

P O L S K A A K A D E M I A N A U K
MONOGRAFIE MATEMATYCZNE

WACŁAW SIERPIŃSKI

ELEMENTARY THEORY
OF NUMBERS

KOMITET REDAKCYJNY

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"The elementary theory of numbers should be one of the very best subjects for early mathematical instruction. It demands very little previous knowledge, its subject matter is tangible and familiar; the processes of reasoning which it employs are simple, general and few; and it is unique among the mathematical sciences in its appeal to natural human curiosity."

G. H. Hardy*

PREFACE

Contemporary mathematics comprises a number of branches whose traditional brief names, established for centuries, give no hint as to their actual scope and subject. This applies also to the Theory of Numbers, which, by the way, owing to its subject and methods, as well as its relation to other sciences, takes a special place among the various branches of mathematics.

The name of the Theory of Numbers might suggest that it is a kind of general theory concerning the notion of number and its generalisations which, starting from integers, introduces successively rational, real and complex numbers, and also some other kinds of numbers, and builds up a theory of operations on these numbers. This, however, is the subject of Higher Arithmetic. The subject of the Theory of Numbers is more special. It is concerned with the properties of integers, while the concept of integers and the theory of operations on them are taken ready-made from Higher Arithmetic and Algebra. However, the Theory of Numbers does not deal exclusively with integers. Many properties of integers have been discovered with the aid of irrational or complex numbers and many theorems about integers can be proved in a much simpler way if one makes use not only of irrational or complex numbers but also of the whole apparatus of the Calculus and the Theory of Functions. The part of the Theory of Numbers which makes extensive use of various parts of Analysis is called the Analytic Theory of Numbers, to be distinguished from the Elementary Theory of Numbers, which does not use the notion of limit.

* Bull. Amer. Math. Soc. 35 (1929), p. 818.

The subject of this book is the Elementary Theory of Numbers, though a number of simple applications of the Analytic Theory of Numbers are also included. The book is prepared on the basis of two of my books issued between the years 1914 and 1959. These are

Teoria Liczb (Theory of Numbers), first edition, Warszawa 1914; second edition, Warszawa 1925; enlarged third edition, Warszawa-Wrocław 1950 (544 pages),

Teoria Liczb, Part II, Warszawa 1959 (487 pages).

To illustrate the progress which the Theory of Numbers has made in the last decade, it is sufficient to recall that the greatest prime number that was known in the year 1950 was $2^{127}-1$ of 39 digits and compare it with the number $2^{9691}-1$ of 2917 digits—the greatest prime number known to-day. In 1950 only 12 perfect numbers were known; to-day we know 21 of them.

In this book I have included various particular results of the Elementary Theory of Numbers that have been found in recent years in many countries.

I would like to express my thanks to Dr A. Hulanicki, who translated the manuscript of the book into English, to Doc. Dr A. Schinzel, who prepared the bibliography and added many valuable suggestions and footnotes concerning the results obtained recently, and to Dr A. Małkowski, who helped me in reading the proofs. It is a pleasure to offer my thanks to Mrs. L. Izertowa, the editor of the book on behalf of PWN, who contributed so much to the preparation of the book so that it can be issued in the present form.

Waclaw Sierpiński

Warsaw, May 1963

CHAPTER I

DIVISIBILITY AND INDETERMINATE EQUATIONS OF FIRST DEGREE

§ 1. Divisibility. By *natural numbers* we mean the numbers 1, 2, ..., by *integers* we mean the natural numbers, the number zero and the negative numbers $-1, -2, -3, \dots$

We say that an integer a is *divisible* by an integer b if there is an integer c such that $a = bc$. We then write

$$b|a.$$

We call b a *divisor* of a and a a *multiple* of b .

We write $b \nmid a$ if b does not divide a .

Since for each integer b we have $0 = 0 \cdot b$, every integer is a divisor of zero. Since for each integer a we have $a = a \cdot 1$, we see that 1 is a divisor of every integer.

Suppose now that x, y, z are integers such that

$$(1) \quad x|y \quad \text{and} \quad y|z.$$

Then there exist integers t and u such that $y = xt$ and $z = yu$. The number $v = tu$ is an integer (as the product of two integers). Thus, since $z = xv$, we obtain $x|z$. This proves that relations (1) imply the relation $x|z$ which means that a divisor of a divisor of an integer is a divisor of that integer. We express this by saying that the relation of divisibility of integers is *transitive*. It follows that if $x|y$, then $x|ky$ for every integer k .

It is easy to prove that a divisor of each of two given integers is a divisor of their sum and their difference. Moreover, if $d|a$ and $d|b$, then, for arbitrary integers x and y , $d|ax+by$.

In fact, the relations $d|a$ and $d|b$ imply that there exist integers k and l such that $a = kd$, $b = ld$, whence $ax+by = (kx+ly)d$ and consequently, since $kx+ly$ is an integer, $d|ax+by$.

Any two of the formulae $a = bc$, $-a = b(-c)$, $a = (-b)(-c)$, $-a = (-b)c$ are equivalent. Hence also any two of the formulae

$$b|a, \quad b|-a, \quad -b|a, \quad -b|-a$$

are equivalent. Consequently while examining divisibility of integers we can restrict ourselves to the investigation of divisibility of natural numbers.

BIBLIOGRAPHY

- Alaoglu, L. and Erdős, P., [1] *On highly composite and similar numbers*, Trans. Amer. Math. Soc. 56 (1944), pp. 448-469.
[2] *A conjecture in elementary number theory*, Bull. Amer. Math. Soc. 50 (1944), pp. 881-882.
- Ankeny, N. C., [1] *Sums of three squares*, Proc. Amer. Math. Soc. 8 (1957), pp. 316-319.
- Anning, N. H. and Erdős, P., [1] *Integral distances*, Bull. Amer. Math. Soc. 51 (1945), pp. 598-600.
- Auluck, F. C., [1] *On Waring's problem for biquadrates*, Proc. Indian Acad. Sci., Sect. A., 11 (1940), pp. 437-450.
- Bachmann, P., [1] *Niedere Zahlentheorie*, Encyklopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen IC 1, pp. 555-581.
[2] *Niedere Zahlentheorie II*, Leipzig 1901.
- [3] *Das Fermatproblem in seiner bisherigen Entwicklung*, Berlin-Lepzig.
- Baker, C. L. and Grunberger, F. J., [1] *The first six million prime numbers*, Madison 1959 (Microcards).
- Bang, A. S., *Über Summen von fünften Potenzen*, Neuvième congrès des math. Scand. 1938, Helsinki 1939, pp. 292-296.
- Bang, T., [1] (Danish) *Large prime numbers*, Nordisk Mat. Tidsskr. 2 (1954), pp. 157-168.
- Beeger, N. G. W. H., [1] *On even numbers m dividing $2^n - 2$* , Amer. Math. Monthly 58 (1951), pp. 553-555.
[2] *Oullen numbers*, Math. Tables Aids to Comp. 8 (1954), p. 188.
- Behrend, F. A., [1] *On sets of integers which contain no three terms in arithmetical progression*, Proc. Nat. Acad. Sci. U. S. A. 32 (1946), pp. 331-332.
- Bell, E. T., [1] *Reciprocal arrays and diophantine analysis*, Amer. J. Math. 55 (1933), pp. 50-66.
- Bendz, T. R., [1] *Över diophantiska ekvationer $x^n + y^n = z^n$* , (Swedish) Diss. Upsala 1901.
- Bieberbach, L., [1] *Über Stiesselsche magische Quadrate I*, Arch. Math. 5 (1954), pp. 4-11.
- Birkhoff, G., [1] *Lattice Theory*, New York 1948.
- Blanuša, D., [1] *Une interprétation géométrique du crible d'Erathostène*, Glasnik Mat. Fiz. Astronom. Drustvo Mat. Fiz. Hrvatske.
- Bochner, S., [1] *Remark on the Euclidean algorithm*, J. London Math. Soc. 9 (1934), p. 4.
- Borel, E., [1] *Les probabilités dénombrables et leurs applications arithmétiques*, Rend. Circ. Mat. Palermo 27 (1909), pp. 247-271.
[2] *Sur les chiffres décimaux de $\sqrt{2}$ et divers problèmes de probabilité en chaîne*, C. R. Acad. Sci. Paris 230 (1950), pp. 591-593.
- Borozdkin, K. G., [1] *K voprosu o postoyannoi I. M. Vinogradova*, Trudy tretego vsesoiuznogo matematicheskogo siedza, vol. I, Moskva 1956, p. 3.
- Bouniakowsky, V., [1] *Notes sur quelques points de l'analyse indéterminée*, Bull. Acad. Sci. St. Pétersbourg 6 (1848), pp. 196-199.
[2] *Sur les diviseurs numériques invariables des fonctions rationnelles entières*, Acad. Sci. St. Pétersbourg Mém. (6), sci. math. et phys. 6 (1857), pp. 305-329.
- Brauer, A., [1] *Über einige spezielle diophantische Gleichungen*, Math. Z. 25 (1926), pp. 499-504.
[2] *On a property of k consecutive integers*, Bull. Amer. Math. Soc. 47 (1941), pp. 328-331.
- Brauer, A. and Reynolds, R. L., [1] *On a theorem of Aubry-Thue*, Canadian J. Math. 3 (1951), pp. 367-374.
- Bredihin, B. M., [1] *Binarnye additivnye problemy neopredelennogo tipa*, Izv. Akad. Nauk. SSSR, Ser. nat. 27 (1963), pp. 439-462, 577-612.
- Brillhart, J., [1] *On the factors of certain Mersenne numbers II*, Math. Comp. 18 (1964), pp. 87-92.
- Brillhart, J. and Johnson, G. D., [1] *On the factors of certain Mersenne numbers*, Math. Comp. 14 (1960), pp. 385-389.
- Bromhead, T., [1] *On square sums of squares*, Math. Gaz. 44 (1960), pp. 210-220.
- Browkin, J., [1] *O pewnej własności liczb trójkątnych* (Polish), Wiadom. Mat. 2 (1957-59), pp. 253-255.
[2] *Solution of certain problem of A. Schinzel*, Prace Mat. 3 (1959), pp. 205-207 (Polish).
- Browkin, J. et Schinzel, A., [1] *Sur les nombres de Mersenne qui sont triangulaires*, C. R. Acad. Sci. Paris 242 (1956), pp. 1780-1781.
- Brown, A. L., [1] *Multiperfect numbers*, Scripta Math. 20 (1954), pp. 103-106.
[2] *Multiperfect numbers—Cousins of the perfect numbers*—No. 1, Recreational Math. Mag 14 (1964), pp. 31-39.
- Brun, V., [1] *La série $\frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \frac{1}{13} + \frac{1}{17} + \frac{1}{19} + \frac{1}{29} + \frac{1}{31} + \frac{1}{41} + \frac{1}{43} + \frac{1}{59} + \frac{1}{61} + \dots$ où les dénominateurs sont "nombres premiers jumeaux"* est convergente ou finie, Bull. Sc. Math. 43 (1919), pp. 100-104, 124-128.
[2] *Le crible d'Erathostène et le théorème de Goldbach*, Vid. Selsk. Skr. I Math. Nat. Kl. Kristiania 1920, No 3.
- Buck, R. C., [1] *Prime representing functions*, Amer. Math. Monthly 53 (1946), p. 265.
- Cantor, G., [1] *Ueber die einfachen Zahlensysteme*, Zeitschr. Math. Phys. 14 (1860), pp. 121-128.
[2] *Zwei Sätze über eine gewisse Zerlegung der Zahlen in unendliche Produkte*, Zeitschr. Math. Phys. 14 (1879), p. 155.
- Carmichael, R. D., [1] *On Euler's φ function*, Bull. Amer. Math. Soc. 13 (1907), pp. 241-243 and *Errata*, ibidem 54 (1948), p. 1192.
[2] *Note of a new number theory function*, Bull. Amer. Math. Soc. 16 (1910), pp. 232-238.
[3] *On composite numbers P , which satisfy the Fermat congruence $a^{P-1} \equiv 1 \pmod{P}$* , Amer. Math. Monthly 19 (1912), pp. 22-27.
[4] *Diophantine analysis*, New York 1915.
[5] *Note on Euler's φ -function*, Bull. Amer. Math. Soc. 28 (1922), pp. 109-110.
- Cassels, J. W. S., [1] *The rational solutions of the Diophantine equation $Y^2 = X^3 - D$* , Acta Math. 82 (1950), pp. 243-273.

