Separation properties in Moore spaces *

by

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Abstract. We prove consistency results about the relation of separation axioms in Moore spaces. Specifically, we prove, under various set theoretic assumptions: 1) separable (or more generally, countable chain condition), countably paracompact Moore spaces are metrizable, 2) there is a normal, collectionwise Hausdorff, non-metrizable Moore space, 3) there is a collectionwise Hausdorff, nonnormal Moore space, and 4) there is a ω_1-collectionwise Hausdorff, not collectionwise Hausdorff Moore space.

The problem of distinguishing normality from metrizability is the old and well known normal Moore space conjecture. In this introduction we explain why we are interested in distinguishing the other separation properties discussed in this paper.

Let us begin with Burton Jones' 1965 account [7] of the origin of the normal Moore space conjecture. "As far as I know the first example of a non-metric Moore space was discovered (probably by Moore himself) in the late 1920's. (A description of two spaces similar to what is called the Cantor tree in [15]). The usual way to see that these spaces were not metric was to observe that each contained a closed separable subspace which ... had no countable topological base. This kind of observation left me mildly restless and it was only after I discovered that neither was normal that I felt that I was closer to the "real reason" for non-metrizability... Among several examples of non-separable non-metric Moore spaces there is one which I had high hopes of proving normal. (A description of the Jones' road space.)"

Using the consistency of Martin's Axiom and the negation of the Continuum Hypothesis, we know that we cannot prove that the ω_1-Cantor tree (a nonmetrizable subspace of the Cantor tree) or the Jones' road space is not normal. And Jones' old result (Lemma 2.1 below) shows that for separable Moore spaces metrizability is the same as ω_1-collectionwise Hausdorff. In the author's opinion the "real reason" why the examples above are not metrizable is that they are not ω_1-collectionwise Hausdorff.

So the Moore space metrization problem can be asked in three ways.

1. What is the "real reason" that nonmetrizable Moore spaces are nonmetrizable?

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2. Are normal Moore spaces metrizable?

3. Are ω_1-collectionwise Hausdorff Moore spaces metrizable?

Of course, Bing [2] has answered question 1. And time has vindicated Jones' choice of question 2.

Countable paracompactness is, technically, a covering property rather than a separation property. But in Moore spaces, it is implied by normality and it implies that two closed sets, one of which is countable, can be separated. And there still is no absolute example distinguishing it from normality (or even metrizability) in Moore spaces. This topic is discussed more fully in [21].

Diagram 1 illustrates the need for work in distinguishing separation properties in Moore spaces. Diagram 2 illustrates the results of this paper and some recent results of Wage and Reed. In the diagrams, double arrows are absolute results, single

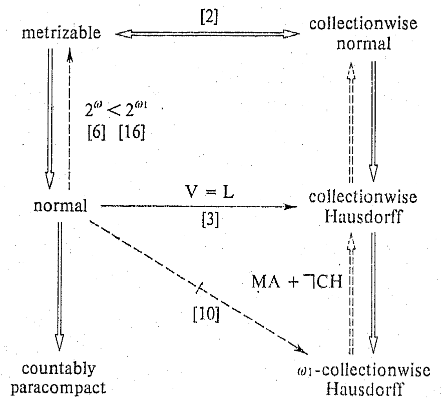

Diagram 1. Separation properties in Moore spaces (up to 1974)

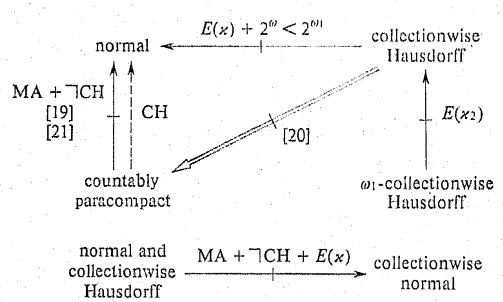

Diagram 2. Results of this and Wage's papers

arrows are consistency results, and dotted arrows are results for the special case of separable (or countable chain condition) spaces.

Although the author was motivated by Moore spaces, these results and techniques have applications in other areas; e.g. the density topology [18], separation properties in products [11], and decompositions of metric spaces [13].

