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Addition and correction to the paper "On stability and products"

Fund. Math. 93 (1976), pp. 81–95

by

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In the paper quoted in the title the second part of Corollary 5.5 was formulated wrongly. Namely it should have the following form:

The class of all ω-stable theories for which α_T is finite is closed under finite droducts.

Now we shall show that "ω-stable" cannot be omitted. The notation and terminology are taken from [1].

Namely, let $\mathfrak{B} = \langle Q \cup (Q \times Q \times Q), W, C, D, R, \sim_n \rangle_{n \in \omega}$, where

Q is the set of rationals numbers,

W is a unary relation and $W(a)$ iff $a \in Q$,

C is a unary relation and $C(a)$ iff $a \notin Q$,

D is a ternary relation and $D(a, b, c)$ iff $W(a)$, $W(b)$ and $\exists q \in Q$, $c = \langle a, b, q \rangle$,

R is a ternary relation and $R(a, b, c)$ iff $D(a, b, c)$ and

$$\begin{cases} a < b \to \exists q \in Q\ c = \langle a, b, q \rangle \text{ and } q \text{ is a natural number,} \\ a \geqslant b \to \exists q \in Q\ c = \langle a, b, q \rangle \text{ and } q \neq 0, \end{cases}$$

$\sim_n$ are equivalence relations on Q with infinitely many classes and $\sim_{n+1}$ divides every equivalence class of $\sim_n$ into infinitely many equivalence classes of $\sim_{n+1}$. Moreover every equivalence class of $\sim_n$ is a dense linear ordering without endpoints (with the ordering taken from Q).

Note that we can define a formula which linearly orders W into type η.

Fact 1. For every $\mathfrak{A} \models \mathrm{Th}(\mathfrak{B})$ and every $p \in SA$ either rank $(p) = 0$, either rank $(p) = 1$, or rank $(p) = \infty$.

Indeed, every type contains one of the following sets of formulas:

1) $\{x_0 = a\}$ for some $a \in A$,

2) $\{D(a,b,x_0),\ \neg R(a,b,x_0),\ x_0 \neq c|\ c \in A,\ \mathfrak{A} \models D(a,b,c) \wedge \neg R(a,b,c)\}$ for some $a, b \in W^{\mathfrak{A}}$,

3) $\{D(a,b,x_0),\ R(a,b,x_0),\ x_0 \neq c|\ c \in A,\ \mathfrak{A} \models D(a,b,c) \wedge R(a,b,c)\}$ for some $a, b \in W^{\mathfrak{A}}$,

4) $\{W(x_0),\ x_0 \neq a|\ a \in A\}$,

5) $\{C(x_0),\ \neg D(a,b,x_0)|\ a,b \in A,\ \mathfrak{A} \models W(a) \wedge W(b)\}$.

By an argument involving automorphisms we can see that in case 1 we have types of rank 0, in cases 2 and 3 types of rank 1 and in cases 4 and 5 types of rank ∞.

So we get

Fact 2. $\alpha_{\mathrm{Th}(\mathfrak{B})} = 2$.

Fact 3. $\alpha_{\mathrm{Th}(\mathfrak{B}\times\mathfrak{B})} > \omega$.

Note that we have no formula which linearly orders an infinite set in $\mathfrak{B}\times\mathfrak{B}$. Now let p be the following type, $p \in S(B\times B)$:

$$\{\neg(x_0 \sim_n \langle a, b\rangle)|\ n \in \omega,\ a, b \in Q\} \cup \{W(x_0)\}.$$

It is easily seen that rank $(p) \geqslant \omega$. On the other hand similar reasoning as in Example 1.5 of [1] shows us that $\mathrm{Th}(\mathfrak{B}\times\mathfrak{B})$ is ω-stable, so rank $(p) < \infty$. From the above facts we get $\alpha_{\mathrm{Th}(\mathfrak{B}\times\mathfrak{B})} > \omega$.

Reference

[1] J. Wierzejewski, *On stability and products*, Fund. Math. 93 (1976), pp. 81–95.

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