- [2] *The rational solutions of the Diophantine equation $Y^2 = X^3 - D$, Addenda and Corrigenda*, Acta Math. 84 (1951), pp. 299.
- [3] *On the equation $a^x - b^y = 1$, II*, Proc. Cambridge Philos. Soc. 56 (1960), pp. 97-103, Corrigendum, ibidem 57 (1961), p. 187.
- [4] *On a Diophantine equation*, Acta Arith. 6 (1960), pp. 47-52.
- Cattaneo, P., [1] *Sui numeri quasiperfetti*, Boll. Un. Mat. Ital. (3) 6 (1951), pp. 59-62.
- Chakrabarti, M. C., [1] *On the limit points of a function connected with the three-square problem*, Bull. Calcutta Math. Soc. 32 (1940), pp. 1-6.
- Champernowne, G. D., [1] *Construction of decimals normal in the scale of ten*, J. London Math. Soc. 8 (1933), pp. 254-260.
- Chen Jing-Run, [1] *The lattice-points in a circle*, Sc. Sinica 12 (1962), pp. 633-649.
- Chen Jing-jung, [1] *Waring's problem for $g(5)$* , Science Record 3 (1959), pp. 327-330.
- Chernick, J., [1] *On Fermat's simple theorem*, Bull. Amer. Math. Soc. 45 (1939), pp. 269-274.
- Chikawa, K., Iséki, K. and Kusakabe, T., [1] *On a problem by H. Steinhaus*, Acta Arith. 7 (1962), pp. 251-252.
- Chikawa, K., Iséki, K., Kusakabe, T. and Shibamura, K., [1] *Computation of cyclic parts of Steinhaus problem for power 5*, Acta Arith. 7 (1962), pp. 253-254 and Corrigendum, ibid. 8 (1963), p. 259.
- Chojnacka-Pniewska, M., [1] *Sur les congruences aux racines données*, Ann. Polon. Math. 3 (1956), pp. 9-12.
- Chowla, S., [1] *An extension of Heilbronn's class-number theorem*, Quart. J. Math. Oxford Ser. 5 (1934), pp. 304-307.
- [2] *There exists an infinity of 3-combinations of primes in A.P.*, Proc. Lahore Philos. Ser. 6, no 2 (1944), pp. 15-16.
- Chowla, S. and Briggs, W. E., [1] *On discriminants of binary quadratic forms with a single class in each genus*, Canadian J. Math. 6 (1954), pp. 463-470.
- Chowla, S., Dunton, M. and Lewis, D. J., [1] *All integral solutions of $2^n - 7 = x^2$ are given by $n = 3, 4, 5, 7, 15$* , Norske Vid. Selsk. Forh. (Trondheim) 33 (1960), Nr. 9.
- Cipolla, M., [1] *Sui numeri composti P , che verificano la congruenza di Fermat $a^{P-1} \equiv 1 \pmod{P}$* , Ann. Mat. Pura. Appl. (3) 9 (1903), pp. 139-160.
- Clement, P. A., [1] *Congruences for sets of primes*, Amer. Math. Monthly 56 (1949), pp. 23-25.
- [2] *Representation of integers in the form: a k -th power plus a prime*, Amer. Math. Monthly 56 (1949), p. 561.
- Cohen, E., [1] *Arithmetical notes V. A divisibility property of the divisor function*, Amer. J. Math. 83 (1961), pp. 693-697.
- Colombo, M., [1] *Tavole numeriche e diagrammi sulla distribuzione delle coppie di numeri primi a differenza fissa*, Ist. Lombardo Sci. Lett. Rend. Cl. Sci. Mat. Nat. (3) 24 (1959), pp. 101-103.
- de Comberousse, C., [1] *Algèbre supérieure 1*, Paris 1887.
- Copeland, A. and Erdős, P., [1] *Note on normal numbers*, Bull. Amer. Math. Soc. 52 (1946), pp. 857-860.
- van der Corput, J. G., [1] *Sur l'hypothèse de Goldbach pour presque tous les nombres pairs*, Acta Arith. 2 (1937), pp. 266-290.
- [2] *Über Summen von Primzahlen und Primzahlquadraten*, Math. Ann. 116 (1939), pp. 1-50.

- [3] *On de Polignac's conjecture*, Simon Stevin 27 (1950), pp. 99-105 (Dutch).
- Coustal, R., [1] *Calcul de $\sqrt[3]{2}$, et reflexion sur une espérance mathématique*, C. R. Acad. Sci. Paris 230 (1950), pp. 431-432.
- Cramér, H., [1] *On the order of magnitude of the difference between consecutive prime numbers*, Acta Arith. 2 (1936), pp. 396-408.
- Crocker, R., [1] *A theorem concerning prime numbers*, Math. Mag. 34 (1960/61), pp. 316, 344.
- Cunningham, A. J. C. and Woodall, H. J., [1] *Factorization of $Q = (2^{\ell} \pm q)$ and $(q^2 \pm 1)$* , Messenger Math. 47 (1917), pp. 1-38.
- Davenport, H., [1] *The Higher Arithmetic, An introduction to the theory of numbers*, London and New York 1952.
- Dénés, P., [1] *Über die Diophantische Gleichung $x^l + y^l = cz^l$* , Acta Math. 88 (1952), pp. 212-251.
- Depman, I. Ya., [1] *Zamečatelnye slavyanske výčisliteli G. Vega i Ya. F. Kulik*, Istor.-Mat. Issled. 6 (1953), pp. 593-604.
- Deshoves, A., [1] *Sur un théorème de Legendre et son application à la recherche de limites qui comprennent entre elles des nombres premiers*, Nouv. Ann. Math. 14 (1855), pp. 281-295.
- Dickson, L. E., [1] *A new extension of Dirichlet's theorem on prime numbers*, Messenger Math. 33 (1904), pp. 155-161.
- [2] *Amicable number triples*, Amer. Math. Monthly 20 (1913), pp. 84-91.
- [3] *Theorems and tables on the sum of the divisors of a number*, Quart. J. Pure Appl. Math. 44 (1913), pp. 264-296.
- [4] *Recent progress on Waring's theorem and its generalizations*, Bull. Amer. Math. Soc. 39 (1933), pp. 701-727.
- [5] *Proof of the ideal Waring theorem for exponents 7-180*, Amer. J. Math. 58 (1936), pp. 521-529.
- [6] *Solution of Waring's problem*, Amer. J. Math. 58 (1936), pp. 530-535.
- [7] *Modern elementary theory of numbers*, Chicago 1939.
- [8] *History of the theory of numbers*, 3 vols., New York 1952.
- Dirichlet, P. G. L., [1] *Sur l'équation $t^k + u^k + v^k + w^k = 4m$* , J. Math. Pures Appl. (2) 1 (1856), pp. 210-214.
- Duparc, H. J. A., [1] *On Carmichael numbers*, Simon Stevin 29 (1952), pp. 21-24.
- [2] *On Mersenne numbers and Poulet numbers*, Math. Centrum Amsterdam, Rapport ZW 1953 - 001 (1953).
- Dyer-Bennet, J., [1] *A theorem on partitions of the set of positive integers*, Amer. Math. Monthly 47 (1940), pp. 152-154.
- Editorial Note, [1] *Editorial Note*, Math. Comp. 15 (1961), p. 82.
- Erdős, P., [1] *Beweis eines Satzes von Tschebyschef*, Acta Litt. Sci. Szeged 5 (1932), pp. 194-198.
- [2] *A theorem of Sylvester and Schur*, J. London Math. Soc. 9 (1934), pp. 282-288.
- [3] *On the normal number of prime factors of $p-1$ and some related problems concerning Euler's φ -function*, Quart. J. Math. Oxford Ser. 6 (1935), pp. 205-213.
- [4] *On the sum and difference of squares of primes*, J. London Math. Soc. 12 (1937), pp. 133-136, 168-171.
- [5] *Note on products of consecutive integers*, J. London Math. Soc. 14 (1939), pp. 194-198.
- [6] *Note on products of consecutive integers II*, J. London Math. Soc. 14 (1939), pp. 245-249.
- [7] *Integral distances*, Bull. Amer. Math. Soc. 51 (1945), p. 996.