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1. Notation. For topological notions we follow [15], and for set theoretic notions, [9]. In addition, we need some notions concerning closed discrete collections of points, and another axiom of set theory.

A space is *$\varkappa$-collectionwise Hausdorff* (*$<\varkappa$-collectionwise Hausdorff*) if every closed discrete collection of $\leqslant\varkappa$-many points ($<\varkappa$-many points) can be simultaneously separated by disjoint open sets.

A space is *$\varkappa$-compact* if every set of cardinality $\varkappa$ has a limit point. An associated cardinal function is defined by $\Delta(X)\leqslant\lambda$ iff there is in X a closed discrete collection of λ-many points.

We denote the collection of regular open sets of X by $\mathrm{RO}(X)$; and the cardinality of $\mathrm{RO}(X)$ by $|\mathrm{RO}(X)|$.

The class of ordinals of cofinality ω is denoted $\mathrm{CF}\omega$.

$E(\varkappa)$ is the assertion that $\varkappa$ is a regular cardinal $>\omega_1$, and that there is a set $E\subset CF\omega\cap\varkappa$ such that a) E is stationary in $\varkappa$ and b) $E\cap\alpha$ is not stationary in α for any $\alpha<\varkappa$.

2. Separable, countably paracompact Moore spaces. For many years, the only result on the normal Moore space conjecture was the following, due to Jones [6].

LEMMA 2.1. *An ω_1-compact Moore space is metrizable.*

LEMMA 2.2. *A normal space X is $2^{d(X)}$-compact. Or, if $\Delta(X)$ is attained, $2^{\Delta(X)}<2^{d(X)}$.*

THEOREM 2.3 ($2^{\omega}<2^{\omega_1}$). *Separable normal Moore spaces are metrizable.*

In this section, we prove an analogue of Lemma 2.2.

LEMMA 2.4. *A countably paracompact space X is $|\mathrm{RO}(X)|^{\omega}$-compact. Or, if $\Delta(X)$ is attained, $\Delta(X)<|\mathrm{RO}(X)|^{\omega}$.*

Proof. We prove the contrapositive. Let $\varkappa=|\mathrm{RO}(X)|^{\omega}$. Let $\{\mathcal{U}_\alpha:\ \alpha<\varkappa\}$, where $\mathcal{U}_\alpha=\{U_{\alpha i}:\ i\in\omega\}$, index the locally finite sequences of regular open sets. Let $Y=\{y_\alpha:\ \alpha<\varkappa\}$ be a closed discrete collection of points. We partition Y into ω pieces Y_i, $i\in\omega$, by placing y_α in Y_m, where m is least such that $y_\alpha\notin U_{\alpha m}$. Let $V_i=X-(Y-Y_i)$. It is easy to verify that the open cover $\{V_i:\ i\in\omega\}$ has no locally finite open refinement.

THEOREM 2.5 (CH). *A separable, countably paracompact Moore space is metrizable.*

Proof. First, note that $|\mathrm{RO}(X)|\leqslant2^{d(X)}=2^{\omega}$; so $|\mathrm{RO}(X)|^{\omega}\leqslant(2^{\omega})^{\omega}=2^{\omega}$, which is ω_1 by CH. By Lemma 2.4, X is ω_1-compact; then by Lemma 2.1, X is metrizable.

Remark 2.6. Assuming MA plus not CH, Mike Wage [19] has constructed a countably paracompact, not normal Moore space. With the same assumption, Mike Reed [21] has constructed a separable such example.

Remark 2.7. The first draft of this paper had $2^{d(X)}$ in place of $|\mathrm{RO}(X)|$. A letter from Frank Tall informed the author of the following facts. Šapirovskiĭ had replaced the $2^{d(X)}$ with $|\mathrm{RO}(X)|$ in Lemma 2.2; his technique can be applied to countable paracompactness; and $|\mathrm{RO}(X)|^{\omega} = |\mathrm{RO}(X)|$.

