Hyperspaces of polyhedra are Hilbert cubes

by

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Abstract. Let $2^X$ be the hyperspace of nonempty closed subsets of a metric continuum $X$, and let $C(X)$ be the space of nonempty subcontinua of $X$, both with the Hausdorff metric. The main results of this paper are that if $P$ is a nondegenerate connected polyhedron, then $2^P$ is homeomorphic to the Hilbert cube $Q$, $C(P) \times Q$ is homeomorphic to $Q$, and if $P$ contains no principal 1-cells, then $C(P)$ is homeomorphic to $Q$. Proofs of these theorems are based on theorems of Schori and West (Hyperspaces of graphs are Hilbert cubes, Pacific J. Math. 53 (1974), pp. 239-251).

§ 1. Introduction. Let $2^X$ be the hyperspace of nonempty closed subsets of a metric continuum $X$, and let $C(X)$ be the space of nonempty subcontinua of $X$, both with the Hausdorff metric. In [4], we announced the following results.

**Theorem 1.1.** $2^X \cong Q$, the Hilbert cube, if and only if $X$ is a nondegenerate Peano space (locally connected metric continuum).

**Theorem 1.2.** $C(X) \times Q \cong Q$ if and only if $X$ is a Peano space, and $C(X) \cong Q$ if and only if $X$ is a nondegenerate Peano space containing no free arcs.

In this paper, we introduce some techniques and apply them to prove the above theorems for polyhedra $X$. In [5], we apply these techniques to prove the above stated general theorems.

We refer the reader to [3], [4], [7], [8], [9] and [10] for background material and previous results on hyperspace problems. In particular, the proofs of the above theorems are based on the recent results of Schori and West [10] that $2^X \cong Q$ for every nondegenerate compact connected graph $G$, and $C(G \cong Q$ for every compact connected local dendron $L$ with a dense set of branch points.

Certain relative versions of these theorems are also obtained. For $A \subseteq 2^X$, let $2_A^X = \{B \subseteq 2^X : A \subseteq B\}$, and for $A \subseteq C(X)$, let $C_A(X) = \{B \subseteq C(X) : A \subseteq B\}$.

**Theorem 1.3.** $2_A^X \cong Q$ if $X$ is a Peano space and $A \neq X$, $C_A(X) \times Q \cong Q$ if $X$ is a Peano space, and $C_A(X) \cong Q$ if $X$ is a Peano space, $A \neq X$, and $X \setminus A$ contains no free arcs.

In §§ 2, 3 and 5, we develop some of the necessary tools (an inverse sequence approximation lemma, and techniques for obtaining near-homeomorphisms between hyperspaces of graphs). These are applied in §§ 4, 6 and 7 to hyperspaces of polyhedra, and will be applied in [5] to complete the proofs of the general results.

* Research supported in part by NSF Grant GP44959.
§ 2. Structure of the proof. A map \( f : X_1 \to X_2 \) between copies of a compact metric space is a near-homeomorphism if it is the uniform limit of (onto) homeomorphisms.

We shall construct inverse sequences satisfying the hypotheses of the following lemma.

**Approximation Lemma 2.1.** Let \( Y \) be a compact metric space, and let \( Q_1 \to Y \to Q_2 \to \cdots \) be an inverse sequence of maps and copies of the Hilbert cube in \( Y \) such that:

(i) \( Q_i \to Y \) (in \( 2^Y \));

(ii) \( \sum_{j=1}^\infty d(f_j, id) < \infty \);

(iii) \( \{ f_1, \ldots, f_j ; j \geq i \} \) is an equi-uniformly continuous family for each \( i \); and

(iv) each \( f_i \) is a near-homeomorphism. Then \( Y \cong Q \).

Thus, for instance, we apply the Approximation Lemma (to be proved below) to the hyperspace \( 2^Y \) of \( Y \) to construct an inverse sequence

\[
2^Y \to 2^Y \to 2^Y \to \cdots
\]

where \( \{ G_i \} \) is a sequence of compact connected graphs in \( Y \) converging to \( Y \) (thus, \( Y \cong Q \) and \( Y \cong Q \to 2^Y \)), and the maps \( \{ f_i \} \) are near-homeomorphisms satisfying conditions (ii) and (iii) of the lemma.