- [8] On some applications of Brun's method, Acta Univ. Szeged Sect. Sci. Math. 13 (1949), pp. 57-63.
- [9] On the converse of Fermat's theorem, Amer. Math. Monthly 56 (1949), pp. 623-624.
- [10] On a new method in elementary number theory which leads to an elementary proof of the prime number theorem, Proc. Nat. Acad. Sci. U.S.A. 35 (1949), pp. 374-384.
- [11] On integers of the form $2^k + p$ and some related problems, Summa Brasil. Math. 2 (1950), pp. 113-123.
- [12] On a Diophantine equation, J. London Math. Soc. 26 (1951), pp. 176-178.
- [13] Arithmetical properties of polynomials, J. London Math. Soc. 28 (1953), pp. 416-425.
- [14] On the product of consecutive integers III, Indagationes Math. 17 (1955), pp. 85-90.
- [15] On consecutive integers, Nieuw. Arch. Wisk. (3) 3 (1955), pp. 124-128.
- [16] On amicable numbers, Publ. Math. Debrecen 4 (1955), pp. 108-111.
- [17] On pseudoprimes and Carmichael numbers, Publ. Math. Debrecen 4 (1956), pp. 201-206.
- [18] Some remarks on Euler's φ function, Acta Arith. 4 (1958), pp. 10-19.
- [19] Solution of two problems of Jankowska, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. 6 (1958), pp. 545-547.
- [20] Some remarks on the functions φ and σ , Bull. Acad. Polon. Sci., Sér. Sci. Math. Astr. Phys. 10 (1962), pp. 617-619.
- Erdős, P. and Mirsky, L., [1] The distribution of values of the divisor function $d(n)$, Proc. London Math. Soc. (3) 2 (1952), pp. 257-271.
- Erdős, P. and Oblath, R., [1] Über diophantische Gleichungen der Form $n! = x^p \pm y^n$ und $n! \pm m! = x^p$, Acta Litt. Sci. Szeged 8 (1936), pp. 241-255.
- Erdős, P. and Rényi, A., [1] Some problems and results on consecutive primes, Simon Stevin 27 (1950), pp. 115-125.
- Erdős, P. and Schinzel, A., [1] Distributions of the values of some arithmetical functions, Acta Arith. 6 (1961), pp. 473-485.
- Erdős, P. and Turán, P., [1] On some sequences of integers, J. London Math. Soc. 11 (1936), pp. 261-264.
- [2] On some new questions on the distribution of prime numbers, Bull. Amer. Math. Soc. 54 (1948), pp. 371-378.
- Escott, E. B., [1] Réponse 1587, Intermédiaire Math. 7 (1900), p. 192.
- [2] Amicable numbers, Scripta Math. 12 (1946), pp. 61-72.
- Estermann, T., [1] Einige Sätze über quadratfreie Zahlen, Math. Ann. 105 (1931), pp. 653-662.
- [2] Note on a paper of A. Rothiewicz, Acta Arith. 8 (1963), pp. 465-467.
- Faber, G., [1] Über die Abzählbarkeit der rationalen Zahlen, Math. Ann. 60 (1905), pp. 196-203.
- Fermat, P., [1] Oeuvres, vol. II, Paris 1894.
- Ferrier, A., [1] Les nombres premiers, Paris 1947.
- Finsler, P., [1] Über die Primzahlen zwischen n und $2n$, Festschrift zum 60. Geburtstag von Prof. Dr. Andreas Speiser, pp. 118-122, Zürich 1945.
- Fluch, W., [1] Verwendung der Zeta-Funktion beim Sieb von Selberg, Acta Arith. 5 (1959), pp. 381-405.
- Franqui, B., and Garcia, M., [1] Some new multiply perfect numbers, Amer. Math. Monthly 60 (1953), pp. 459-462.
- [2] 57 new multiply perfect numbers, Scripta Math. 20 (1954), pp. 169-171.
- Fröberg, C. E., [1] Some Computations of Wilson and Fermat Remainders, Math. Tables Aids Comp. 12 (1958), p. 281.

- [2] Investigation of the Wilson remainders in the interval $3 < p < 50000$, Ark. Mat. 4 (1963), pp. 479-499.
- Frücht, K., [1] Statistische Untersuchung über die Verteilung von Primzahl-Zwillingen, Anz. Öster. Akad. Wiss. Math. Nat. Kl. (1950), pp. 226-232.
- Fueter, R., [1] Über kubische diophantische Gleichungen, Comment. Math. Helv. 2 (1930), pp. 69-89.
- Gabard, E., [1] Sur deux factorisations, Mathesis 63 (1954), pp. 117-119.
- [2] Trois factorisations inédites, Mathesis 63 (1954), p. 285.
- [3] Factorisations et équation de Pell, Mathesis 67 (1958), pp. 218-220.
- Gabowicz, A., [1] O rozwijanach równania $x^3+y^3+z^3-t^3=1$ w liczbach naturalnych, Wiadom. Mat. 7 (1963), pp. 63-64.
- Gelfond, A. O., [1] Ob odnom obščem svojstve sistem sčíslenia, Izv. Akad. Nauk SSSR, Ser. Mat. 23 (1959), pp. 809-814.
- Georgiev G., [1] On the solution in rational numbers of certain diophantine equations, Prace Mat. 1 (1955), pp. 201-238 (Polish).
- Gerono, C. G., [1] Note sur la résolution en nombres entiers et positifs de l'équation $x^m = y^n + 1$, Nouv. Ann. Math. (2) 9 (1870), pp. 469-471, 10 (1871), pp. 204-206.
- Gillies, D. B., [1] Three new Mersenne primes and the statistical theory, Math. Comp. 18 (1964), pp. 93-95.
- Ginsburg, J., [1] The generators of a Pythagorean triangle, Scripta Math. 11 (1945), p. 188.
- Giusga, G., [1] Su una presumibile proprietà caratteristica dei numeri primi, Ist. Lombardo Sci. Lett. Rend. Cl. Sci. Mat. Nat. (3) 14 (1950), pp. 511-528.
- Glisher, J. W. C., [1] Number divisor tables, Cambridge 1940.
- Godwin, H. J., [1] A note on $x^3+y^3+z^3=1$, J. London Math. Soc. 32 (1957), pp. 501-503.
- Golomb, S., [1] Sets of primes with intermediate density, Math. Scand. 3 (1955), pp. 264-274.
- Golubew, W. A., [1] Abzählung von "Vierlingen" von 2000000 bis 3000000 und von "Fünflingen" von 0 bis 2000000, Anz. Österr. Akad. Wiss. Math. Nat. Kl. 93 (1956), pp. 153-157.
- [2] Abzählung von "Vierlingen" und "Fünflingen" bis zu 5000000 und von Sechslingen von 0 bis 14000000, Anz. Österr. Akad. Wiss. Math. Nat. Kl. 94 (1957), pp. 82-87.
- [3] Abzählung von "Vierlingen" und "Fünflingen" bis zu 10000000, Einige Formeln, Anz. Österr. Akad. Wiss. Math. Nat. Kl. 94 (1957), pp. 274-280.
- [4] Abzählung von "Vierlingen" und "Fünflingen" bis zu 15000000, Anz. Österr. Akad. Wiss. Math. Nat. Kl. 96 (1959), pp. 227-232.
- [5] Primezahlen der Form x^3+1 , Anz. Österr. Akad. Wiss. Math. Nat. Kl. 95 (1958), pp. 9-13; 96 (1959), pp. 126-129; 97 (1960), pp. 312-323; 98 (1961), pp. 59-63.
- Goodstein, E., [1] A note on magic squares, Math. Gaz. 24 (1940), p. 117.
- Gradstein, I. S., [1] O nečetnych soversennych čislach, Mat. Sb. 32 (1925), pp. 476-510.
- Grosswald, E., Calloway, A., and Calloway, J., [1] The representation of integers by three positive squares, Proc. Amer. Math. Soc. 10 (1959), pp. 451-455.
- Grunbe, F., [1] Ueber Einige Euler'sche Sätze aus des Theorie der quadratischen Formen, Zeitschr. Math. Phys. 90 (1874), pp. 492-519.
- Gupta, H., [1] Congruence properties of $\sigma(n)$, Math. Student 13 (1945), pp. 25-29.
- [2] A table of values of $N_2(t)$, Res. Bull. East Punjab Univ. 1952 no. 20, pp. 13-93.
- Hadwiger, H., [1] Ungelöste Probleme Nr 24, Elem. Math. 13 (1958), p. 85.

- Hall, M., Jr., [1] *On the sum and product of continued fractions*, Ann. of Math. (2) 48 (1947), pp. 966-993.
- Hausner, M. and Sachs, D., [1] *On the congruence $2^n \equiv 2(p^2)$* , Amer. Math. Monthly 70 (1963), p. 996.
- Hanly, V. S., [1] *A proposition equivalent to Dirichlet's theorem*, Amer. Math. Monthly 64 (1957), p. 742.
- Hardy, G. H. and Littlewood, E. J., [1] *Some problems of "Partitio numerorum" III*, Acta Math. 44 (1923), pp. 1-70.
- Hardy, G. H. and Wright, E. M., [1] *An introduction to the theory of numbers*, Oxford 1954.
- Harris, O. C., [1] *A modification of the sieve of Eratosthenes*, Amer. Math. Monthly 60 (1953), pp. 325-326.
- Hasse, H., [1] *Vorlesungen über Zahlentheorie*, Berlin-Göttingen-Heidelberg 1950.
- [2] *Cyclic projective planes*, Duke Math. J. 4 (1947), pp. 1079-1090.
- Hausdorff, F., [1] *Grundzüge der Mengenlehre*, Leipzig 1914.
- Haussner, R., [1] *Über die Verteilung von Lücken und Primzahlen*, J. Reine Angew. Math. 168 (1932), p. 192.
- Hecke, E., [1] *Eine neue Art von Zetafunktionen und ihre Berechnungen zur Verteilung der Primzahlen*, Math. Z. 1 (1918), pp. 299-318 and ibidem 6 (1920), pp. 11-51.
- Heilbronn, H. and Linfoot, E. H., [1] *On the imaginary quadratic corpora of class-number one*, Quart. J. Math. Oxford Ser. 5 (1934), pp. 150-160, 293-301.
- Hemer, O., [1] *On the Diophantine equation $y^2 - k = x^3$* , Diss., Upsala 1952.
- [2] *Note on the Diophantine equation $y^2 - k = x^3$* , Ark. Mat. 3 (1954), pp. 67-77.
- Hill, J. D., [1] *Solution of the problem 8449*, Amer. Math. Monthly 58 (1951), pp. 298-299.
- Hurwitz, A., [1] *Sur la décomposition des nombres en cinq carrés*, C. R. Acad. Sci. Paris 98 (1884), pp. 504-507.
- [2] *Über eine besondere Art der Kettenbruch-Entwicklung reeller Größen*, Acta Math. 12 (1889), pp. 367-405.
- Hornfeck, B. und Wirsing, E., [1] *Über die Häufigkeit vollkommener Zahlen*, Math. Ann. 133 (1957), pp. 431-438.
- Iséki, K., [1] *A problem of number theory*, Proc. Japan Acad. 36 (1960), pp. 578-583.
- [2] *Necessary results for computation of cyclic parts in Steinhaus problem*, Proc. Japan Acad. 36 (1960), pp. 650-651.
- Jacobi, C., [1] *De compositione numerorum e quatuor quadratis*, J. Reine Angew. Math. 12 (1834), pp. 167-172.
- Jakóbczyk, F., [1] *Les applications de la fonction $\lambda_g(n)$ à l'étude des fractions périodiques et de la congruence chinoise $2^n - 2 \equiv 0 \pmod{n}$* , Ann. Univ. Mariae Curie-Skłodowska, Sect. A, 5 (1951), pp. 97-138.
- Jankowska, S., [1] *Les solutions du système d'équations $\varphi(x) = \varphi(y)$ et $\sigma(x) = \sigma(y)$ pour $x < y < 10000$* , Bull. Acad. Polon. Sci. Sc. Math. Astr. Phys. 6 (1958), pp. 541-543.
- Jesmanowicz, L., [1] *Kilka uwagi o liczbach pitagorejskich*, Wiadom. Mat. 1 (1956), pp. 196-202.
- de Joncourt, E., [1] *De Natura et Praeclaro Usu Simplicissimae Speciei Numerorum Trigonali*, Hagae 1762.
- Jordan, C., [1] *Traité des substitutions*, Paris 1870.
- Józefiak, T., [1] *Ciekawostka o liczbach trójkątowych*, Matematyka 13 (1960), p. 327.
- [2] *On a hypothesis of L. Jesmanowicz concerning Pythagorean numbers*, Prace Mat. 5 (1961), pp. 119-123.