3. Some Moore spaces suggested by Starbird. Perhaps the favorite example of a nonmetrizable Moore space is the Niemytskii plane. The points of the space are the points of the plane. Points not on the x-axis are isolated. A neighborhood of a point on the x-axis is an open disc in the upper half-plane tangent to the x-axis at the point, plus the point itself. In this section we use T, a subspace consisting of $Y = \{y_\alpha: \alpha<\omega_1\}$, ω_1 points on the x-axis, plus D, all the isolated points. It is well known ([15]), for example) that T is not normal if $2^{\omega}<2^{\omega_1}$ and T is normal if MA plus not CH.

Let M be a metric space. In the product $T\times M$, we make points (d, m) isolated; points (y_α, m) have the usual product neighborhoods. Mike Starbird suggested the class of subspaces of such spaces as a good place to search for absolute examples of normal nonmetrizable Moore spaces.

In this section, the metric space will be $B_\varkappa$, the Baire space of weight $\varkappa$; equivalently, the usual product of countably many copies of the discrete space of cardinality $\varkappa$. We consider points of $B_\varkappa$ to be functions f from ω to $\varkappa$. Let f^* be the least ordinal strictly greater than every element of the range of f. Assume that $\varkappa$ is regular and greater than ω_1. Let $A\subset \mathrm{CF}\omega \cap \varkappa$ be stationary. By the Ulam-Solovay theorem, A can be split into ω_1 disjoint stationary sets, A_α, $\alpha<\omega_1$. We will consider spaces of the form

$$X = \{(d,f)\colon d\in D,\ f\in B_\varkappa\} \cup \{(y_\alpha,f)\colon y_\alpha\in Y,\ f\in B_\varkappa,\ f^*\in A_\alpha\}.$$

Theorem 3.1. a) *X is normal if T is normal.*

b) *X is not collectionwise normal.*

c) *X is normal only if T is normal.*

d) *If A witnesses $E(\varkappa)$, then X is collectionwise Hausdorff.*

Corollary 3.2. a) *($E(\varkappa)$ for some $\varkappa$). There is a collectionwise Hausdorff, not collectionwise normal Moore space.*

b) *(MA plus not CH plus $E(\varkappa)$ for some $\varkappa$). There is a normal, collectionwise Hausdorff, not collectionwise normal Moore space.*

c) *($2^{\omega}<2^{\omega_1}$ plus $E(\varkappa)$ for some $\varkappa$). There is a collectionwise Hausdorff, not normal Moore space.*

Proof of 3.1 a). The proof will show that this, Starbird's Lemma, holds in more general situations. We start with an easy lemma.

Lemma 3.3. *Let $\{U_i\colon i\in\omega\}$, $\{V_i\colon i\in\omega\}$ be open covers of H, K respectively, satisfying $\overline{U}_i\cap K = \emptyset = \overline{V}_i\cap H$ for all i. Then there are disjoint open U, V covering H, K respectively.*

Proof. Define $U = \bigcup\{U_i-(\overline{V}_0\cup\overline{V}_1 \dots \cup \overline{V}_i)\colon i\in Y\}$. Define V similarly.

Let H, K be disjoint closed subsets of X. It is sufficient to consider the case where $H\cup K\subset Y\times B_\varkappa$.

Let $\mathscr{B} = \bigcup\{\mathscr{B}_i\colon i\in\omega\}$ be a σ-discrete base for $B_\varkappa$.

For $B\in\mathscr{B}$, define $H(B) = \{y_\alpha\colon \{y_\alpha\}\times B\cap H\neq\emptyset,\ \{y_\alpha\}\times B\cap K=\emptyset\}$, and $K(B)$ similarly. Using the assumption that T is normal, define disjoint open $U(B)$, $V(B)$ such that $V(B)\supset H(B)$, $V(B)\supset K(B)$. Set $U_i = \bigcup\{U(B)\times B\colon B\in\mathscr{B}_i\}$, V_i similarly, and apply Lemma 3.3.

Proof of 3.1 b), c). We start with a combinatorial lemma.

Lemma 3.4. *Let $S\subset B_\varkappa$. If $\{f^*\colon f\in S\}$ is stationary, there is a cub C such that $\mathrm{CF}\omega\cap C\subset\{f^*\colon f\in \overline{S}\}$.*

Proof. Let Σ be the set of functions σ from a natural number to $\varkappa$ such that $\{f^*\colon \sigma\subset f\in S\}$ is stationary. By assumption $\Sigma\neq\emptyset$. Using the Pressing Down Lemma, we can show that there is a function $\theta\colon \Sigma\times\varkappa\to\Sigma$ such that (i) $\theta(\sigma,\alpha)\supset\sigma$, (ii) range $\theta(\sigma,\alpha)\not\subset\alpha$. Let C be the set of γ such that $\alpha<\gamma$, range $\sigma\subset\gamma$ implies range $\theta(\sigma,\alpha)\subset\gamma$.