Each map \( f_i : 2^{G_{i-1}} \to 2^{G_i} \) is induced by a map \( \varphi_i : \Gamma \to \varphi_s(A), \) i.e.,

\[
f_i(A) = \bigcup \{ \varphi(A) : \alpha \in A \}.
\]

The particular type of map \( \varphi \), used for this purpose (a C-monotone piecewise-linear map) is discussed in § 3 and § 5, where it is shown that the induced maps \( f_i \) are near-homeomorphisms.

**Proof of Lemma 2.1.** If we denote \( \text{invlim}(Q_s, f_s) \) by \( Q_s \), then the fact that \( Y \cong Q_s \) follows from [1], Theorem I. As an aid to the reader we outline this proof.

Define \( h : Q_s \to Y \) as follows. For \( (q) \in Q_s \), the sequence \( (q) \) in \( Y \) is Cauchy by Condition (ii) and hence converges to a point \( q \in Y \). Let \( h(q) = p \). Condition (ii) also implies that \( h \) is continuous. With an easy proof by contradiction, Condition (iii) implies that \( h \) is one-to-one and Condition (i) implies that \( h \) is onto. Thus, \( h \) is a homeomorphism and hence \( Q_s \cong Y \).

Since each \( Q_s \cong Q \) and each \( f_i \) is a near-homeomorphism, it follows by Morton Brown's theorem [2] that \( Q_s \cong Q \) and hence \( Y \cong Q \).

§ 3. Piecewise-linear induced maps on hyperspaces of graphs. Let \( \Gamma \) be a compact connected graph and for every compact connected subgraph \( S \) of \( \Gamma \) let \( q_S \) be the minimum-path-length metric. For \( D = \text{diam}(S, \varphi) \), let \( e_S : C(S) \times I \to C(S) \) be the expansion homotopy defined by \( e_S(A, t) = \{ x \in S : q_S(x, A) \leq t \} \). Thus \( e_S(A, 0) = A \) and \( e_S(A, 1) = S \) for each \( A \in C(S) \). In the following \( \Gamma \) will always denote a compact connected graph.

**Definition 3.1.** A map \( \varphi : \Gamma \to C(\Gamma) \) of length \( \varphi \) is simplicial if for each vertex \( v \in \Gamma \), \( \varphi(v) \) is a connected subgraph (possibly degenerate) of \( \Gamma \), and for each edge \( e \in \Gamma \), either:

(i) \( \varphi(e) \) is a linear map onto an edge of \( \Gamma \), or

(ii) \( \varphi(e) \subseteq \varphi(e) \subset \varphi(v) \), where \( \varphi = \{ v, w \} \), and \( \varphi(e) = \varphi(e) \subseteq \varphi(v) \), for every \( e \in I \).

**Definition 3.2.** A map \( \varphi : \Gamma \to C(\Gamma) \) is piecewise-linear if there exist triangulations of \( \Gamma \) and \( \Gamma \) with respect to which \( \varphi \) is simplicial.

**Remark.** If \( \varphi : \Gamma \to C(\Gamma) \) is piecewise-linear, then there exist arbitrarily fine subdivisions of \( \Gamma_1 \) and \( \Gamma_2 \) with respect to which \( \varphi \) is simplicial.

We are now ready to introduce C-monotone maps. Let \( \Gamma_2 \to C(\Gamma_2) \) be the collection of degenerate subcontinua, and let \( \varphi_1 \) is a monotone map onto \( \Gamma_1 \).

**Definition 3.3.** A piecewise-linear map \( \varphi : \Gamma \to C(\Gamma) \) is C-monotone if:

(i) \( \varphi(\Gamma_1) \) is a monotone map onto \( \Gamma_2 \), and

(ii) for each \( x \in \Gamma_1 \) there exists a subcontinuum \( C_x \) such that for all \( x \in C_x \), \( C_x \cap \Gamma_1 \not= \emptyset \), and \( \varphi(y) \subseteq \varphi(x) \) for each \( y \in C_x \).

C-monotone piecewise-linear maps \( \Gamma_1 \to C(\Gamma) \) may be regarded as generalizations of monotone piecewise-linear maps \( \Gamma_1 \to \Gamma_2 \). The following examples may serve to clarify the above definitions.

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Fig. 1
Every map \( \varphi: \Gamma_1 \to C(\Gamma_2) \) induces hyperspace maps \( f: 2^{\Gamma_1} \to 2^{\Gamma_2} \) and \( g: C(\Gamma_1) \to C(\Gamma_2) \). Furthermore, if \( \varphi(p) = q \in \Gamma_2 \), then \( \varphi \) induces relative hyperspace maps \( f_p: 2^{\Gamma_1}_p \to 2^{\Gamma_2}_q \) and \( g_p: C(\Gamma_1)_p \to C(\Gamma_2)_q \).