- Kanold, H. J., [1] *Untere Schranken für teilerfremde befreundete Zahlen*, Arch. Math. 4 (1953), pp. 399-401.
- [2] *Über Zahlentheoretische Funktionen*, J. Reine Angew. Math. 195 (1955), pp. 180-181.
- [3] *Über mehrfach volkommene Zahlen II*, J. Reine Angew. Math. 197 (1957), pp. 82-96.
- Kapferer, H., [1] *Über die diophantischen Gleichungen $Z^3 - Y^2 = 3^3 2^2 X^{1+2}$ und deren Abhängigkeit von der Fermatschen Vermutung*, S. B. Heidelberg Akad. Wiss. Math. Nat. Kl. 1933, pp. 32-37.
- Khatri, M. N., [1] *Triangular numbers and Pythagorean triangles*, Scripta Math. 21 (1955), p. 94.
- Khinchin, A. Ya., [1] *Three pearls of number theory*, Rochester 1952.
- Killgrave, R. B. and Ralston, K. E., [1] *On a conjecture concerning primes*, Math. Tables Aids Comp. 13 (1959), pp. 121-122.
- Klee, V. L. Jr., [1] *On the equation $\varphi(x) = 2m$* , Amer. Math. Monthly 53 (1946), pp. 327-328.
- [2] *Some remarks on Euler's totient*, Amer. Math. Monthly 54 (1947), p. 332.
- [3] *On a conjecture of Carmichael*, Bull. Amer. Math. Soc. 53 (1947), pp. 1183-1186.
- [4] *A generalization of Euler's φ function*, Amer. Math. Monthly 55 (1948), pp. 358-359.
- Knödel, W., [1] *Carmichaelsche Zahlen*, Math. Nachr. 9 (1953), pp. 343-350.
- Ko Chao, [1] *Decompositions into four cubes*, J. London Math. Soc. 11 (1936), pp. 218-219.
- [2] *Note on the Diophantine equation $x^a y^b = z^c$* , J. Chinese Math. Soc. 2 (1940), pp. 205-207.
- [3] *Zametka o pitagorowych cięślach*, Acta Sc. Nat. Univ. Szechuan, 1958, pp. 73-80 (Chinese).
- [4] *On a conjecture of Jesmanowicz*, Acta Sc. Nat. Univ. Szechuan, 1958, pp. 81-90 (Chinese).
- [5] *On a Diophantine equation $(a^2 - b^2)^x + (2ab)^y = (a^2 + b^2)^z$* , Acta Sc. Nat. Univ. Szechuan, 1959, pp. 25-34 (Chinese).
- Kraftchik, M., [1] *Théorie des nombres II*, Paris 1926.
- [2] *Recherches sur la Théorie des Nombres II, Factorisation*, Paris 1929.
- [3] *Théorie des nombres III, Analyse diophantienne et applications aux cuboïdes rationnels*, Paris 1947.
- [4] *Introduction à la Théorie des Nombres*, Paris 1952.
- Krishnamani, A. A., [1] *On isoperimetric Pythagorean triangles*, Tôhoku Math. J. 27 (1926), pp. 332-348.
- Kubilyus, I., [1] *O rozloženii prostych cisel na dva kvadrata*, Dokl. Akad. Nauk. SSSR (NS) 77 (1961), pp. 791-794.
- Kuhn, P., [1] *Neue Abschätzungen auf Grund der Viggo Brunschen Siebmethode*, Toltfe Skandinaviska Matematikerkongressen, Lund 1953, pp. 160-168.
- Kühnel, U., [1] *Verschärfung der notwendigen Bedingungen für die Existenz von ungeraden vollkommenen Zahlen*, Math. Z. 52 (1949), pp. 202-211.
- Kulik, J. Ph., Poletti, L. et Porter, R. J., [1] *Liste des nombres premiers du onzième million (plus précisément de 100000741 à 10999997) d'après des tables manuscrites*, Amsterdam 1951.
- Kulikowski, T., [1] *Sur l'existence d'une sphère passant par un nombre donné de points aux coordonnées entières*, Enseignement Math. (2) 5 (1959), pp. 89-90.
- Lagrange, J. L., [1] *Oeuvres*, vol. II, Paris 1868.

Lamé, G., [1] *Note sur la limite du nombre des divisions dans la recherche du plus grand commun diviseur entre deux nombres entiers*, C. R. Acad. Sci. Paris 19 (1844), pp. 867-870.

Landau, E., [1] *Ueber die Einteilung der positiven ganzen Zahlen in vier Klassen nach der Mindestzahl der zu ihrer additiven Zusammensetzung erforderlichen Quadrate*, Arch. Math. Phys. (3) 13 (1908), pp. 305-312.

[2] *Vorlesungen über Zahlentheorie I*, Leipzig 1927.

[3] *Handbuch der Lehre von der Verteilung der Primzahlen*, 2 vols., 2d ed. with an appendix by P. T. Bateman, New York 1953.

Lebesgue, H., [1] *Sur certaines démonstrations d'existence*, Bull. Soc. Math. France 45 (1917), pp. 132-144.

Lebesgue, V. A., [1] *Sur l'impossibilité en nombres entiers de l'équation $x^m = y^2 + 1$* , Nouv. Ann. Math. 9 (1850), pp. 178-181.

[2] *Note sur quelques équations indéterminées*, Nouv. Ann. Math. (2) 8 (1869), pp. 452-456, 559.

Leech, J., [1] *Some solutions of Diophantine equations*, Proc. Cambridge Philos. Soc. 53 (1957), pp. 778-780.

[2] *On $A^4 + B^4 + C^4 + D^4 = E^4$* , Proc. Cambridge Philos. Soc. 54 (1958), pp. 554-555.

[3] *Note on the distribution of prime numbers*, J. London Math. Soc. 32 (1957), pp. 56-58.

Legendre, A. M., [1] *Essai sur la théorie des nombres*, Paris 1798.

Lehmer, D. H., [1] *A further note on the converse of Fermat's theorem*, Bull. Amer. Math. Soc. 34 (1928), pp. 54-56.

[2] *On Euler's totient function*, Bull. Amer. Math. Soc. 38 (1932), pp. 745-757.

[3] *On imaginary quadratic fields whose class-number is unity*, Bull. Amer. Math. Soc. 39 (1933), pp. 360.

[4] *On Lucas's test for the primality of Mersenne's numbers*, J. London Math. Soc. 10 (1935), pp. 162-165.

[5] *On the converse on Fermat's theorem*, Amer. Math. Monthly 43 (1936), pp. 347-354.

[6] *On the partition of numbers into squares*, Amer. Math. Monthly 55 (1948), pp. 476-481.

[7] *On a conjecture of Krishnaswami*, Bull. Amer. Math. Soc. 54 (1948), pp. 1185-1190.

[8] *On the converse of Fermat's theorem II*, Amer. Math. Monthly 56 (1949), pp. 300-309.

[9] *On the Diophantine equation $x^3 + y^3 + z^3 = 1$* , J. London Math. Soc. 31 (1956), pp. 275-282.

[10] *Tables concerning the distribution of primes up to 37 millions*, 1957, mimeographed. Deposited in the UMT File, cf. Math. Tables Aids Comp. 13 (1959), pp. 56-57.

[11] *On the exact number of primes less than a given limit*, Illinois J. Math. 3 (1959), pp. 381-388.

Lehmer, D. H. and Lehmer, E., [1] *Density of primes having various specified properties*, Communication to Section II B of the Inter. Congress of Math. (Edinburgh 1958).

Lehmer, D. N., [1] *Factor table for the first ten millions containing the smallest factor of every number not divisible by 2, 3, 5 or 7 between the limits 0 and 10017000*, New York 1956.

Lerch, M., [1] *Zur Theorie der Fermatschen Quotienten* $\frac{a^{p-1}-1}{p} = q(a)$, Math. Ann. 60 (1905), pp. 471-490.

Leszczyński, B., [1] *O równaniu $n^x + (n+1)^y = (n+2)^z$* , Wiadom. Mat. 3 (1959-60), pp. 37-39 (Polish).

LeVeque, W. J., [1] *The distribution of multiplicative functions*, Michigan Math. J. 2 (1953-54), pp. 179-192.

[2] *Topics in number theory*, 2 vols., Reading 1956.

Lietzmann, W., [1] *Lustiges und merkwürdiges von Zahlen und Formen*, Göttingen 1930.

Lind, C. E., [1] *Untersuchungen über die rationalen Punkte der ebenen kubischen Kurven von Geschlecht Eins*, Diss., Uppsala 1940.

