the conjugates of G_1 and as two such conjugates have only the identity in common it follows that G contains at least α^2 substitutions which transform into itself each of the systems of imprimitivity in question whenever G_1 is abelian. There cannot be more than α^2 such substitutions in G since the order of G is $(n-\alpha)n$ while that of the primitive group in question cannot be less thon $^n/_\alpha(^n/_\alpha-1)$. Hence the theorem *when G_1 is regular and of degree $n-\alpha$, $\alpha>1$, G contains $^n/_\alpha$ systems of imprimitivity which it transforms either according to the group of order 2 or according to a multiply transitive group. When G_1 also abelian G contains exactly α^2 substitutions which transform each of these systems into itself and it contains an additional invariant subgroup of index $^n/_\alpha-1$.*

The latter of these two invariant subgroups is the identity when $^n/_\alpha=2$ but it includes the former without being identical with it for all other values of this fraction. It may be observed that the existence of these two invariant subgroups depends only upon the fact that G_1 involves α substitutions which do not permute any of the systems of imprimitivity under consideration. This condition is always satisfied when G_1 is abelian but it may also be satisfied when G_1 is non-abelian. Some of the considerations above are simplified by the following theorem: *If a regular group H of degree n is transformed into itself by a substitution s of degree $n-\alpha$ in the same letters then s is commutative with exactly α of the substitutions of H whenever $\alpha>0$. When $\alpha=0$ s must be commutative with at least one of the substitutions of H besides the identity.*

A part of this theorem is included in the evident statement, if a transitive group of degree n is transformed into itself by any substitution in the same letters whose degree is not $n-1$ then this substitution must be commutative with at least one substitution of the group besides the identity.

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