**Theorem 3.5.** Let \( \varphi: \Gamma_1 \to C(\Gamma_2) \) be a C-monotone piecewise-linear map. Then all induced maps \( f, g, f_p, g_p \) stabilize near-homeomorphisms (i.e., \( f \times \text{id}_{\Gamma_2}, f_p \times \text{id}_{\Gamma_2}, g_p \times \text{id}_{\Gamma_2} \) are near-homeomorphisms).

The proof of Theorem 3.5 that we give is relatively short but uses a good deal of recently established and powerful apparatus. Our original proof of Theorem 3.5, on which our announcement in [4] was based, was more elementary but much longer and used rather involved constructions of \( Q \)-factor decompositions.

A closed subset of the Hilbert cube has trivial shape if it is contractible in each neighborhood of itself, and a surjection between Hilbert cubes is cell-like if each point inverse has trivial shape. The next theorem is a powerful theorem originally proved by T. A. Chapman with a more direct and shorter proof supplied by A. Fathi in [6].

**Theorem (Chapman).** A cell-like map between Hilbert cubes is a near-homeomorphism.

Proof of Theorem 3.5. By the previous theorem, it is sufficient to show that each point-inverse has trivial shape, but we shall show that each point inverse is contractible. For each \( K \in C(\Gamma_1) \), let \( K^* = \{ x \in \Gamma_1 : \varphi(x) = K \} \). Then by condition (i) of Definition 3.3, \( K^* \cap \Gamma_1 = \{ x \in \Gamma_1 : \varphi(x) = K \} = \{ \varphi(\Gamma_1)^{-1}(K) \} \) is connected, and by Condition (ii) each component of \( K^* \) meets \( K^* \cap \Gamma_1 \). Thus \( K^* \) is connected. Now consider \( A \in 2^{\Gamma_1} \), and let \( K(A) \) be the set of components of \( A \). It is clear that \( \{ K^* \cap \Gamma_1, K \in K(A) \} \) is the set of components of \( A \).

For each \( E \in 2^{\Gamma_1} \), if \( f(B) = A \) we have \( B \subset A^* \) and \( B \cap K^* \neq \emptyset \) for each component \( K^* \) of \( A^* \). Thus there exists an "expansion homotopy"

\[
E: f^{-1}(A) \times I \to f^{-1}(A)
\]

such that \( E_0(B) = B \) and \( E_1(B) = A^* \) for each \( B \in f^{-1}(A) \). (Specifically, we can set \( E_0(B) = \{ x \in A^* : \varphi(x) = K \cap \Gamma_1 \} \), where \( \varphi(x) = K \cap \Gamma_1 \).)

Recall that \( B \) must meet each component of \( A^* \). The same argument shows that the other induced maps are also cell-like.

§ 4. Hyperspaces of polyhedra. In this section, we state the Subdivision lemma, postponing its proof to Section 5, and use it to prove our main result for polyhedra. By a geometric cell complex \( K \) we mean a finite collection of convex cells intersecting only along common faces. For \( i \geq 0 \), the \( i \)-skeleton of \( K \), \( K_i \), is the collection of all \( i \)-dimensional faces of \( K \). This should not be confused with the earlier use of the notation \( K_i \), in the proof of Theorem 3.5.

**Subdivision Lemma 4.1.** If \( K \) is a cell complex and \( s > 0 \), then there exists a subdivision \( L \) of \( K \) and a C-monotone piecewise-linear map \( \varphi: L \to C(K) \) such that

- (i) \( \text{mesh} L < s \);
- (ii) \( \varphi(x) < s \) if \( x \in P \), a cell of \( K \); and
- (iii) \( \text{diam} \varphi(R) < s \) for each cell \( R \) of \( L \).

(Here we consider \( \varphi(R) = (C(K), d^*) \) where \( d^* \) is the induced Hausdorff metric.)

**Theorem 4.2.** If \( K \) is a nondegenerate connected compact polyhedron, then \( 2^K = \emptyset \) and \( C(K) \times Q = Q \).