Lindenbaum, A., [1] *Sur les ensembles dans lesquels toutes les équations d'une famille donnée ont un nombre de solutions fixé d'avance*, Fund. Math. 20 (1933), pp. 1-29.

Linnik, Yu. V., [1] *On the representation of large numbers as sums of seven cubes*, Mat. Sb. N. S. 12 (1943), pp. 220-224.

[2] *Elementarnoe rešenie problemy Waringa po metodu Šnirelmana*, Mat. Sb. N. S. 12 (1943), pp. 225-230.

Liouville, J., [1] *Sur l'équation $1 \cdot 2 \cdot 3 \cdots (p-1) + 1 = p^m$* , J. Math. Pures Appl. (2) 1 (1856), pp. 351-352.

Ljunggren, W., [1] *Zur Theorie der Gleichung $x^2 + 1 = Dy^4$* , Avh. Norske Vid. Akad. Oslo I, 1942, no 5.

[2] *Über einige Arcustangengleichungen die auf interessante unbestimmte Gleichungen führen*, Ark. Mat. Astr. Fys. 39A no 13 (1943).

[3] *On the Diophantine equation $x^2 + p^2 = y^n$* , Norske Vid. Selsk. Forh. (Trondheim) 16 (1943), pp. 27-30.

[4] *Solution complète de quelques équations du sixième degré à deux indéterminées*, Arch. Math. Naturvid. 48 (1946), pp. 177-212.

[5] *New solution of a problem proposed by E. Lucas*, Norsk. Mat. Tidsskr. 34 (1952), pp. 65-72.

[6] *On the Diophantine equation $y^2 - k = x^3$* , Acta Arith. 8 (1963) pp. 451-463.

Locher-Ernst, L., [1] *Bemerkungen über die Verteilung der Primzahlen*, Elem. Math. 14 (1959), pp. 1-5.

Lochs, G., [1] *Die ersten 968 Kettenbrüchner von π* , Monatsh. Math. 67 (1963), pp. 311-316.

Lucas, E., [1] *Question 1180*, Nouv. Ann. Math. (2) 14 (1875), p. 336.

[2] *Théorie des nombres*, vol. I, Paris 1891.

Lu Wen-Twan, [1] *On Pythagorean numbers $4n^2 - 1, 4n, 4n^2 + 1$* , Acta Sci. Nat. Univ. Szechuan 1959, pp. 39-42 (Chinese).

Mahler, K., [1] *On the fractional parts of the powers of a rational number (II)*, Mathematika 4 (1957), pp. 122-124.

Mason, Th. E., [1] *On amicable numbers and their generalizations*, Amer. Math. Monthly 28 (1921), pp. 195-200.

Makowski, A., [1] *Sur quelques problèmes concernant les sommes de quatre cubes*, Acta Arith. 5 (1959), pp. 121-123.

[2] *Remark on a paper of Erdős and Turán*, J. London Math. Soc. 34 (1959), p. 480.

[3] *O pewnej funkcji liczbowej*, Matematyka 10 (1959), pp. 145-157.

[4] *On some equations involving functions $\varphi(n)$ and $\sigma(n)$* , Amer. Math. Monthly 67 (1960), pp. 668-670; Correction, ibidem 68 (1961), p. 650.

[5] *Remarques sur les fonctions $\theta(n), \varphi(n)$ et $\sigma(n)$* , Mathesis 69 (1960), pp. 302-303.

[6] *Partitions into unequal primes*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. 8 (1960), pp. 125-126.

- [7] Three consecutive integers cannot be powers, Colloq. Math. 9 (1962), p. 297.
- [8] Generalization of Morrow's D numbers, Simon Stevin 36 (1962), pp. 71.
- Melnikov, I. G., [1] Otkrytie Eilerom udobnykh čisel, Ist. Mat. Issled. 13 (1960), pp. 187-216.
- Meyl, A., [1] Question 1194, Nouv. Ann. Math. (2) 17 (1878), pp. 464-467.
- Miller, J. C. P. and Woollett, M. F. C., [1] Solutions of the Diophantine equation $x^3+y^3+z^3=k$, J. London Math. Soc. 30 (1955), pp. 101-110.
- Moessner, A., [1] A magic square of triangular numbers, Math. Student 10 (1942-43), p. 95.
- [2] Magic squares, Math. Student 19 (1951), pp. 124-126.
- [3] All-prime magic squares, Scripta Math. 18 (1953), p. 303.
- Mordell, L. J., [1] The Diophantine equation $y^2-k=x^3$, Proc. London Math. Soc. (2) 13 (1913), pp. 60-80.
- [2] Note on the integer solutions of the equation $Ey^2 = Ax^3+Bx^2+Cx+D$, Messenger of Math. 51 (1922), pp. 169-171.
- [3] On the four integer cubes problem, J. London Math. Soc. 11 (1936), pp. 208-218, Addendum, ibidem 12 (1937), p. 80, and Corrigendum, ibidem 32 (1957), p. 383.
- [4] On sums of three cubes, J. London Math. Soc. 17 (1942), pp. 139-144.
- [5] On the integer solutions of the equation $x^2+y^2+z^2+2xyz=n$, J. London Math. Soc. 28 (1953), pp. 500-510 and Corrigendum, ibidem 32 (1957), p. 383.
- [6] On intervals containing an affinely equivalent set of n integers mod k , Proc. Amer. Math. Soc. 5 (1954), pp. 854-859.
- [7] On the representation of a number as a sum of three squares, Rev. Math. Pures Appl. 3 (1958), pp. 25-27.
- [8] The Diophantine equation $2^n = x^2 + 7$, Ark. Mat. 4 (1962), pp. 455-460.
- Moret-Blanc, [1] Question 1175, Nouv. Ann. Math. (2) 15 (1876), pp. 44-46.
- Morrow, D. C., [1] Some properties of D numbers, Amer. Math. Monthly 58 (1951), pp. 324-330.
- Moser, L., [1] Some equations involving Euler's totient function, Amer. Math. Monthly 56 (1949), pp. 22-23.
- [2] On the Diophantine equation $1^n+2^n+\dots+(m-1)^n=m^n$, Scripta Math. 19 (1953), pp. 84-88.
- [3] On non-averaging sets of integers, Canadian J. Math. 5 (1953), pp. 245-252.
- [4] On the theorems of Wilson and Fermat, Scripta Math. 22 (1957), p. 288.
- Moszyński, K. et Swianiewicz, J., [1] Sur une équation cubique, Mathesis 69 (1960), p. 218.
- Müller, M., [1] Über die Approximation reeller Zahlen durch die Näherungsbrüche ihres regelmässigen Kettenbruches, Arch. Math. 6 (1955), pt. 253-258.
- Nagell, T., [1] Zur Arithmetik der Polynome, Abh. Math. Sem. Univ. Hamburg 1 (1922), pp. 184-188.
- [2] Sur l'impossibilité de quelques équations à deux indéterminées, Norsk Mat. Forenings Skrifter I Nr 13 (1923).
- [3] Einige Gleichungen von der Form $ay^2+by+c=dx^3$, Norske Vid. Akad. Skrifter, Oslo I, 1930, no. 7.
- [4] Løste oppgaver, Norsk Mat. Tidsskr. 30 (1948), pp. 62-64.
- [5] Introduction to the number theory, New York and Stockholm 1951.
- [6] Sur un théorème d'Axel Thue, Ark. Mat. 3 (1951), pp. 489-498.
- [7] On a special class of Diophantine equations of the second degree, Ark. Mat. 3 (1954), pp. 51-65.
- [8] Verallgemeinerung eines Fermatschen Satzes, Arch. Math. 5 (1954), pp. 153-159.

- [9] Contributions to the theory of a category of Diophantine equations of the second degree with two unknowns, Nova Acta Soc. Sci. Upsal. (4) 16 (1954), No 2.
- [10] On the Diophantine equation $x^k+8D=y^n$, Ark. Mat. 3 (1955), pp. 103-112.
- [11] The Diophantine equation $x^2+7=2^n$, Ark. Mat. 4 (1961), pp. 182-185.
- Norton, K., [1] Remarks on the number of factors of an odd perfect number, Acta Arith. 6 (1961), pp. 365-374.
- Obláth, R., [1] Une propriété des puissances parfaites, Mathesis 65 (1956), pp. 356-364.
- O'Keefe, E. S., [1] Verification of a conjecture of Th. Skolem, Math. Scand. 9 (1961), pp. 80-82.
- Ore, O., [1] Theory of equivalence relations, Duke Mathematical Journal. 9 (1942), pp. 573-627.
- [2] Number theory and its history, New York 1948.
- Palama, G., [1] Il problema di Waring, Boll. Un. Mat. Ital. (3) 12 (1957), pp. 83-100.
- Pall, G., [1] On sums of squares, Amer. Math. Monthly 40 (1933), pp. 10-18.
- Pan Cheng-Tun, [1] On the least prime in an arithmetical progression, Sci. Record N. S. 1 (1957), pp. 311-313.
- Patz, W., [1] Tafel der regelmässigen Kettenbrüche und ihrer vollständigen Quotienten für die Quadratwurzeln aus den natürlichen Zahlen von 1-10000, Berlin 1955.
- Pawlak, Z. and Wakulicz, A., [1] Use of expansions with a negative basis in the arithmometer of a digital computer, Bull. Acad. Polon. Sci., Cl. III, 5 (1957), pp. 233-236.
- Peano, G., [1] Formulaire de Mathématique, Torino 1901.
- Pépin, T., [1] Sur certains nombres complexes de la forme $a+b\sqrt{-c}$, J. Math. Pures Appl. (3) 1 (1875), pp. 317-372.
- Perron, O., [1] Die Lehre von den Kettenbrüchen I, Stuttgart 1954.
- Picard, S., [1] Sur les ensembles de distances des ensembles de points d'un espace euclidien, Paris 1939.
- Pillai, S. S., [1] On some empirical theorem of Scherk, J. Indian Math. Soc. 17 (1927-28), pp. 164-171.
- [2] On some functions connected with $\varphi(n)$, Bull. Amer. Math. Soc. 35 (1929), pp. 832-836.
- [3] On Waring's problem II, J. Indian Math. Soc. (N. S.) 2 (1936), pp. 16-44.
- [4] On m consecutive integers III, Proc. Indian Acad. Sci., Sect A 12 (1940), pp. 6-12.
- [5] On m consecutive integers II, Proc. Indian Acad. Sci., Sect A 11 (1940), pp. 73-80.
- [6] On m consecutive integers IV, Bull. Calcutta Math. Soc. 36 (1944), pp. 99-101.
- [7] On the smallest primitive root of a prime, J. Indian Math. Soc. (N. S.) 8 (1944), pp. 14-17.
- [8] On the equation $2^x-3^y=2^x+3^y$, Bull. Calcutta Math. Soc. 37 (1945), pp. 15-20.
- Pipping, N., [1] Neue Tafeln für das Goldbachsche Gesetz nebst Berichtigungen zu den Haussnerschen Tafeln, Comment. Phys. Math. 4 (1927-29), no. 4.
- [2] Über Goldbachsche Spaltungen grosser Zahlen, Comment Phys. Math. 4 (1927-29), no. 10.
- [3] Die Goldbachsche Vermutung und der Goldbach-Vinogradovsche Satz, Acta Acad. Aboens. 11 no 4 (1938).