Corollary 3.5. *Let $R\subset\mathrm{CF}\omega\cap\varkappa$ be stationary. Let $R' = \{f\colon f^*\in R\}$. Let U be an open set of $B_\varkappa$ containing R'; let $F = B_\varkappa - U$. Then, $\{f^*\colon f\in F\}$ is not stationary. Equivalently, there is a cub C such that $f^*\in C$ implies $f\in U$.*

Note that if for some f, $T\times\{f\}\subset X$, the statement 3.1 b), c) are obvious. Loosely speaking, our plan is to use Corollary 3.5 to choose an f such that, in effect, $T\times\{f\}\subset X$.

We now prove 3.1 c). (3.1 b) is similar). Let H, K be any disjoint subsets of Y. Let $\mathscr{U} = \{U_\alpha\colon \alpha\in\omega_1\}$ be a family of open sets such that

(i) U_α is the union of basic open rectangles, one for each $(y_\alpha, f)\in X$.

(ii) If $y_\alpha\in H$, $y_\beta\in K$ then $U_\alpha\cap U_\beta = \emptyset$.

Apply Corollary 3.5 to each $U\in\mathscr{U}$ to get cubs $C(U)$. Let $C = \bigcap\{C(U)\colon U\in\mathscr{U}\}$. C is cub because $\varkappa>\mathrm{card}\,\mathscr{U}$. Choose $f\in B_\varkappa$, $f^*\in C$. Then we can use $\mathscr{U}$ to show that $H\times\{f\}$ and $K\times\{f\}$ can be separated in $T\times\{f\}$, and hence H, K can be separated in T.

Proof of 3.1. d). Let $Z\subset X$ be a closed discrete collection of points. Define:

$$Z' = \{f\colon (y_\alpha,f)\in Z,\ y_\alpha\in Y\},$$

$$Z|\beta = \{f\in Z'\colon f^*<\beta\},$$

$$Z^* = \{f^*\colon f\in Z'\}.$$

It will be sufficient to show that Z' is σ-discrete. For then $X' = D\times B_\varkappa\cup Z$ has a σ-discrete base. Because X' is metrizable, Z can be separated in X', and hence in X.

We will show by induction on β, $\beta\leqslant\varkappa$, that $Z|\beta$ is σ-discrete. The induction is easy for $\beta = 0$, $\beta = \delta+1$, and cf $\beta = \omega$. For cf$\beta>\omega$ we need to know that there is C cub in β disjoint from Z^*. If $\beta<\varkappa$, this follows from $E(\varkappa)$; for $\beta = \varkappa$, we prove the following lemma.

LEMMA 3.6. *Z^* is not stationary in $\varkappa$.*

Proof. For each α, $\{y_\alpha\}\times B_\varkappa$ is metrizable, *a fortiori* collectionwise Hausdorff. It then follows from the Pressing Down Lemma that for each α $\{f^*\colon (y_\alpha, f)\in Z\}$ is not stationary. Because $\varkappa>\omega_1$, Z^*, the union of ω_1 not stationary sets, is not stationary.

Now, because C is cub in β and disjoint from Z^*, we can define for all $f\in Z|\beta$ ordinals $\gamma(f)$, $\gamma^+(f)$ such that

(i) $\gamma(f)<f^*<\gamma^+(f)$,

(ii) there is no $\gamma'\in C$ strictly between $\gamma(f)$ and $\gamma^+(f)$.

By induction hypothesis, $Z|\gamma$ is σ-discrete. So we can write $\{f\in Z|\gamma\colon \gamma = \gamma^+(f)\} = \bigcup\{W_{\gamma ik}\colon i, k\in\omega\}$ such that $f, g\in W_{\gamma ik}$ implies

(iii) $f|i\neq g|i$,

(iv) there is $m<i$ such that $f(m)\geqslant\gamma(f)$.