Proof. As remarked earlier, we apply the Approximation lemma 2.1 by inductively constructing a sequence \( \{ \varphi_i: \Gamma_{i+1} \to C(\Gamma_i) \} \) of C-monotone-piecewise-linear maps, where \( \Gamma_1 \) is the 1-skeleton of \( K \). We use an arbitrary metric \( d \) on \( K \) and the induced Hausdorff metric \( d^* \) on \( 2^K \).

Suppose that subdivisions \( K_0, \ldots, K_i \) and the corresponding C-monotone maps \( \varphi_0, \ldots, \varphi_i \) have been constructed, with mesh \( \text{mesh} K_i < 2^{-i} \), for each \( i \). Let \( f_1, \ldots, f_i \) be the hyperspace maps induced by \( \varphi_1, \ldots, \varphi_i \), respectively. For each \( i \leq m < n \) define \( f_m = f_n \circ f_{n-1} \circ \ldots \circ f_{i+1} \). Choose \( \delta < s \) such that for \( A, B \in 2^K \) with \( d^*(A, B) < \delta \), we have \( d^*(f_n(A), f_n(B)) < 1/n \) for each \( n \). By 4.1, take a subdivision \( K_{i+1} \) of \( K_i \), with respect to \( s \) minimum \( 2^{-i+1} \), and this completes the inductive construction.

Obviously, this construction of the inverse sequence

\[
2^{\Gamma_1} \overset{f_1}{\to} 2^{\Gamma_2} \overset{f_2}{\to} \ldots
\]

satisfies Conditions (i) and (iv) of the Approximation lemma. For \( x \in \Gamma_{i+1} \), we have \( \varphi(x) < s \) where \( P \) is a face of \( K_i \), containing \( x \), and since mesh \( \text{mesh} K_i < 2^{-i} \) it follows that \( d^*(f_i(x), \text{id}) < 2^{-i} \). Thus, Condition (iii) is satisfied. To verify Condition (ii), let \( x \in K_i \), \( \ell \geq 0 \), and \( k \geq 1 \) be given. Choose \( j \) such that \( 1/j < \ell \). Choose \( [0, 1] \) such that for \( x, y \in K_i \), \( d(x, y) < \ell \), there exist interesting faces \( P_x \) and \( P_y \) of \( K_i \) containing \( x \) and \( y \), respectively. Now consider points \( x, y \in \Gamma_i \), \( \ell \geq 1/j \), and \( d(x, y) < \ell \). With \( P_x \) and \( P_y \) as above, we have \( f_{n+1}(x) = f_n(x) = P_x \) and \( f_{n+1}(y) = f_n(y) = P_y \), and it follows from the construction of \( K_{i+1} \) and \( \varphi_i \), that \( d^*(f_{n+1}(x), f_{n+1}(y)) < 1/j \). Thus for \( A, B \in 2^K \), with \( d^*(A, B) < \ell \), we have \( d^*(f_{n+1}(A), f_{n+1}(B)) < 1/j \), and therefore \( d^*(f_n(A), f_n(B)) < 1/j < \ell \). This shows that for each \( k \), the sequence of maps \( \{ f_i : \ell \geq k \} \) is equi-uniformly continuous. Thus \( 2^K = \emptyset \).

To obtain the result \( C(K) \times Q = Q \), we consider the same sequence of 1-skeletons \( \{ \Gamma_i \} \) and piecewise-linear maps \( \{ \varphi_i \} \) and form the inverse sequence

\[
C(\Gamma_i) \times Q \overset{d^*}{\leftarrow} C(\Gamma_i) \times Q \overset{d^*}{\leftarrow} \ldots
\]

where the maps \( \{ \varphi_i \} \) are those induced by \( \{ \varphi_i \} \). Each \( C(\Gamma_i) \times Q = Q \) (by Lemma 4.1, [10]), and each map \( \varphi_i \) is a near-homeomorphism. Since mesh \( K_i = 0, C(\Gamma_i) = C(K) \).
and therefore $C(P) \times Q = C(K) \times Q$. Since Conditions (ii) and (iii) of the Approximation lemma are clearly satisfied, we conclude that $C(K) \times Q \approx Q$.

§ 5. Proof of the Subdivision lemma. Let $K$ be a compact connected polyhedron. We shall view $K$ and its subdivisions as geometric cell complexes. For $t \geq 0$, the $t$-skeleton of $K$, denoted $K^t$, is the collection of all $t$-dimensional faces of $K$. For each face $P$ of $K$, let $P$ be an arbitrarily chosen point in the interior of $P$; and consider $P$ as a cone over its boundary $P$ with cone point $P$. We use the canonical coordinates given by the map $C_P: \mathbb{R}^t \to I \to P$, where $C_P(b, \ell) = (1 - \ell) b + \ell P$.