- [4] Goldbachsche Spaltung der geraden Zahlen für $x = 60000 \cdot 99998$, Acta Acad. Aboens. 12 no 11 (1940).
- Pocklington, H. C., [1] Some diophantine impossibilities, Proc. Cambridge Philos. Soc. 17 (1914), pp. 108-121.
- Podsypanin, V. D., [1] Ob odnom svoistve čieel Pisagora, Izv. Vyss. Učebn. Zaved. Matematika 1962, no 4 (29), pp. 130-133.
- van der Pol, B., and Speziali, P., [1] The primes in $k(\varrho)$, Indagationes Math. 13 (1951), pp. 9-15.
- de Polignac, A., [1] Six propositions arithmologiques déduites du crible d'Eratostène, Nouv. Ann. Math. 8 (1849), pp. 423-429.
- Pollock, F., [1] On the extension of the principle of Fermat's theorem of the polygonal numbers to the higher orders of series whose ultimate differences are constant. With a new theorem proposed, applicable to all the orders, Proc. Roy. Soc. London 5 (1851), pp. 922-924.
- Pólya G. and Szegő G., [1] Aufgaben und Lehrsätze aus der Analysis, Bd II, Berlin 1925.
- Porges, A., [1] A set of eight numbers, Amer. Math. Monthly 52 (1945), pp. 379-382.
- Postnikov, M. M., [1] Magičeskie kvadraty, Moskva 1964.
- Poulet, P., [1] La chasse aux nombres, Fasc. 1, Bruxelles 1929.
- [2] Table de nombres composés vérifiant le théorème de Fermat pour le module 2 jusqu'à 100000000, Sphinx 8 (1938), pp. 42-52.
- [3] 43 new couples of amicable numbers, Scripta Math. 14 (1948), p. 77.
- [4] Suites de totalités en départ de $n < 2000$, Hectographed copy in possession of D. H. Lehmer, cf. Math. Tables Aids Comp. 3 (1948), p. 120.
- Prachar, K., [1] Primzahlverteilung, Berlin-Göttingen-Heidelberg 1957.
- Rado, R., [1] Some solved and unsolved problems in the theory of numbers, Math. Gaz. 25 (1941), pp. 72-77.
- Ramanujan, S., [1] Problem 465, J. Indian Math. Soc. 5 (1913), p. 120.
- [2] On the expression of a number in the form $ax^2 + by^2 + cz^2 + du^2$, Proc. Cambridge Philos. Soc. 19 (1917), pp. 11-21.
- Rankin, R. A., [1] On the representation of a number as the sum of any number of squares, and in particular of twenty, Acta Arith. 7 (1962), pp. 399-407.
- Reitwiesner, G. W., [1] An ENIAC determination of π and e to more than 2000 decimal places, Math. Tables Aids Comp. 4 (1950), pp. 11-15.
- Ricci, G., [1] Sull'andamento della differenza di numeri primi consecutivi, Riv. Mat. Univ. Parma 5 (1954), pp. 3-54.
- [2] Sull'insieme dei valori di condensazione del rapporto $(p_{n+1} - p_n)/\ln p_n$ ($n = 1, 2, 3, \dots$), Riv. Mat. Univ. Parma 6 (1955), pp. 353-361.
- Richert, H. E., [1] Über Zerlegungen in paarweise verschiedene Zahlen, Norsk Mat. Tidsskr. 31 (1949), pp. 120-122.
- [2] Über Zerfällungen in ungleiche Primzahlen, Math. Z. 52 (1950), pp. 342-343.
- Riesel, H., [1] A new Mersenne prime, Math. Tables Aids Comp. 12 (1958), p. 60.
- [2] Note on the congruence $a^{p-1} \equiv 1 \pmod{p^2}$, Math. Comp. 18 (1962), pp. 149-150.
- Robinson, R. M., [1] Mersenne and Fermat numbers, Proc. Amer. Math. Soc. 5 (1954), pp. 842-846.
- [2] A report on primes of the form $k \cdot 2^n + 1$ and on factors of Fermat numbers, Proc. Amer. Math. Soc. 9 (1958), pp. 673-681.

- de Rocquigny, G., [1] Question 1408, Intermédiaire Math. 5 (1898), p. 268.
- Rosser, J. B., [1] The n -th prime is greater than $n \log n$, Proc. London Math. Soc. (2) 45 (1939), pp. 21-44.
- Rosser, J. B., and Schoenfeld, L., [1] Approximate formulas for some functions of prime numbers, Illinois J. Math. 6 (1962), pp. 64-89.
- Roth, K. F., [1] On certain sets of integers, J. London Math. Soc. 28 (1953), pp. 104-109.
- Rotkiewicz, A., [1] Sur les nombres composés n qui divisent $a^{n-1} - b^{n-1}$, Rend. Circ. Mat. Palermo (2) 8 (1959), pp. 115-116.
- [2] Sur les nombres pairs n pour lesquels les nombres $a^n b - ab^n$, respectivement $a^{n-1} - b^{n-1}$, sont divisibles par n , Rend. Circ. Mat. Palermo (2) 9 (1959), pp. 341-342.
- [3] Sur les nombres pairs n qui divisent $(a+2)^{n-1} - a^{n-1}$, Rend. Circ. Mat. Palermo (2) 9 (1960), pp. 78-80.
- [4] On the properties of the expression $a^n - b^n$, Prace Mat. 6 (1961), pp. 1-20 (Polish).
- [5] Démonstration arithmétique d'existence d'une infinité de nombres premiers de la forme $nk + 1$, Enseignement Math. (2) 7 (1962), pp. 277-280.
- [6] Sur les nombres pseudopremiers de la forme $ax+b$, C. R. Acad. Sci. Paris 257 (1963), pp. 2601-2604.
- Salem, R. and Spencer, D. C., [1] On sets of integers which contain no three terms in arithmetical progression, Proc. Nat. Acad. U. S. A., 28 (1942), pp. 561-563.
- [2] On sets of integers which do not contain a given number of terms in arithmetical progression, Nieuw. Arch. Wisk. 23 (1952), pp. 133-143.
- Salzer, H., [1] On numbers expressible as the sum of four tetrahedral numbers, J. London Math. Soc. 20 (1945), pp. 3-4.
- Salzer, H. and Levine, N. J., [1] Tables of integers not exceeding 10000000 that are not expressible as the sum of four tetrahedral numbers, Math. Tables Aids Comp. 12 (1958), pp. 141-144.
- Sansone, G. and Cassels, J. W. S., [1] Sur le problème de M. Werner Münich, Acta Arith. 7 (1962), pp. 187-190.
- Sardi, S., [1] Sulle somme dei divisori dei numeri, Giorn. Mat. Battaglini 7 (1869), pp. 112-116.
- Sathe, L. G., [1] On a problem of Hardy on the distribution of integers having a given number of prime factors, J. Indian Math. Soc. N. S. 17 (1953), pp. 63-141, 18 (1954), pp. 27-81.
- Schäffer, J. J., [1] The equation $1^p + 2^p + 3^p + \dots + n^p = m^q$, Acta Math. 95 (1956), pp. 155-189.
- Scherk, H. F., [1] Bemerkungen über die Bildung der Primzahlen aus einander, J. Reine Angew. Math. 10 (1833), pp. 201-208.
- Schinzel, A., [1] Sur la décomposition des nombres naturels en somme de nombres triangulaires distincts, Bull. Acad. Polon. Sci. Cl. III, 2 (1954), pp. 409-410.
- [2] Sur une propriété du nombre de diviseurs, Publ. Math. Debrecen 2 (1954), pp. 261-262.
- [3] Generalization of a theorem of B. S. K. R. Somayajulu on the Euler's function $\varphi(n)$, Ganita 5 (1954), pp. 123-128.
- [4] On the equation $x_1 x_2 \dots x_n = t^k$, Bull. Acad. Polon. Sci. Cl. III, 3 (1955), pp. 17-19.
- [5] Sur un problème concernant la fonction φ , Czechoslovak Math. J. 6 (1956), pp. 164-165.
- [6] Sur l'équation $\varphi(x) = m$, Elem. Math. 11 (1956), pp. 75-78.
- [7] Sur les diviseurs naturels des polynômes, Matematiche (Catania) 12 (1957), pp. 18-22.