Then, $\bigcup\{W_{\gamma ik}\colon \gamma\in C\}$ is discrete, demonstrating that $Z|\beta$ is σ-discrete.

Remark 3.7. Any not ω_1-collectionwise Hausdorff Moore space could have been used for T.

Remark 3.8. The proof of 3.1 presented above does not seem to show that X is countably paracompact iff T is countably paracompact.

Remark 3.9. At about the same time, Alster and Pol [1] independently constructed a consistent example of a collectionwise Hausdorff not normal Moore space under weaker set theoretic assumptions. The history of this problem is discussed in the introduction of [20], in which Wage constructs an absolute example of a collectionwise Hausdorff not countably paracompact Moore space.

Remark 3.10. In the first version of this paper, the examples were constructed separately, using an additional set theoretic assumption beyond the assumptions of Corollary 3.2 in the definition of the spaces. The author is very grateful for a letter from Roman Pol containing the definition of X used above and the following result (which is Corollary 6 of [13]).

LEMMA 3.11 ($E(\varkappa)$). *There is a decomposition of $B_\varkappa$ into ω_1 pieces R_α, $\alpha<\omega_1$, such that*

(i) *if $U_\alpha\supset R_\alpha$ is open, then $\bigcap\{U_\alpha\colon \alpha<\omega_1\}\neq\emptyset$,*

(ii) *every selector (i.e. choice function, or transversal) is σ-discrete.*

Proof. As in the proof of 3.1 b), d). Pol's proof of i) uses Arthur Stone's notion of σ-LW($<t$) rather than Corollary 3.5.

Remark 3.12. An absolute example of a normal, collectionwise Hausdorff, not collectionwise normal space is given in [4] (and also in [15] and [12]).

4. $<\varkappa$-collectionwise Hausdorff, not $\varkappa$-collectionwise Hausdorff Moore spaces.

THEOREM 4.1 ($E(\varkappa)$). *There is a $<\varkappa$-collectionwise Hausdorff, not $\varkappa$-collectionwise Hausdorff Moore space.*

Proof. We define a "wide" Cantor tree [15]. Let E witness $E(\varkappa)$. For $\lambda\in E$, choose $f_\lambda\colon \omega\to\lambda$ with the range of f cofinal in λ. Let $D = \{f_\lambda|n\colon \lambda\in E,\ n\in\omega\}$. The points of the space S are $D\cup\{f_\lambda\colon \lambda\in E\}$. Points of D are isolated; the nth basic open neighborhood of f_λ is $\{f_\lambda\}\cup\{f_\lambda|m\colon m\geqslant n\}$. It is straightforward to use the Pressing Down Lemma to show that S is not $\varkappa$-collectionwise Hausdorff. That S is $<\varkappa$-collectionwise Hausdorff is proved as in 3.1d).

Remark 4.2. Let E' be the set E with the topology inherited from $\varkappa$ with the order topology. With a proof paralell to that of Theorem 4.1 above, Istvan Juhász [8] independently showed that E' is a first countable nonmetrizable space all of whose subspaces of cardinality $<\varkappa$ are metrizable. S also has this property. It is interesting to note, in light of Bing's theorem, that S is Moore and E' is collectionwise normal.

Remark 4.3. Another way to construct a $<\varkappa$-collectionwise Hausdorff, not $\varkappa$-collectionwise Hausdorff Moore space is to apply Mike Reed's technique [14] to E'.

Remark 4.4. Teodor Przymusiński [12] constructs an absolute example of a $<\varkappa$-collectionwise normal, not $\varkappa$-collectionwise Hausdorff space.

5. Consistency. All the combinations of axioms used in this paper are consistent. Jensen [5], assuming V = L, shows that $E(\varkappa)$ holds for all regular, not weakly compact cardinals $\varkappa$ greater than ω_1. It is easy to show that $E(\varkappa)$ is preserved under ccc extensions; in particular, the Solovay–Tennenbaum extension forcing MA plus not CH. Finally, Baumgartner [22] has shown that in the Lévy model, collapsing a weakly compact cardinal to ω_2, $E(\omega_2)$ is false.

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