As in § 3, on every subcontinuum $S$ of the $1$-skeleton $K^1$ we use the minimum path-length metric $e_0$. For $D = \text{diam}(S, e_0)$, let $e_0: C(S) \times I \to C(S)$ be the expansion homotopy defined by $e_0(A, t) = \{x \in S : e_0(x, A) \leq tD\}$. Thus $e_0(A, 0) = A$ and $e_0(A, 1) = S$ for each $A \in C(S)$.

**Lemma 5.1.** For every cell complex $K$ there exists an unique map $\alpha: K \Rightarrow C(K^t)$ such that

(i) $\alpha(x) = \{x\}$ for each $x \in K^1$;
(ii) $\alpha(C_P(b, t)) = e_0(\alpha(b), t)$ for each cell $P$ with $\text{dim} P > 1$;
(iii) $\alpha(P) = C(P)$ for each cell $P$.

**Proof.** The conditions define the map for $K$ and the extension of the map to the rest of $K$ is by the obvious induction on the $K^t$.

For each $n \geq 1$ we construct the $n$-th radial-transverse subdivision $K(n)$ of $K$ by inductively describing the $n$-th subdivision $K(n)^t$ of the $t$-skeletons of $K$, $t \geq 0$. With $K(n)^t = K^t$, let $K(n)^{t+1}$ be the cell subdivision of $K(n)^t$ given by the convex cells $\{C_p \times [m/n, (m+1)/n]\}$, $P \in K(n)^t$, $\sigma \in C(n)^t$, $0 \leq m < n$. Thus $K(1)$ is simply a barycentric subdivision of $K$ and in constructing $K(n)^t$, each element of $K^t$ is subdivided into $2^n$ subintervals.

Clearly the mesh $K(n)^t$ tends to $0$ as $n \to \infty$ where we can use an arbitrary metric on $K$. The $1$-skeleton $K^1(n)$ of the radial-transverse subdivision $K(n)$ is the union of two subcomplexes, $R(n)$ (the radial segments) and $T(n)$ (the transverse segments), where $R(n) = \{C_P \times [m/n, (m+1)/n]\}$, $P$ is a cell of $K$, $\sigma$ is a vertex of $K(n)$ in $P$, $0 \leq m < n$, and $T(n) = \{C_P \times [m/n, (m+1)/n]\}$, $P$ is a cell of $K(n)$ with $\text{dim} P > 1$, $\sigma \in \Gamma(n)$, $0 \leq m < n$. Thus $R(n)$ covers all the vertices of $K(n)$, and also the $1$-skeleton $K^1$.

We now restate and prove the Subdivision lemma 4.1.

**Subdivision Lemma.** If $K$ is a cell complex and $t > 0$, then there exists a subdivision $L$ of $K$ and a $C$-monotone piecewise-linear map $\varphi: L^t \to C(K^t)$ such that

(i) $\varphi(x) = \{x\}$ for each $x \in K^1$;
(ii) $\varphi(C_P(b, t)) = e_0(\varphi(b), t)$ for each cell $P$ of $K$;
(iii) $\varphi(P) = C(P)$ for each cell $P$.

**Proof.** Let $\varepsilon > 0$ and let $\varphi$ be the minimum path-length metric of $K^t$ and $\varphi^0$ the induced Hausdorff metric on $C(K^t)$. By the uniform continuity of the map a frontier.

Lemma 5.1, and the fact that mesh $K(n)^t$ tends to $0$ as $n \to \infty$, pick $n$ sufficiently large such that if $a, b$ belong to the same cell of $K(n)^t$, then $\varphi^0(a, b) < \varepsilon$. Let $L = K(n)^t$ and define $\varphi^t: L^t \to C(K^t)$ as follows. We have $L^t = \Gamma(n) = R(n) \cup T(n)$, $\varphi(T(n)) = \sigma \in \Gamma(n)$, and $\varphi(R(n)) = \sigma \in \Gamma(n)$ and $\varphi^0(\varphi(T(n), \varphi(R(n))) < \varepsilon$ for each cell $P$ of $K(n)$.