- [8] Sur l'existence d'un cercle passant par un nombre donné de points aux coordonnées entières, *Enseignement Math.* (2) 4 (1958), pp. 71-72.
- [9] Sur l'équation diophantienne $x^ay^b = z^c$, *Acta Sc. Nat. Univ. Szechuan* 1958, pp. 81-83 (Chinois).
- [10] Sur l'équation $\varphi(x+k) = \varphi(x)$, *Acta Arith.* 4 (1958), pp. 181-184.
- [11] Sur les nombres composés n qui divisent $a^n - a$, *Rend. Circ. Mat. Palermo* (2) 7 (1958), pp. 1-5.
- [12] Sur les sommes de trois carrés, *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys.* 7 (1959), pp. 307-309.
- [13] Sur une conséquence de l'hypothèse de Goldbach, *Bulgar Akad. Nauk. Izv. Mat. Inst.* 4 (1959), pp. 35-38.
- [14] Sur l'équation diophantienne $\sum_{k=1}^n A_k x_k^{\theta_k} = 0$, *Prace Mat.* 4 (1960), pp. 45-49 (Polish).
- [15] Remarks on the paper "Sur certaines hypothèses concernant les nombres premiers", *Acta Arith.* 7 (1961), pp. 1-8.
- [16] On the composite integers of the form $c(ak+b)! \pm 1$, *Nordisk Mat. Tidskr.* 10 (1962), pp. 8-10.
- Schinzel, A. and Sierpiński, W., [1] Sur quelques propriétés des fonctions $\varphi(n)$ et $\sigma(n)$, *Bull. Acad. Polon. Sci. Cl. III*, 2 (1954), pp. 463-465.
- [2] Sur les sommes de quatre cubes, *Acta Arith.* 4 (1958), pp. 20-30.
- [3] Sur certaines hypothèses concernant les nombres premiers, *Acta Arith.* 4 (1958), pp. 185-208, and *Corrigendum*, *ibidem* 5 (1960), p. 259.
- [4] Sur les congruences $x^a \equiv c \pmod{m}$ et $x^a \equiv b \pmod{p}$, *Collect. Math.* 11 (1959), pp. 153-164.
- Schinzel, A. et Wakulicz, A., [1] Sur l'équation $\varphi(x+k) = \varphi(x)$, II, *Acta Arith.* 5 (1960), pp. 425-426.
- Schmidt, W. M., [1] Über die Normalität von Zahlen zu verschiedenen Basen, *Acta Arith.* 7 (1962), pp. 299-309.
- Schnirelman, L., [1] Über additive Eigenschaften von Zahlen, *Math. Ann.* 107 (1933), pp. 649-690.
- Schoenberg, I. J., [1] Über asymptotische Verteilung reeller Zahlen mod 1, *Math. Z.* 28 (1928), pp. 171-200.
- Scholomiti, N. C., [1] An expression for the Euler φ -function, *Amer. Math. Monthly* 61 (1954), pp. 36-37.
- Scholz, A. and Schoenberg, B., [1] Einführung in die Zahlentheorie, Berlin 1955.
- Schuh, F., [1] Can $n-1$ be divisible by $\varphi(n)$ when n is composite?, *Mathematica*, Zutphen B 12 (1944), pp. 102-107 (Dutch).
- Schur, I., [1] Über die Kongruenz $x^m + y^m = z^m \pmod{p}$, *Jber. Deutsch. Math. Verein.* 25 (1916), pp. 114-117.
- [2] Einige Sätze über Primzahlen mit Anwendung auf Irreduzibilitätsfragen, *S. B. Preuss. Akad. Wiss., Phys. Math. Kl.* 23 (1929), pp. 1-24.
- Segal, S. L., [1] On $\pi(x+y) < \pi(x)+\pi(y)$, *Trans. Amer. Math. Soc.* 104 (1962), pp. 523-527.
- Segre, B., [1] A note on arithmetical properties of cubic surfaces, *J. London Math. Soc.* 18 (1943), Abt. 1, pp. 24-31.
- Selberg, A., [1] An elementary proof of the prime-number theorem, *Ann. of Math.* (2) 50 (1949), pp. 305-313.
- [2] Note on a paper by L. G. Sathe, *J. Indian Math. Soc. (N. S.)* 18 (1953), pp. 83-87.
- Selfridge, J. L., [1] Euler's function, *Amer. Math. Monthly* 70 (1963), p. 101.

- Selfridge, J. L. and Hurwitz, A., Fermat numbers and Mersenne numbers, *Math. Comp.* 18 (1964), pp. 146-148.
- Selmer, E. S., [1] A special summation in the theory of prime numbers and its application to "Brun's sum", *Norsk Mat. Tidsskr.* 24 (1942), pp. 74-81.
- [2] The Diophantine equation $ax^3+by^3+cz^3=0$, *Acta Math.* 85 (1951), pp. 203-302.
- [3] The Diophantine equation $ax^3+by^3+cz^3=0$. Completion of the tables, *Acta Math.* 92 (1954), pp. 191-197.
- [4] The rational solutions of the Diophantine equation $\eta^2 = \xi^3 - D$ for $|D| \leq 100$, *Math. Scand.* 4 (1956), pp. 281-286.
- Selmer, E. S. and Nesheim, G., [1] Tafel der Zwillingssprimzahlen bis 200000, *Norske Vid. Selsk. Forh. (Trondheim)* 15 (1942), pp. 95-98.
- Sexton, C. R., [1] Counts of twin primes less than 100000, *Math. Tables Aids Comp.* 8 (1954), pp. 47-49.
- [2] Computo del numero delle coppie di numeri primi gemelli comprese fra 100000 et 1100000, distinte secondo cifre terminali, *Boll. Un. Mat. Ital.* (3) 10 (1955), pp. 99-101.
- [3] Abzählung der Vierlingen von 1000000 bis 2000000, *Anz. Österr. Akad. Wiss. Math.-Nat. Kl.* 92 (1955), pp. 236-239.
- Shanks, D., [1] A sieve method for factoring numbers of the form n^2+1 , *Math. Tables Aids Comp.* 13 (1959), pp. 78-86.
- [2] Quadratic residues and the distribution of primes, *Math. Tables Aids Comp.* 13 (1959), pp. 272-284.
- [3] A note on Gaussian twin primes, *Math. Comp.* 14 (1960), pp. 201-203.
- Shanks, D. and Wrench, W. J., Jr., [1] Calculation of π to 100000 decimals, *Math. Comp.* 16 (1962), pp. 76-99.
- Shapiro, H. S. and Slotnick, D. L., [1] On the mathematical theory of error-correcting codes, *IBM J. Res. Develop.* 3 (1959), pp. 25-34.
- Sierpiński, W., [1] Sur un problème du calcul des fonctions asymptotiques, *Prace Mat.-Fiz.* 17 (1906), pp. 77-118 (Polish).
- [2] Sur les rapports entre les propriétés fondamentales du symbole de Legendre, *C. R. Soc. Sci. Lettr. Varsovie* 2 (1909), pp. 260-273.
- [3] Sur quelques algorithmes pour développer les nombres réels en séries, *C. R. Soc. Sci. Lettr. Varsovie* 4 (1911), pp. 56-77 (Polish).
- [4] Sur un algorithme pour développer les nombres réels en séries rapidement convergentes, *Bull. Acad. Sci. Cracovie, Cl. Sci. Math. Série A*, 1911, pp. 113-117.
- [5] Démonstration élémentaire d'un théorème de M. Borel sur les nombres absolument normaux et détermination effective d'un tel nombre, *Bull. Soc. Math. France* 45 (1917) pp. 125-132.
- [6] Remarque sur une hypothèse des Chinois concernant les nombres $(2^n - 2)/n$, *Colloq. Math.* 1 (1947), p. 9.
- [7] Działania nieskończoności, Warszawa-Wrocław 1948.
- [8] Remarques sur la décomposition des nombres en sommes des carrés de nombres impairs, *Colloq. Math.* 2 (1949), pp. 52-53.
- [9] Contribution à l'étude des restes cubiques, *Ann. Soc. Polon. Math.* 22 (1949), pp. 269-272.
- [10] Un théorème sur les nombres premiers, *Matematiche (Catania)* 5 (1950), pp. 66-67.
- [11] Sur les puissances du nombre 2, *Ann. Soc. Polon. Math.* 23 (1950), pp. 246-251.
- [12] Teoria liczb, Warszawa-Wrocław 1950.

- [13] Une proposition de la géométrie élémentaire équivalente à l'hypothèse du continu, C. R. Acad. Sci. Paris 252 (1951), pp. 1046-1047.
- [14] Sur une propriété des nombres premiers, Bull. Soc. Roy. Sci. Liège 21 (1952), pp. 537-539.
- [15] Remarques sur les racines d'une congruence, Ann. Polon. Math. 1 (1954), pp. 89-90.
- [16] Sur une propriété des nombres naturels, Ann. Mat. Pura Appl. (4) 39 (1955), pp. 69-74.
- [17] Sur une propriété de la fonction $\varphi(n)$, Publ. Math. Debrecen 4 (1956), pp. 184-185.
- [18] Sur quelques problèmes concernant les points aux coordonnées entières, Enseignement Math. (2) 4 (1958), pp. 25-31.
- [19] Sur les nombres premiers de la forme n^n+1 , Enseignement Math. (2) 4 (1958), pp. 211-212.
- [20] Sur les ensembles de points aux distances rationnelles situés sur le cercle, Elem. Math. 14 (1959), pp. 25-27.
- [21] Cardinal and Ordinal Numbers, Warszawa 1959.
- [22] Sur l'équivalence de deux hypothèses concernant les nombres premiers, Bulgar. Akad. Nauk. Izv. Mat. Inst. 4 (1959), pp. 3-6.
- [23] Sur les sommes égales des cubes distincts de nombres naturels, Bulgar. Akad. Nauk. Izv. Mat. Inst. 4 (1959), pp. 7-9.
- [24] Sur les nombres premiers ayant des chiffres initiaux et finaux donnés, Acta Arith. 5 (1959), pp. 265-266.
- [25] Teoria liczb, Część II, Warszawa 1959 (Polish).
- [26] Sur les nombres dont la somme des diviseurs est un puissance du nombre 2. The Golden Jubilee Commemoration Volume (1958-59), Calcutta Math. Soc., pp. 7-9.
- [27] Sur un problème concernant les nombres $k \cdot 2^n + 1$, Elem. Math. 15 (1960), pp. 73-74, and Corrigendum, ibidem 17 (1962), p. 85.
- [28] Sur les nombres impairs admettant une seule décomposition en une somme de deux carrés de nombres naturels premiers entre eux, Elem. Math. 16 (1961), pp. 27-30.
- [29] Sur les nombres triangulaires carrés, Bull. Soc. Roy. Sci. Liège 30 (1961), pp. 189-194, and Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. Nr 65 (1961).
- [30] Démonstration élémentaire d'un théorème sur les sommes de trois nombres premiers distincts, Glasnik Mat.-Fiz. Astronom. Drustvo Mat. Fiz. Hrvatske, Ser. II, 16 (1961), pp. 87-88.
- [31] Sur une propriété des nombres triangulaires, Elem. Math. 17 (1962), p. 28.
- [32] Sur une propriété des nombres tétraédraux, Elem. Math. 17 (1962), pp. 29-30.
- [33] Sur quelques conséquences d'une hypothèse de M. A. Schinzel, Bull. Soc. Roy. Sci. Liège 31 (1962), pp. 317-320.
- [34] Sur un problème de A. Mąkowski concernant les nombres tétraédraux, Publ. Inst. Math. Belgrade 2 (1962), pp. 115-119.
- [35] Pythagorean triangles, New York 1962.
- [36] Sur une propriété des nombres naturels, Elem. Math. 19 (1964), pp. 27-29.
- Sispanov, S., [1] On pseudo-prime numbers, Boll. Mat. 14 (1941), pp. 99-106 (Spanish).
- Skolem, T., [1] Unlösbarkeit von Gleichungen deren entsprechende Kongruenz für jeden Modul lösbar ist, Avh. Norske Vid. Akad. Oslo I, no 4 (1942).
- [2] Diophantine Gleichungen, New York 1950.
- [3] On certain distributions of integers in pairs with given differences, Math. Scand. 5 (1957), pp. 57-68.