In the notation of the $C$-monotone Definition 3.3, we have $(L^t)^0 = \{x \in L^t : \varphi(x)$ is degenerate $\}$ is precisely the subset $K^t$ of $L^t$, thus $\varphi(\varepsilon)$. $(L^t)^0 \Rightarrow L^t$ is actually a homeomorphism. For a point $x = C_P(v, [0, 1])$ in an edge $C_P(v, [m/n, (m+1)/n])$ of $T(n)$, a subcontinuum $C_n$ satisfying Condition (ii) of the $C$-monotone definition is given by $C_n = C_P(v, [0, 1])$, and for a point $x = (1-t) + t \ell$ in an edge $\tau = C_P(\sigma, [m/n, (m+1)/n])$, we may take $C_n = \{x = (1-t) + t \ell : 0 \leq \ell \leq t\} \cup \{\sigma \in C(n)^t, 0 \leq m < n\}$, where $v \in P^t$ is the vertex of $K(n)$ such that $C_P(v, [m/n, (m+1)/n]) = c \in \ell$. It is easily seen that $\varphi$ satisfies Conditions (i) and (ii) of the Subdivision lemma.

Regarding Condition (iii), we define $\varphi(x) = \{x\} \in C(n)^t$ for each subcontinuum $x$ of $K$. For a cell $C(\sigma) \subseteq C(K(n)^t)$, there exists $\tau = (c, d) \in \ell$ where $c$ is a vertex of an edge of $R(n)$ containing $x$ and $d$ is a vertex of an edge containing $y$ and $z$ so that using the triangle inequality $\varphi^0(\varphi(x), \varphi(y)) < \varepsilon$.

§ 6. $C(K)$ for polyhedra with no principal 1-cells.

**Lemma 6.1.** Let $S = \text{invlim}(X_n, f_n)$ and $T = \text{invlim}(Y_n, g_n)$, where all the spaces are compact metric and for each $n$ let $h_n: X_n \to Y_n$ be a map such that $h_n \circ f_n = g_n = h_n \circ f_n$. If for each $n$, both $f_n$ and $h_n$ are near-homeomorphisms, then the induced map $h = \text{lim} h_n: S \to T$ is a near-homeomorphism.

**Theorem 6.2.** If $K$ is a nondegenerate compact connected polyhedron with no principal 1-cells, then $C(K) = Q$.

**Proof.** We proceed essentially as before in constructing the sequence $[K_i]$ of radial-transverse subdivision, but add at the $i$th stage of the construction the arbitrary finite collection of stickers to $F_i$, and to each of its predecessors $F_{i-1}, \ldots, F_1$. These stickers are obtained from $F_{i+1}$ and do not change the homology of the graphs $F_i, \ldots, F_1$. In this manner, we eventually add countably many stickers to each $F_i$, and obtain (upon forming the closures) a sequence $[F_i^e]$ of compact connected local dendrites whose sets of branch points are dense. Thus each $C(F_i^e) = Q$ (by Theorem 5.7, [10]). We construct an inverse sequence

$$\cdots \to C(F_i^e) \to C(F_{i+1}^e) \to \cdots$$

to which the Approximation lemma applies, and thereby obtain the desired results.

Let $[K_i]$ be the sequence of radial-transverse subdivisions constructed in the proof of Theorem 4.2. We may assume that for each $i$, $K_{i+1} = K(n)$ for some
for the result $C(K) \approx Q$, since otherwise $C(K)$ would at some point locally look like $C(I) \approx \mathbb{I}^2$.

§ 7. The relative hyperspaces $2^X$ and $C_0(K)$.

Theorem 7.1. Let $K$ be a nondegenerate compact connected polyhedron with $p \in K$. Then $2^p \approx Q$, $C_0(K) \times Q \approx Q$, and $C_0(K) \approx Q$ if $K$ has no principal 1-cell.

Proof. We may assume $p$ is a vertex of $K$. The arguments are exactly the same as for Theorems 4.2 and 6.2, with all induced hyperspace maps $f$ and $g$ replaced by their restrictions $f_p$ and $g_p$. We also use the results from [10] that $2^p \approx Q$, and $C_0(\mathbb{I}^2) \times Q \approx Q$, for every nondegenerate compact connected graph $\Gamma'$, and that $C_0(\mathbb{I}^2) \approx Q$, for every compact connected local dendron $\Gamma^*$ with a dense set of branch points.

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Accepted par la Redaction le 31. 1. 1976