- Skolem, T., Chowla, S., Lewis, D. J., [1] The Diophantine equation $2^{n+2} - 7 = 8^2$ and related problems, Proc. Amer. Math. Soc. 10 (1959), pp. 663-669.
- Srinivasan, A. K., [1] Practical numbers, Current Sci. 17 (1948), pp. 179-180.
- Steiger, F., [1] Über die Grundlösung der Gleichung $a^2 + b^2 + c^2 = d^2$, Elem. Math. 11 (1956), pp. 105-108.
- Steinhaus, H., [1] Zadanie 498, Matematyka 10 (1957), No 2, p. 58 (Polish).
- Stemmler, R. M., [1] The ideal Waring theorem for exponents 401-200000, Math. Comp. 18 (1964), pp. 144-146.
- Stéphanos, G., [1] Sur une propriété remarquable des nombres incommensurables, Bull. Soc. Math. France 7 (1879), pp. 81-83.
- Stern, M. A., [1] Über eine der Theilung von Zahlen ähnliche Untersuchung und deren Anwendung auf die Theorie der quadratischen Reste, J. Reine Angew. Math. 61 (1863), pp. 66-94.
- Steuerwald, R., [1] Ein Satz über natürliche Zahlen mit $\sigma(N) = 3N$, Arch. Math. 5 (1954), pp. 449-451.
- Stewart, B. M., [1] Sums of functions of digits, Canadian J. Math. 12 (1960), pp. 374-380.
- [2] Sums of distinct divisors, Amer. J. Math. 76 (1954), pp. 779-785.
- Storchi, E., [1] Alcuni criteri di divisibilità per i numeri di Mersenne e il carattere 6⁶⁰, 12⁶⁰, 24⁶⁰, 48⁶⁰ dell'intero 2, Boll. Un. Mat. Ital. (3) 10 (1955), pp. 363-375.
- Strauss, E., [1] Eine Verallgemeinerung der dekadischen Schreibweise nebst funktionentheoretischer Anwendung, Acta Math. 11 (1887), pp. 13-18.
- Subba Rao, K., [1] An interesting property of numbers, Math. Student 27 (1959), pp. 57-58.
- Swift, E., [1] Solution of the problem 213, Amer. Math. Monthly 22 (1915), pp. 70-71.
- Swift, J. D., [1] Note on discriminants of binary quadratic forms with a single class in each genus, Bull. Amer. Math. Soc. 54 (1848), pp. 560-561.
- Sylvester, J. J., [1] On arithmetical series, Messenger Math. 21 (1892), pp. 1-19, 87-120.
- Szele, T., [1] Une généralisation de la congruence de Fermat, Mat. Tidskr. B, 1948, pp. 57-59.
- Tardy, P., [1] Transformazione di un prodotto di n fattori, Ann. Sc. Mat. e Fis. 2 (1851), pp. 287-291.
- Tchacaloff, L. et Karanicoloff, C., [1] Résolution de l'équation $Ax^m + By^n = z^p$ en nombres rationnels, C. R. Acad. Sci. Paris 210 (1940), pp. 281-283.
- Teilhet, P. F., [1] Équations indéterminées, Intermédiaire Math. 12 (1905), pp. 209-210.
- Teuffel, R., [1] Beweise für zwei Sätze von H. F. Scherk über Primzahlen, Jber. Deutsch. Math. Verein. 58 (1955), Abt. 1, pp. 43-44.
- Thue, A., [1] Et par antydning til en talteoretisk methode, Vid. Selsk. Forhandlinger Christiania 1902 No 7 (Norwegian).
- [2] Über die Unlösbarkeit der Gleichung $ax^2 + bx + c = dy^n$ in grossen ganzen Zahlen x und y , Arch. Math. Naturvid. 34 (1917). No. 16.
- Tietze, H., [1] Tafel der Primzahl-Zwillinge unter 300000, S.-B. Math. Nat. Kl. Bayer Akad. Wiss. 1947, pp. 57-62.
- Trost, E., [1] Aufgabe 79, Elem. Math. 6 (1951), pp. 18-19.
- [2] Bemerkung zu einem Satz über Mengen von Punkten mit ganzzahligen Entfernung, Elem. Math. 6 (1951), pp. 59-60.
- [3] Primzahlen, Basel-Stuttgart 1953.

- Turán, P., [1] *Results of number theory in the Soviet Union*, Mat. Lapok 1 (1950), pp. 243-266 (Hungarian).
- Turski, S., [1] *Décomposition de nombres entiers en sommes de carrés de nombres impairs*, Bull. Soc. Roy. Sci. Liège 2 (1933), pp. 70-71.
- Uhler, H. S., [1] *Many figure approximations to $\sqrt{2}$ and distribution of digits in $\sqrt{2}$ and $1/\sqrt{2}$* , Proc. Nat. Acad. Sci. U. S. A. 37 (1951), pp. 63-67.
- [2] *A brief history of the investigations on Mersenne numbers and the latest immense primes*, Scripta Math. 18 (1952), pp. 122-131.
- [3] *On the 16th and 17th perfect numbers*, Scripta Math. 19 (1953), pp. 128-131.
- Ulam, S., [1] *A collection of mathematical problems*, New York and London 1960.
- Uspensky, J. V. and Heaslet, M. A., [1] *Elementary Number Theory*, New York and London 1939.
- Vahlen, Th., [1] *Beiträge zu einer additiven Zahlentheorie*, J. Reine Angew. Math. 112 (1893), pp. 1-36.
- Vijayaraghavan, T., [1] *The general rational solution of some Diophantine equations of the form $\sum_{r=1}^{k+1} A_r X_r^r = 0$* , Proc. Indian Acad. Sci., Sect. A, 12 (1940), pp. 284-289.
- Wakulicz, A., [1] *On the equation $x^3 + y^3 = 2z^3$* , Colloq. Math. 5 (1957), pp. 11-15.
- Walker, G. W., [1] *A problem in partitioning*, Amer. Math. Monthly 59 (1952), p. 253.
- Walsh, C. M., [1] *Fermat's Note XIV*, Ann. of Math. 29 (1928), pp. 412-432.
- Wang, Y., [1] *On sieve methods and some of their applications*, Sci. Sinica 8 (1959), pp. 357-381.
- [2] *On the representation of large integer as a sum of a prime and an almost prime*, Sci. Sinica 11 (1962), pp. 1033-1054.
- [3] *On sieve methods and some of their applications*, Sci. Sinica 11 (1962), pp. 1607-1624.
- Ward, M., [1] *A type of multiplicative diophantine systems*, Amer. J. Math. 55 (1933), pp. 67-76.
- [2] *Euler's problem on sums of three fourth powers*, Duke Math. J. 15 (1948), pp. 827-837.
- Watson, G. L., [1] *A proof of the seven-cube theorem*, J. London Math. Soc. 26 (1951), pp. 153-156.
- [2] *Sums of eight values of a cubic polynomial*, J. London Math. Soc. 27 (1952), pp. 217-224.
- Watson, G. N., [1] *The problem of the square pyramid*, Messenger Math. 48 (1918), pp. 1-22.
- Wertheim, G., [1] *Anfangsgründe der Zahlenlehre*, Braunschweig 1902.
- Western, A. E., [1] *Note on the magnitude of the difference between successive primes*, J. London Math. Soc. 9 (1934), pp. 276-278.
- Whitten, S., [1] *Tables of the totient and reduced totient function*, Manuscript deposited in UMT File, cf. Math. Tables Aids Comp. 4 (1950), pp. 29-31.
- Wieferich, A., [1] *Über die Darstellung der Zahlen als Summen von Biquadraten*, Math. Ann. 66 (1908), pp. 106-108.
- von Wijngarden, A., [1] *A table of partitions into two squares with an application to rational triangles*, Indagationes Math. 12 (1950), pp. 313-325.
- Willey, M., [1] *Solution of the problem E 68*, Amer. Math. Monthly 41 (1934), p. 330.

- Williams, G. T., [1] *Numbers generated by the function $e^{ax} - 1$* , Amer. Math. Monthly 52 (1945), pp. 323-327.
- Wirsing, E., [1] *Bemerkung zu der Arbeit über vollkommen Zahlen*, Math. Ann. 137 (1959), pp. 316-318.
- Wrathall, C. P., [1] *New factors of Fermat numbers*, Math. Comp. 18 (1964), pp. 324-325.
- Wunderlich, M., [1] *Certain properties of pyramidal and figurate numbers*, Math. Comp. 16 (1962), pp. 482-486.
- Yanney, B. F., [1] *Another definition of amicable numbers and some of their relation to Dickson's amicables*, Amer. Math. Monthly 30 (1923), pp. 311-315.
- Yin Wen-Lin, [1] *Note on the representation of large integers as sums of primes*, Bull. Acad. Polon. Sci., Cl. III, 4 (1956), pp. 793-795.
- [2] *Dirichlet's divisor problem*, Sci. Record N. S. 3 (1959), pp. 6-8.
- Zahlen, J. P., [1] *Sur les nombres premiers à une suite d'entiers consécutifs*, Euclides (Madrid) 8 (1948), pp. 115-121.
- Zarankiewicz, K., [1] *O liczbach trójkątowych*, Matematyka 2 (1949), Nr 4, pp. 1-7 and Nr 5, pp. 1-8 (Polish).

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Elementary theory of numbers

CORRIGENDUM

Page, line	Instead of	should be
6 ¹²	$2^{9891}-1$ of 2917 digits	$2^{11213}-1$ of 8376 digits
6 ¹⁴	21	23
112 ₆	$F_{19}=2^{2^{19}}+1$	$F_{17}=2^{2^{17}}+1$
112 _{9.7}	Recently the number $2^{161}-1$ was completely factorized into a product of two primes. One of its prime factors has 13 digits, the other has 18 digits.	
156 ⁷	1, 2, ..., a	
157 ⁹	m	n
194 ₈	a_2x+b_2	a_2y+b_2
205	$p l$	$p \mid l$
formula (26)		